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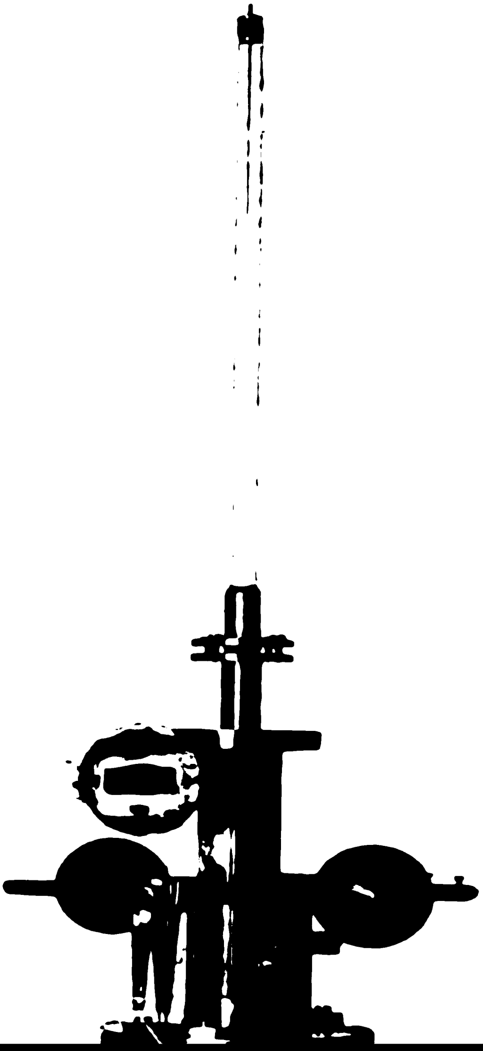

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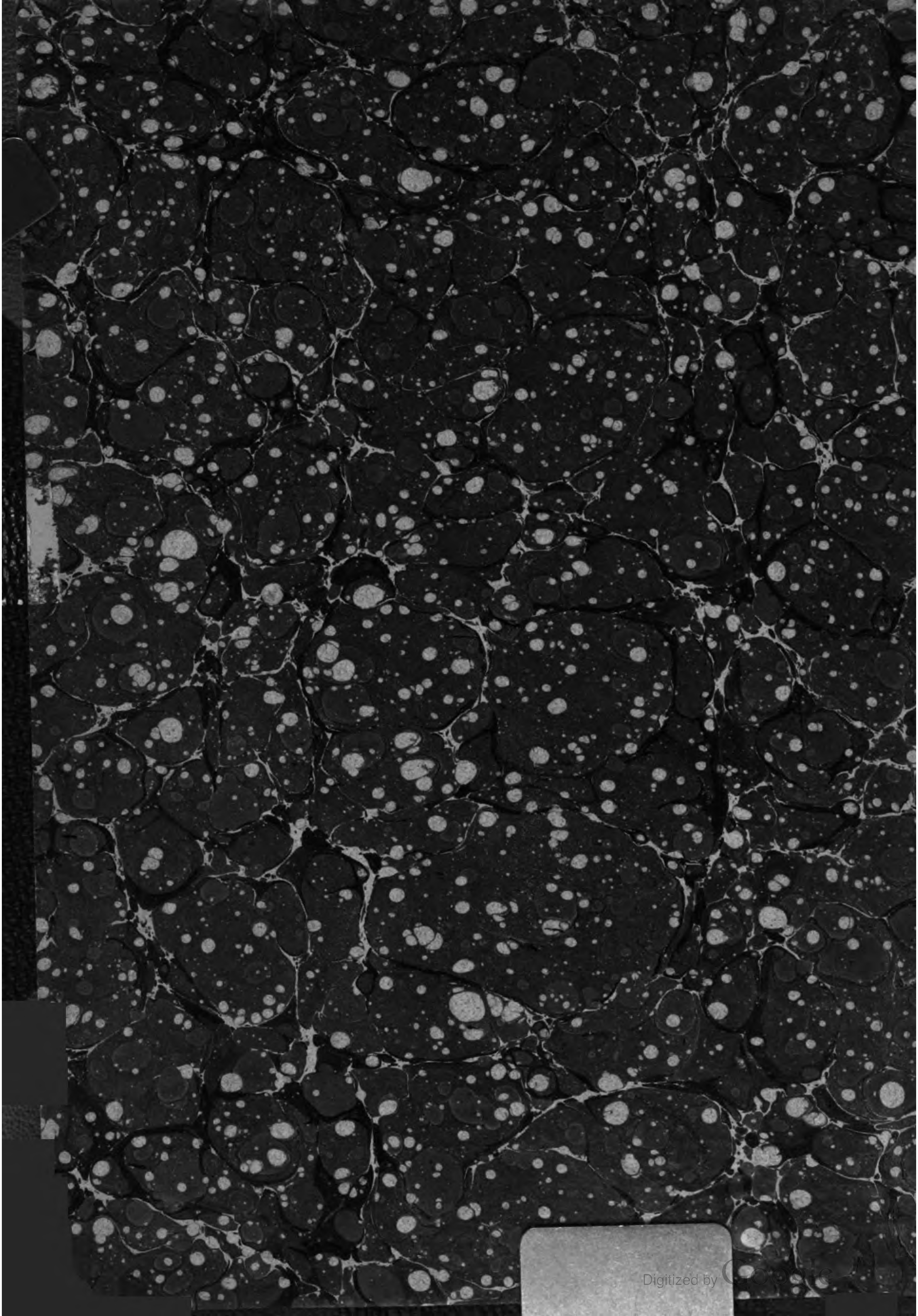

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THE

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CONDUCTED BY

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AUGUST KUNDT

Volume II. *July–August, 1894.* *Number 1.*

THE PHYSICAL REVIEW.

A BOLOMETRIC STUDY OF LIGHT STANDARDS.

By Clayton H. Sharp and W. R. Turnbull.

I.

Introduction.

THE two great essentials for a light standard are constancy during use and exact reproducibility.

It is evident even to a most casual observer that the standard candles which are in use do not possess the first of these essentials. The English standard candle has been charged with fluctuations of 40 per cent, and it is only by snuffing and measuring carefully the height of flame that anything approaching uniform results has been obtained.

The Carcel lamp has stood the test of time but little better than the best of standard candles. It has disadvantages entailed by the use of a light-absorbing chimney and a rapidly charring wick; while of its reproducibility, within a reasonable margin of error, there is serious question. Although the Carcel lamp furnishes a very steady source of light, it is known to show slow changes in the amount of light emitted, rising slowly to a maximum as the lamp and oil become heated, and then falling off with the charring of the wick.

Among the more modern standards that of Methven is faulty in that it requires a chimney, employs as an illuminant an indeterminate mixture of gases, and is affected also by variations in the pressure of the gas and by draughts of air in the room. The Vernon-Harcourt pentane burner and the Hefner-Alteneck amyl-acetate lamp, while avoiding this difficulty, run into others perhaps equally serious. The open flame of the latter is not only of a decidedly reddish hue, but is also extremely sensitive to draughts and vibrations of the air. On this account it is only with the greatest care that the height of the flame can be adjusted according to the prescription.

Many attempts have been made to determine the fluctuations of the various standard lights by comparing them by means of a photometer with a source of constant light, as, for example, a glow-lamp at constant voltage. This method is very faulty on account of fundamental difficulties of a physiological or psychological nature which are met in the use of even the most perfect photometer. It requires care and deliberation to make a photometer setting which can be relied upon as being correct within a narrow limit of error, while the fluctuations in the intensity of the light may be quite rapid.

If one seeks to determine these fluctuations by independent settings, succeeding each other as rapidly as reliable settings can be made, the extreme fluctuations may escape observation. The consequence is that no very definite idea can be obtained of the form of the curve which would be traced by the changes of the light intensity. On the other hand, if continuous observations are made with the photometer, it is possible to follow up the fluctuations only very tardily, irregularly, and imperfectly.

In any case the personal error of the observer enters so largely into photometric results that the perplexing question is continually arising, Have I really observed a fluctuation of the light, or have I merely made a bad setting? This remark applies, of course, only to the smaller fluctuations, and not to the large ones commonly observed in the case of candles.

In order to determine these fluctuations, an instrument is needed which will not only serve to measure radiant energy, but which will also respond instantly to any change in the amount of radiation meeting it. Such an instrument is found in the bolometer.

Suggestions as to the use of the bolometer in photometry have not been infrequent. Lummer and Kurlbaum[1] have described a large double-faced bolometer of platinum foil, constructed by them for photometric work. It is difficult to see how this bolometer, intended as it was to be used among the fickle draughts and changing temperatures of a photometer room, would serve to get a continuous curve of the variations of a light standard, by the method described below.

In order to get a complete knowledge of the performance of any light source, it is necessary to obtain a continuous record of its behavior during a considerable period of time. If we make the assumption that light radiation from a given source varies in direct ratio to the total radiation, a properly constructed bolometer furnishes a suitable instrument for obtaining the record desired. The bolometer strip must be so thin that its temperature will follow almost instantly all changes in the amount of radiation meeting it, and the galvanometer must be nearly dead beat. Then if the working conditions are such that it is certain that all observed galvanometer deflections are due to changes in the radiation from the light source, and to nothing else, a plat of galvanometer deflections will represent very truly the variations in the intensity of the light.

The proper adjustment of conditions involves the very careful exclusion of draughts of air from the bolometer strips and from the other parts of the apparatus; and the maintenance of a nearly constant temperature in the room. Moreover, it necessitates the taking of observations at such a time or place that the galvanometer shall not be subjected to any irregular magnetic influences. In order to fulfil these conditions, the apparatus described below was constructed.

II.

Description of Apparatus.

After numerous unsatisfactory experiments with gold leaf lightly plated with nickel, with pure nickel, and with the thinnest tin foil, the bolometer strips were made as follows: —

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[1] **Lummer und Kurlbaum, Weid. Ann., Vol. 46, p. 204; Zeitsch. für Instrumentenkunde, Vol. 12, p. 81.**

A piece of Swedish iron wire, of No. 30 B. & S. gauge, was passed through jeweler's rollers until its thickness was about 0.045 mm., and its width 1.5 mm. It was then placed in dilute sulphuric acid, in which potassium bichromate had been dissolved, and a current was passed through it in such a manner as to dissolve the iron. The potassium bichromate was introduced into the solution to dispose of the hydrogen bubbles which would ordinarily have clung to the metallic surface and which would have caused it to be dissolved unevenly.

In this way a strip was obtained which was about 0.025 mm. in thickness and still moderately strong. From this strip were cut two pieces, each about 6 cm. in length, to constitute two arms of a Wheatstone's bridge.

To carry the strips so obtained, a light oblong frame, *F* (Fig. 1), of thin wood was made, and to it were fastened small bits of sheet brass, *b*, *b*, *b*, to which the strips and the copper wires intended to connect them with the other arms of the bridge could be soldered. The strips, *S*, *S'*, were then bent and placed over the frame, so that each strip crossed the frame twice. The free ends of each strip were displaced laterally from each other so that, when viewed from the front, the portion of the strip on one side of the frame hid only very little of the portion on the other side of the frame.

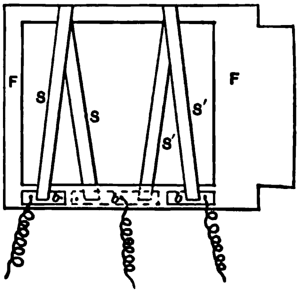

Fig. 1.

After the two strips had been arranged on the frame symmetrically with respect to each other, the one which was to receive radiation was carefully smoked on both sides. To accomplish this smoking without undue heating of the strip, a piece of sheet metal, through which a small hole had been punched, was held over a candle flame so that the flame was caused to smoke. The smoke passed through the hole, over which a tube was held to direct the current. The strip was passed back and forth over the top of the tube. In this way a very delicate strip can be blackened without injury. In their completed state the strips had a resistance of about 0.5 ohm each.

The frame holding the strips was mounted in a wooden box about 20 cm. long and 5 cm. × 6 cm. in cross-section. A number of suitable cardboard screens were placed in the front portion of the box to shield one of the strips from radiation and to protect both of them from draughts. To close the box at its other end a piece of bright tin was cut equal in size to the cross-section of the box. This was soldered fast to a heavy block of brass, to the other end of which was soldered another piece of tin which covered the end of the box. When this arrangement was placed in the box, the first piece of tin came close behind the bolometer strips, and its bright surface acted as a mirror in reflecting upon the back of the exposed strips many rays which would otherwise have been lost.

By this means the efficiency of the bolometer was nearly doubled. The tinned surface did not tarnish perceptibly during the course of the investigation. It was not smooth enough to reflect a distinct image, and the light reflected from it was to a large extent scattered. The use of a plane-surfaced mirror in such a position would not be allowable, since any slight change in the angle of incidence would cause a different amount of light to be reflected upon the bolometer strip. The use of the irregular-surfaced plate, since it diffuses the light, can scarcely affect the accuracy of the results to an appreciable degree in such work as has been done with this bolometer. Nevertheless, this arrangement is to be recommended only where great sensitiveness is desired rather than the most exact comparison of results.

Other sources of error which might affect the exact comparability of results have also been ignored. Among these may be mentioned the error which is introduced in assuming that the sensitiveness of the bolometer is independent of the absolute temperature of the strips. The brass block served to keep the temperature of the tin screen from rising to any marked degree above that of the air in the room, by conducting any excess of heat to the outer piece of tin, the surface of which was smoked.

The compensating resistance, forming the other arms of the bridge, was made of german-silver wire arranged inside a small wooden box. Fig. 2 gives the top and side views of this box.

The german-silver wire was strung back and forth on tacks nailed in the two sides of the box, as shown in the side view. The middle portion, *S*, of this wire was drawn tightly, lengthwise of the box, and a sliding contact piece, *C*, was arranged to press against this wire. This contact piece was carried on a little block of wood, *A*, running on ways, *F*, *F'*, and driven by a fine-threaded screw, *E*, which could be turned by means of a large disk head, *D*, outside of the box. One terminal of the battery (which consisted of two gravity cells in multiple with each other) was connected with corresponding ends of the bolometer strips. The other terminal was connected with the sliding contact piece just mentioned. In order to adjust the balance of the bridge, it was necessary only to turn the screw and so to move the contact piece along the stretched wire, until a position was found where no current flowed through the galvanometer. This method was found to be very delicate and convenient.

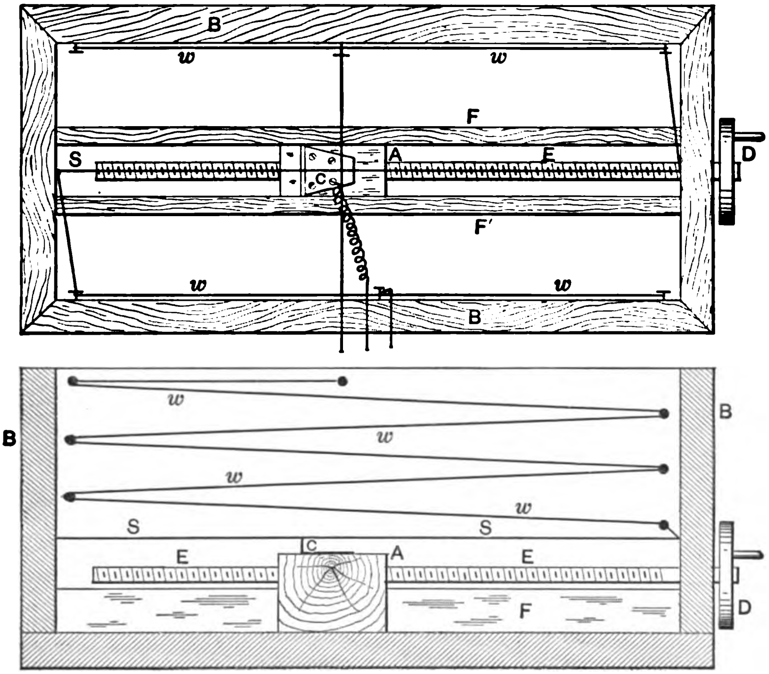

Fig. 2.

The compensating resistance was connected with the bolometer strips by means of copper wires. All metallic junctions which might give rise to thermal currents were, as far as possible, so arranged that currents produced in this way in any part of the bridge would be opposed by similar thermal currents produced in the opposite part of the bridge; this condition being secured by bringing corresponding thermo-junctions as near to each other as it was convenient to do.

The galvanometer employed was of the four-coil type. It was constructed by Professor W. S. Franklin, after the same general plan as has been followed by Snow,[1] Paschen,[2] and others. When the two front coils were in multiple with each other and the two rear coils similarly connected, and the two pairs were connected in series, the resistance was found to be 190 ohms. The moving parts consisted of four little magnets of piano wire, each about 5 mm. long, and a mirror of thin cover glass, 4 mm. wide by 7 mm. long, all mounted on a slender rod of glass and suspended by a very fine quartz fiber. Any oscillations of the needles were very strongly and effectively damped by the air resistance to the light-moving parts, — a very essential condition to the correct operation of the instrument when used to get the variations of a rapidly fluctuating source of radiant energy.

The scale was divided into 100 half-inch divisions, each of which was, in turn, divided into tenths. The distance of the scale from the galvanometer was 100 scale divisions, *i.e.* 50 inches. With the telescope used, fifths of the smallest divisions could be estimated. In speaking of scale divisions, the half-inch divisions will always be meant.

To determine at any time the sensitiveness of the arrangement, a plan was followed which was first proposed by Knut Ångström.[3] In multiple with the exposed bolometer strip was placed a resistance of 1000 ohms. By removing a plug and throwing 5000 ohms more into the shunt circuit, the galvanometer needle was deflected in the same direction as when the strip was heated. Only the first throws of the needle were taken in observations for sensitiveness, — a procedure which a comparison of first throws and final deflections, at different degrees of sensitiveness, showed to be justifiable. The ratio of first throws to permanent deflections was found to be 1.5.

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[1] B. W. Snow, Wied. Ann., Vol. 47, p. 213; Phys. Review, Vol. 1, p. 2.

[2] F. Paschen, Wied. Ann., Vol. 48, p. 272; Zeitschr. für Instrumentenkunde, Vol. 13, 1893, p. 13.

[3] Ångström, Oefvers. af kongl. Vetenskaps-Akad. Förhandl., 1888, Vol. 6, p. 379.

No attempt has been made to correct the determinations of sensitiveness for any variation of the ratio of resistance of the bolometer strip and that of the shunt, due to temperature changes. This would be a simple correction to apply if only one could ascertain the temperature of the strips at any time. Since, however, this temperature is a function not only of the temperature of the room, but also of the strength of the main current and of the radiating power of the surface of the strip, a direct determination of it is impracticable. A value for this correction combined with the correction for the change in sensitiveness due to change in absolute temperature of the strip, could have been ascertained by comparing deflections given by a source of constant radiation, under different conditions of room temperature and current strength; but for the present work such refinements were deemed unnecessary, since the error introduced by not applying these corrections would form only a small proportional part of the quantities to be measured.

For observations requiring only a low degree of sensitiveness, the galvanometer coils were connected in the multiple-series arrangement described above. For higher degrees of sensitiveness all the coils were put in multiple, the increase in sensitiveness produced by this change being about twofold. With either connection the fine adjustment of sensitiveness was obtained by changing the distance of a controlling magnet. In nearly all cases the sensitiveness of the bolometer was such as to permit the controlling magnet to be placed so as to strengthen the earth's field, and the period of the galvanometer needle was correspondingly short.

The constants of the galvanometer and of the bolometer were as follows: With the galvanometer coils in the multiple-series connection and the sensitiveness adjusted to give a throw of the needle of 16.2 scale divisions on putting the 5000 ohms in the shunt (this being the standard sensitiveness for candles), the period of the needles was 6 seconds for a complete vibration. A deflection of one scale division corresponded to a current of 68×10^{-10} amperes.

This deflection of one scale-division corresponded to a temperature rise in the bolometer strips of 0°.00657 C. If we reduce this to millimeter divisions on a scale placed at a distance of 1 m. from the mirror, we see that one millimeter deflection corresponded to a current of 68×10^{-11} amperes, and the corresponding rise in temperature of the strip was 0°.00066 C.

This temperature sensitiveness is much smaller than has usually been employed in bolometer work, but it was amply sufficient for the purpose. That the bolometer itself was one of high sensitiveness is evident from the fact that this degree of sensitiveness was attained with galvanometer needles swinging in a strengthened field and with a galvanometer of 190 ohms resistance. The conditions for maximum sensitiveness of the bridge would have required a galvanometer resistance of only 0.5 ohms.

The reason for the great sensitiveness lay in the nature of the strips employed. Their area was considerable, the temperature coefficient of the iron was high, and the current through it was large, ranging from 0.15 to 0.20 amperes, the size of the strips permitting the use of a large current without undue heating. As a result of the strong field in which the galvanometer needles swung, the drift due to magnetic changes was usually imperceptible.

The bolometer, compensating resistance, and battery were all placed in an interior room, with thick brick walls, and having communication with the outer room only by a door, *D*, Fig. 3.

The temperature of this room changed very slowly, and it was quite free from draughts. The bolometer box, *B*, was placed upon a shelf fastened to the door of the room, and looked out, through a hole in the door, upon the outer room. A double screen, *S*, of tin, arranged to slide up and down on the outside of this door, covered up the bolometer strips when desired. The box, *B'*, containing the compensating resistance, was also fastened to the inside of the door. The end of the box through which the screw projected, fitted into a hole in the door, so that the screw could be turned and the bridge balanced from the outer room. The wires, *g*, *g*, led to the galvanometer, and *s*, *s* led to the box which was in multiple with the exposed strip. For further protection from draughts the bolometer box and connecting wires were covered with cotton waste.

Under this arrangement it was necessary to enter the inner room only occasionally to attend to the batteries, and, therefore, the bolometric resistances were subjected to no sudden changes of temperature. As a consequence, drift due to changes of temperature of these resistances was very small and quite regular. Unfortunately, however, it was necessary to place the box containing the resistance which was in multiple with the exposed strip in the outer room, near the galvanometer telescope. The resistance of this box, and of the wires leading to it, changed slightly with temperature changes in the outer room. As a consequence a slight and regular drift would always ensue upon lighting the gas in the outer room preparatory to beginning work.

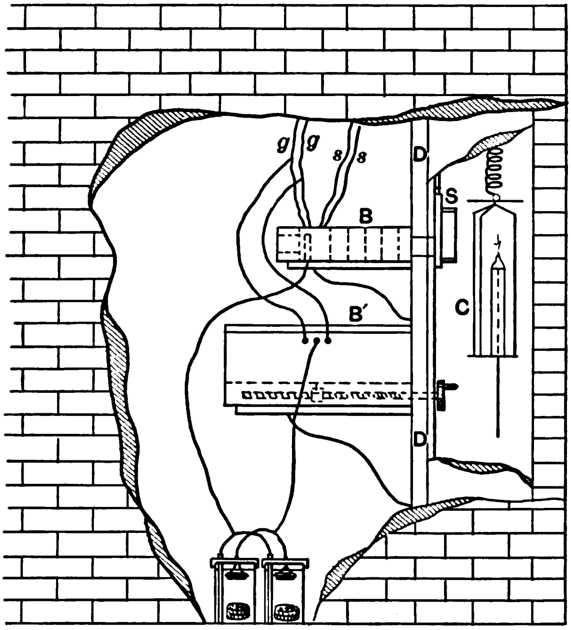

Fig. 3.

Since only a very small proportion of the current flowed through this outer circuit, this drift was never large, and would gradually cease as the temperature of the room became constant. Various expedients were adopted for minimizing this temperature change; such as opening a window slightly to admit enough cold air to counterbalance the heating due to the gaslights; closing the valves in the radiator pipes, etc. In every case correction was made for the drift and for any change of sensitiveness which might take place, on the assumption that the change had been a perfectly regular one. Whenever it would seem that this assumption could result in any considerable error, the observations were rejected.

In getting the variations of candles, a special device was required to keep the top of the burning candle at constant height in front of the bolometer strip. A suggestion of Mr. C. H. Bierbaum, M.E., resulted in the construction of the following simple and effective arrangement. A spiral spring, about 60 cm. long and 4 cm. in diameter, was attached to a small scale-pan, and the spring was cut off to such a length that when a candle was put on the scale-pan, the elongation was just equal to the length of the candle. The spring would then take up as fast as the top of the candle was lowered by burning. A small piece of sheet metal served to protect the spring from the heat of the candle. In order to keep the scale-pan from swinging sidewise and from oscillating up and down, a couple of wires were passed vertically through holes on opposite sides of it, and served as loosely fitting guides, which, without interfering with the take-up of the spring, effectually damped any vibrations. The adjustment of a spring to any candle could be made in a few minutes with sufficient accuracy so that the height of the top of a candle burning on the scale-pan would not vary over 1 mm. in an hour. *C*, Fig. 3, shows this arrangement in place.

III.

Method of Taking Observations.

All determinations of the variations of standards were made at times when the laboratory and its surroundings were very quiet. Most of them were made between the hours of seven and twelve in the evening; a few were made on holidays, when the laboratory was closed for general work. Before beginning a set of observations the sensitiveness was adjusted, and the galvanometer was carefully watched for a considerable time to get the amount of its swings due to currents of air about the bolometer strips or to changes in the earth's field. The sensitiveness was also tested from time to time in order to find whether this was variable or not. If swings of the galvanometer greater in amount than two-tenths of a scale division, or any irregular changes in sensitiveness were detected, no run was attempted. Since two-tenths of a scale division amounted to only about 0.5 per cent of the total deflection in most cases, and since the variation in the light sources themselves were many times two-tenths of a scale division in the case of all the standards except the Carcel lamp, this amount of error was considered entirely negligible.

After taking these preliminary observations, which usually required about an hour's time, the bolometer was exposed to the source of light. After waiting a few minutes for the strip and the surrounding parts to be thoroughly heated, the galvanometer deflections were read rapidly by one person, and were plotted by another as fast as they were read, the time being taken from a watch. In this way curves were traced which represent very truly all the changes in the radiation of the light source. This method of recording observations was found to be much preferable to the ordinary one of noting galvanometer readings and plotting them afterwards, both in point of saving of time and in accuracy attainable. It made possible, not only more numerous readings, but also the giving of oral instructions to the recorder, and the consequent plotting of curves which, in correctness of their minor as well as of their major details, would be but little surpassed by a photographic record.

An unfortunate feature in these curves is that the scale of abscissas, representing times, is so small. As a result of this it has been difficult to represent with the greatest accuracy the true slope of those portions of the curves which correspond to very rapid changes in radiation. In examining the curve it should be borne in mind that a very steep line may cover several seconds of time, and that it is quite possible that a photometric setting might be made during the time that a candle is executing just such a variation as is represented by the steepest parts of one of the candle curves. The number of galvanometer readings plotted during the space of five minutes was usually from 50 to 100.

At the end of a run the strips and surrounding parts were given time completely to lose the heat imparted to them, after which readings for zero and sensitiveness were taken. To correct the curves for changes in zero and in sensitiveness, a plan was followed which was justified only by the fact that the error introduced by employing it was entirely insignificant. Change in sensitiveness was reduced to ordinates of the curve by means of the equation: —

$$\frac{\text{change in sensitiveness}}{\text{initial sensitiveness}} \times \text{approximate mean ordinate} = \text{correction to ordinate.}$$

This was combined with the drift, and a point was placed on the cross-section paper having its ordinate such as to represent the combined effect of drift and change of sensitiveness, its abscissa being the time of taking observations. This point was joined by a straight line with a point on the axis of X at the time of taking the initial zero readings, thus assuming that the drift and change of sensitiveness had been regular during the run. Then, if the initial sensitiveness happened to be unequal to the sensitiveness which was taken as standard, by an equation similar to the one given above, a correction to all the ordinates was computed to reduce the curve to the standard sensitiveness. If the distance of the light source from the strip was other than that taken as standard, a similar correction was computed for this. These two reductions were then combined, and a line was drawn parallel to the first correction line and distant from it by an amount equal to these combined corrections. The mean ordinates of the curves, reckoned from the new base line so constituted, represent the relative quantities of radiation of the sources of light. The curves were then traced on bond paper, the axis of abscissas on which was put so as to fall on the correction line on the original cross-section paper. Since the corrections which have been spoken of above were all small (except in the case of curves XVI. and XVI. (*a*)), neither the proportionality of the variations to the total deflection, nor the slope of the lines of the curves, has been materially affected.

Before setting out on the work it was feared that the inertia of the moving parts of the galvanometer might have a decided effect on the accuracy of the results obtained, in that it would tend to exaggerate the rapid changes. This fear was found to be groundless. In the first place, the galvanometer was, as has been said, very nearly dead beat. Although the bolometer strip followed almost instantly every fluctuation of the source of radiation, the fluctuations themselves were not usually so rapid that the galvanometer needles were driven with any considerable velocity. Consequently the tendency of the needles to oscillate in their natural period was very slight. That the inertia of the moving parts had very little, if any, influence on the shape of the curves was shown by the fact that at the end of most swings the needle would pause for some seconds before beginning its return; while it rarely happened that the angular velocity did not change during a swing.

It has been impossible to draw the curves so as to show clearly these variations which involved only a few seconds of time, since, as has been said, the scale to which the time abscissas are plotted is very small compared with the scale of ordinates. These important peculiarities of the movements of the galvanometer needle were clearly perceptible, however, to one who observed the galvanometer deflections for only a little time; while their character made it perfectly evident that the swings of the galvanometer needles in their natural period exercised only an inappreciable influence on the slopes of the curves and the magnitudes of the variations recorded.

On the plates the smallest divisions of abscissas represent five minutes of time. The corresponding divisions of ordinates are five divisions of the galvanometer scale. In order to economize space, no attention has been given to the height of the curves relative to the base line on the plates. The total ordinates for each curve are indicated by such a symbol as this $\left.\begin{matrix}\text{I}\\ \text{II}\end{matrix}\right\}40$, indicating that the true X-axis for curves I. and II. should be placed 10 divisions below the base line on the plate. Consequently on the various plates curves having the same ordinates are plotted one above the other, the true ordinate being indicated in the way mentioned above.

The data by which the curves have been corrected are given in Table I.

Table II. gives, in its first column, the approximate abscissas of the portion of the curve in question, and in its second column the mean ordinates of these portions. The third column contains the deviations of the mean ordinates of the portions from the mean ordinate of the whole curve. The fourth column contains these deviations reduced to percentages of the whole. The fifth contains similar deviations from the mean ordinate of all the curves of the particular standard under consideration, and in the sixth these also are reduced to percentages.

The standard distance and sensitiveness for the candle and Hefner lamp were 26 cm. and 16.2, respectively; for the Carcel lamp they were 56.7 cm. and 15, and for the Methven screen they were 50 cm. and 40, respectively.

TABLE I.

	Number of curve.	Times of taking zero and sensitiveness readings.		Time of lighting.	Time of taking the curve.		Hourly rate of consumption.	Distance from bolometer.	Initial sensitiveness.	Correction for		Reduction to standard		
										Drift.	Change of sensitiveness.	Sensitiveness.	Distance.	Rate.
		h. m.	h. m.	h. m.	h. m.	h. m.	grams.	cm.						
ENGLISH CANDLE.	I.	8 47 and	10 01	8 50	8 50 to	9 35	7.910	25.3	16.2	+0.9	+0.2	0	−2.3	−0.70
	II.	9 41	10 50	9 33	9 50	10 35	7.767	26.0	16.1	−0.4	0	+0.3	0	+0.05
	III.	8 00	9 15	8 00	8 00	9 00	7.842	26.0	16.1	−2.9	+0.8	+0.3	0	−0.34
	III. (*a*)	9 38	10 30	8 00	9 45	10 15	7.842	26.0	16.08	+0.3	0	+0.3	0	−0.34
	IV.	7 50	9 05	7 50	7 50	8 50	7.692	25.3	16.15	+1.0	−0.1	+0.1	−2.1	+0.45
	IV. (*a*)	9 35	10 30	7 50	9 45	10 15	7.692	25.3	16.4	−0.1	0	−0.5	−2.3	+0.45
GERMAN CANDLE.	V.	7 55	9 10	7 55	7 55	8 55	8.60	25.3	16.27	+1.1	−1.6	−0.3	−2.8	—
	V. (*a*)	9 50	10 40	7 55	9 55	10 25	8.60	25.3	16.3	−1.7	+1.0	−0.3	−2.9	—
	V. (*b*)	8 21	9 25	8 10	8 25	9 10	—	26.0	16.2	−5.7	+4.7	0	0	—
	VI.	8 05	9 20	8 05	8 05	9 05	—	26.0	16.15	+1.3	−0.3	+0.15	0	—
	VI. (*a*)	9 30	10 31	8 05	9 46	10 16	—	26.0	16.2	−0.3	−0.15	0	0	—

HEFNER LAMP.	VII.	8 05	8 57	7 15	8 10	8 40	—	25.0	16.33	−3.1	+1.8	−0.4	−2.2	—
	VII. (*a*)	9 05	10 00	7 15	9 10	9 45	—	25.0	16.1	−1.3	−1.9	+0.3	−2.3	—
	VIII.	10 17	11 00	9 57	10 25	10 45	—	26.0	16.2	−3.2	+0.5	0	0	—
	IX.	8 08	9 05	7 13	8 20	8 50	—	26.0	16.2	−0.3	+0.1	0	0	—
	X.	8 42	9 20	—	8 50	9 00	—	36.8	32.5	−0.3	−0.3	−0.1	0	—
	XI.	9 08	10 26	8 55	9 14	9 34	—	26.0	16.2	−3.0	+0.8	0	0	—
	XI. (*a*)	9 08	10 26	8 55	9 38	9 40	—	26.0	16.2	−3.0	+0.8	0	0	—
	XI. (*b*)	9 08	10 26	8 55	9 46½	9 49½	—	26.0	16.2	−3.0	+0.8	0	0	—
	XI. (*c*)	9 08	10 26	8 55	9 55	9 58	—	26.0	16.2	−3.0	+0.8	0	0	—
	XII.	7 53	8 35	7 40	8 00	8 20	—	26.0	16.2	−2.3	+0.4	0	0	—
	XII. (*a*)	8 40	9 25	7 40	8 50	9 10	—	26.0	16.2	−0.7	0	0	0	—
	XIII.	10 11	10 50	9 38	10 15	10 35	—	26.0	16.2	−0.5	+1.3	0	0	—
	XIV.	10 00	10 35	9 53	10 08	10 18	—	26.0	16.2	+0.7	0	0	0	—
CARCEL LAMP.	XV.	10 14	11 03	9 40	10 18	10 48	47.05	56.7	15.0	+0.7	−0.4	0	0	−4.5
	XVI.	8 38	9 38	8 10	8 45	9 25	43.05	56.7	28.2	+2.8	−0.4	−33.0	0	−0.7
	XVI. (*a*)	10 00	10 38	8 10	10 03	10 23	44.14	56.7	28.4	0	+0.2	−35.6	0	−1.5
	XVII.	8 13	8 55	7 59	8 20	8 40	46.85	56.7	15.03	+0.6	0	−0.1	0	−3.0
	XVII. (*a*)	10 54	11 20	7 59	10 59	11 09	40.00	56.7	15.0	0	0	0	0	+1.3
METHVEN SCREEN.	XVIII.	9 15	10 06	—	9 21	9 51	—	49.7	39.8	+4.7	−0.2	+0.1	−0.3	—
	XIX.	9 22	10 00	—	9 30	9 50	—	50.0	39.6	+2.5	−1.0	+0.2	0	—
	XX.	9 00	9 51	—	9 06	9 36	—	50.0	40.2	−0.5	+0.2	−0.2	0	—
	XXI.	8 17	8 59	—	8 24	8 44	—	50.0	40.1	+0.6	+0.1	−0.1	0	—
	XXII.	9 01	9 37	—	9 09	9 19	—	50.0	40.0	+0.6	0	0	0	—

TABLE II.

	Time: Minutes on curve.	**Mean ordinates and their mean.**	**Deviations from the mean ordinate of the curve.**	**Deviations reduced to percentages.**	**Deviations from mean ordinate of all the English candle curves.**	**Deviations from general mean reduced to percentages.**
CURVE I.	15–20	41.68	−1.04	−2.43	+0.59	+1.44
	20–25	44.68	+1.96	+4.58	+3.59	+8.72
	25–30	40.84	−1.88	−4.40	−0.25	−0.61
	30–35	43.39	+0.67	+1.57	+2.30	+5.59
	35–40	43.71	+0.99	+2.32	+2.62	+6.37
	40–45	42.04	−0.68	−1.59	+0.95	+2.31
		42.72	Correction for rate of burning = − 0.70.			
CURVE II.	45–50	39.24	−1.77	−4.32	−1.85	−4.50
	50–55	37.85	−3.16	−7.70	−3.24	−7.87
	55–60	39.12	−1.89	−4.61	−1.97	−4.79
	60–65	40.44	−0.57	−1.39	−0.65	−1.58
	65–70	40.76	−0.25	−0.61	−0.33	−0.80
	70–75	42.85	+1.84	+4.48	+1.76	+4.28
	75–80	43.52	+2.51	+6.12	+2.43	+5.92
	80–85	43.20	+2.19	+5.34	+2.11	+5.13
	85–90	42.11	+1.10	+2.68	+1.02	+2.48
		41.01	Correction for rate of burning = +0.05.			
CURVE III.	15–20	43.04	−0.54	−1.25	+1.95	+4.74
	20–25	42.88	−0.70	−1.62	+1.79	+4.35
	25–30	42.82	−0.76	−1.76	+1.73	+4.21
	30–35	42.18	−1.40	−3.23	+1.09	+2.65
	35–40	43.23	−0.35	−0.80	+2.14	+5.20
	40–45	44.80	+1.22	+2.80	+3.71	+9.02
	45–50	43.58	0	0	+2.49	+6.06
	50–55	44.30	+0.72	+1.65	+3.21	+7.80
	55–60	45.38	+1.80	+4.13	+4.29	+10.45
		43.58	Correction for rate of burning = −0.34.			
CURVE III. (*a*).	60–65	43.25	+0.23	+0.53	+2.16	+5.25
	65–70	43.52	+0.50	+1.16	+2.43	+5.91
	70–75	42.82	−0.20	−0.47	+1.73	+4.21
	75–80	43.68	+0.66	+1.54	+2.59	+6.30
	80–85	42.82	−0.20	−0.47	+1.73	+4.21
	85–90	42.04	−0.98	−2.28	+0.95	+2.31
		43.02	Correction for rate of burning = −0.34.			

TABLE II. (*continued*).

	Time: Minutes on curve.	**Mean ordinates and their mean.**	**Deviations from the mean ordinate of the curve.**	**Deviations reduced to percentages.**	**Deviations from mean ordinate of all the English candle curves.**	**Deviations from general mean reduced to percentages.**
CURVE IV.	15–20	38.65	+1.42	+3.82	−2.44	−5.94
	20–25	39.84	+2.61	+7.01	−1.25	−3.04
	25–30	38.07	+0.84	+2.26	−3.02	−7.35
	30–35	35.76	−1.47	−3.87	−5.33	−12.95
	35–40	35.72	−1.51	−4.05	−5.37	−13.08
	40–45	37.04	−0.19	−0.51	−4.05	−9.84
	45–50	35.84	−1.39	−3.75	−5.25	−12.78
	50–55	37.16	−0.07	−0.02	−3.93	−9.55
	55–60	36.96	−0.27	−0.72	−4.13	−10.05
		37.23	Correction for rate of burning = +0.45.			

CURVE IV. (*a*).	60–65	38.26	−1.47	−3.70	−2.83	−6.88
	65–70	39.04	−0.69	−1.74	−2.05	−4.99
	70–75	38.26	−1.47	−3.70	−2.83	−6.88
	75–80	40.84	+1.11	+2.80	−0.25	−0.62
	80–85	40.84	+1.11	+2.80	−0.25	−0.62
	85–90	41.16	+1.43	+3.60	+0.07	+0.17
		39.73	Correction for rate of burning = +0.45.			

Mean ordinate of all the English candle curves is 41.09.

Mean ordinate of all the English candle curves corrected for rate of burning is 41.05.

Mean ordinate of all the English candle curves corrected for rate and reduced to true deflections is 41.06.

					German candle curves.	
CURVE V.	15–20	49.48	−0.18	−0.36	−1.03	−2.05
	20–25	48.62	−1.04	−2.10	−1.89	−3.77
	25–30	49.20	−0.46	−0.92	−1.31	−2.60
	30–35	49.44	−0.22	−0.44	−1.07	−2.12
	35–40	49.40	−0.26	−0.52	−1.11	−2.20
	40–45	49.20	−0.46	−0.92	−1.31	−2.60
	45–50	49.52	−0.14	−0.28	−0.99	−1.96
	50–55	51.96	+2.30	+4.64	+1.45	+2.88
	55–60	50.12	+0.46	+0.92	−0.39	−0.77
		49.66				

TABLE II. (*continued*).

	Time: Minutes on curve.	**Mean ordinates and their mean.**	**Deviations from the mean ordinate of the curve.**	**Deviations reduced to percentages.**	**Deviations from mean ordinate of all the German candle curves.**	**Deviations from general mean reduced to percentages.**
CURVE V. (*a*).	60–65	52.80	+1.43	+2.78	+2.29	+4.53
	65–70	51.00	−0.37	−0.72	+0.49	+0.97
	70–75	49.44	−1.93	−3.76	−1.07	−2.12
	75–80	50.24	−1.13	−2.20	−0.27	−0.53
	80–85	50.68	−0.69	−1.35	+0.17	+0.34
	85–90	54.08	+2.71	+5.27	+3.57	+7.07
		51.37				
CURVE V. (*b*).	50–55	53.14	−0.78	−1.45	+2.63	+5.21
	55–60	55.41	+1.49	+2.77	+4.90	+9.71
	60–65	52.53	−1.39	−2.58	+2.02	+4.00
	65–70	52.82	−1.10	−1.88	+2.31	+4.58
	70–75	52.62	−1.30	−2.40	+2.11	+4.18
	75–80	53.30	−0.62	−1.15	+2.79	+5.53
	80–85	55.22	+1.30	+2.42	+4.71	+9.32
	85–90	56.33	+2.41	+4.47	+5.82	+11.52
		53.92				
CURVE VI.	15–20	45.24	−3.67	−7.35	−5.27	−10.43
	20–25	46.63	−2.28	−4.62	−3.88	−7.69
	25–30	48.17	−0.74	−1.51	−2.34	−4.63
	30–35	49.20	+0.29	+0.59	−1.31	−2.60
	35–40	49.84	+0.93	+1.90	−0.67	−1.33
	40–45	49.52	+0.61	+1.25	−0.99	−1.96
	45–50	49.88	+0.97	+1.99	−0.63	−1.25
	50–55	50.20	+1.29	+2.64	−0.31	−0.61
	55–60	57.48	+2.57	+5.16	+0.97	+1.92
		48.91				
CURVE VI. (*a*).	60–65	48.93	+0.14	+0.28	−1.58	−3.13
	65–70	49.20	+0.41	+0.84	−1.31	−2.60
	70–75	47.78	−1.01	−2.07	−2.73	−5.40
	75–80	48.52	−0.27	−0.55	−1.99	−3.94
	80–85	48.97	+0.18	+0.37	−1.54	−3.05
	85–90	49.36	+0.57	+1.17	−1.15	−2.28
		48.79				

Mean ordinate of all German candle curves is 50.51
Reduced to true deflections = 50.40

TABLE II. (*continued*).

	Time: Minutes on curve.	Mean ordinates and their mean.	Deviations from the mean ordinate of the curve.	Deviations reduced to percentages.	Deviations from mean ordinate of all the Hefner lamp curves.	Deviations from general mean reduced to percentages.
CURVE VII.	0–5	38.81	−0.50	−1.27	+0.17	+0.44
	5–10	40.02	+0.71	+1.81	+1.38	+3.57
	10–15	39.64	+0.33	+0.84	+1.00	+2.59
	15–20	39.19	−0.12	−0.31	+0.55	+1.42
	20–25	39.03	−0.28	−0.71	+0.39	+1.01
	25–30	39.19	−0.12	−0.31	+0.55	+1.42
		39.31				
CURVE VII. (*a*).	30–35	39.96	+0.44	+1.11	+1.32	+3.42
	35–40	40.27	+0.75	+1.90	+1.63	+4.22
	40–45	39.83	+0.31	+0.79	+1.19	+3.08
	45–50	39.54	+0.02	+0.05	+0.90	+2.33
	50–55	39.09	−0.43	−1.09	+0.45	+1.16
	55–60	39.03	−0.49	−1.24	+0.39	+1.01
	60–65	39.19	−0.33	−0.84	+0.55	+1.42
	65–70	39.27	−0.25	−0.63	+0.63	+1.63
		39.52				
CURVE VIII.	70–75	38.91	−1.12	−2.80	+0.27	+0.70
	75–80	39.83	−0.20	−0.50	+1.19	+3.08
	80–85	40.54	+0.51	+1.28	+1.90	+4.92
	85–90	40.86	+0.83	+2.08	+2.22	+5.74
		40.03				
CURVE IX.	0–5	38.26	−0.54	−1.39	−0.38	−0.98
	5–10	38.62	−0.18	−0.46	−0.02	−0.05
	10–15	38.97	+0.17	+0.44	+0.33	+0.85
	15–20	38.87	+0.07	+0.18	+0.23	+0.59
	20–25	39.04	+0.24	+0.62	+0.40	+1.03
	25–30	39.03	+0.23	+0.59	+0.39	+1.01
		38.80				
CURVE X.	30–35	38.89	0	0	+0.25	+0.65
	35–40	39.16	+0.27	+0.70	+0.52	+1.34
	40–45	38.62	−0.27	−0.70	−0.02	−0.05
		38.89				

TABLE II. (*continued*).

	Time: Minutes on curve.	Mean ordinates and their mean.	Deviations from the mean ordinate of the curve.	Deviations reduced to percentages.	Deviations from mean ordinate of all the Hefner lamp curves.	Deviations from general mean reduced to percentages.
CURVE XI.	45-50	39.42	−0.83	−2.06	+0.78	+2.02
	50-55	40.50	−0.10	−0.25	+1.51	+3.91
	55-60	40.50	+0.25	+0.62	+1.86	+4.81
	60-65	40.92	+0.67	+1.67	+2.28	+5.90
		40.25				
CURVES XI. (*a*), (*b*), and (*c*).	70	39.37	−0.24	−0.61	+0.73	+1.89
	75-80	40.00	+0.39	+0.98	+1.36	+3.52
	85-90	39.46	−0.15	−0.38	+0.82	+2.12
		39.61				
CURVE XII.	0-5	36.47	−0.90	−2.27	−2.17	−5.62
	5-10	37.21	−0.16	−0.43	−1.43	−3.70
	10-15	37.85	+0.48	+1.28	−0.79	−2.05
	15-20	37.94	+0.57	+1.52	−0.70	−1.81
		37.37				
CURVE XII. (*a*).	25-30	35.53	−0.67	−1.85	−3.11	−8.03
	30-35	35.73	−0.47	−1.30	−2.91	−7.52
	35-40	36.38	+0.18	+0.50	−2.26	−5.84
	40-45	37.18	+0.98	+2.71	−1.46	−3.78
		36.20				
CURVE XIII.	50-55	38.84	+0.15	+0.39	+0.20	+0.52
	55-60	38.74	+0.05	+0.13	+0.10	+0.26
	60-65	38.20	−0.49	−1.27	−0.44	−1.14
	65-70	39.00	+0.31	+0.80	+0.36	+0.93
		38.69				
CURVE XIV.	75-80	38.46	−0.16	−0.41	−0.18	−0.47
	80-85	38.78	+0.16	+0.41	+0.14	+0.36
		38.62				

Mean ordinate of first five minutes of Hefner lamp curves = 38.645.
Reduced to true deflections = 38.66.

Table II. (*continued*).

	Time: Minutes on curve.	**Mean ordinates and their mean.**	**Deviations from the mean ordinate of the curve.**	**Deviations reduced to percentages.**	**Deviations from mean ordinate of all the Carcel lamp curves.**	**Deviations from general mean reduced to percentages.**
Curve XV.	0–5	27.11	−0.49	−1.78	−1.16	−4.11
	5–10	27.37	−0.23	−0.83	−0.90	−3.19
	10–15	27.62	+0.02	+0.07	−0.65	−2.30
	15–20	27.68	+0.08	+0.29	−0.59	−2.09
	20–25	27.85	+0.25	+0.91	−0.42	−1.49
	25–30	27.98	+0.38	+1.34	−0.29	−1.03
		27.60				
Curve XVI.	0–5	29.00	−0.37	−1.26	+0.73	+2.58
	5–10	29.09	−0.28	−0.95	+0.82	+2.90
	10–15	29.19	−0.18	−0.61	+0.92	+3.26
	15–20	29.32	−0.05	−0.17	+1.05	+3.72
	20–25	29.35	−0.02	−0.07	+1.08	+3.83
	25–30	29.61	+0.24	+0.82	+1.34	+4.74
	30–35	29.61	+0.24	+0.82	+1.34	+4.74
	35–40	29.76	+0.39	+1.33	+1.49	+5.28
		29.37				
Curve XVI. (*a*).	40–45	29.00	+0.37	+1.29	+0.73	+2.59
	45–50	28.75	+0.12	+0.42	+0.48	+1.70
	50–55	28.65	+0.02	+0.07	+0.38	+1.34
	55–60	28.14	−0.49	−1.72	−0.13	−0.46
		28.63				
Curve XVII.	60–65	25.90	−0.92	−3.44	−2.37	−8.39
	65–70	26.86	+0.04	+0.15	−1.41	−4.97
	70–75	27.27	+0.45	+1.68	−1.00	−3.54
	75–80	27.27	+0.45	+1.68	−1.00	−3.54
		26.82				
Curve XVII. (*a*).	80–85	27.82	−0.24	−0.86	−0.45	−1.59
	85–90	28.30	+0.24	+0.86	+0.03	+0.11
		28.06				

TABLE II. (*continued*).

	Time: Minutes on curve.	Mean ordinates and their mean.	Deviations from the mean ordinate of the curve.	Deviations reduced to percentages.	Deviations from mean ordinate of all the Carcel lamp curves.	Deviations from general mean reduced to percentages.
CURVE XVIII.	0-5	25.95	+0.09	+0.35	−0.66	−2.48
	5-10	25.82	−0.04	−0.15	−0.79	−2.97
	10-15	25.57	−0.29	−1.12	−1.04	−3.91
	15-20	25.57	−0.29	−1.12	−1.04	−3.91
	20-25	25.95	+0.09	+0.35	−0.66	−2.48
	25-30	26.31	+0.45	+1.74	−0.30	−1.13
		25.86				

Mean ordinate of all the Carcel lamp curves = 28.27.

					Methven screen curves.	
CURVE XIX.	35-40	23.87	−0.91	−3.67	−2.74	−10.30
	40-45	24.89	+0.11	+0.44	−1.72	−6.47
	45-50	24.98	+0.20	+0.81	−1.63	−6.13
	50-55	25.38	+0.60	+2.42	−1.23	−4.63
		24.78				
CURVE XX.	60-65	25.31	−1.07	−4.06	−1.30	−4.88
	65-70	25.82	−0.56	−2.12	−0.79	−2.97
	70-75	26.24	−0.14	−0.53	−0.37	−1.39
	75-80	26.59	+0.21	+0.80	−0.02	−0.08
	80-85	26.82	+0.44	+1.67	+0.21	+0.79
	85-90	27.48	+1.10	+4.17	+0.87	+3.27
		26.38				
CURVE XXI.	0-5	28.10	−0.63	−2.19	+1.49	+5.60
	5-10	28.97	+0.24	+0.84	+2.36	+8.87
	10-15	28.81	+0.08	+0.28	+2.20	+8.27
	15-20	29.06	+0.33	+1.15	+2.45	+9.27
		28.73				
CURVE XXII.	25-30	28.87	−0.16	−0.55	+2.26	+8.51
	30-35	29.19	+0.16	+0.55	+2.58	+9.70
		29.03				

Mean ordinate of all the Methven screen curves = 26.61.

IV.

Discussion of the Results Obtained.

Plate I. exhibits the curves given by the English standard candles. In curves III. and IV. the candles were lighted at their tops. The wicks flared up and gave a high point on each curve, which in the case of curve IV. is not shown on the plate. The flames then increased gradually to their normal size, which was reached after about 15 minutes. Curves III. (*a*) and IV. (*a*) are continuations of III. and IV. The candles were allowed to burn during the interval between the curves, which in the case of III. and III. (*a*) was 45 minutes, and in the case of IV. and IV. (*a*) was 55 minutes. During III. the room was rather more draughty than during III. (*a*), and the effect of the draughts is seen in the much larger numbers of small irregularities in the former than in the latter curve. During the interval between IV. and IV. (*a*) the height of the flame of the candle was found to vary between 43 and 48 mm.

Curve I. was taken with the portion of candle left over from IV. and IV. (*a*). It was lighted, its wick being already charred and its crater formed, and readings were taken immediately. Curve II. was taken with the lower half of the candle used in getting III. and III. (*a*). The bottom, *i.e.* the larger end of this, was hollowed out to expose the wick and readings were taken after the candle had been burning long enough to come to its normal light-giving power. The agreement in the amount of radiation of the candle burned in this way, with the amount when burned from the smaller end, shows that the variation in the diameter of the candle has little if any influence on the intensity of the light emitted.

One marked peculiarity which characterizes, to a greater or less degree, all these curves, is the succession of sudden drops followed by gradual rises to a maximum. In the case of the drop in curve IV. at 55 minutes, the change amounted to 15 per cent of the total deflection, and in other instances the change was nearly or quite as large. The reason for these drops is to be looked for in the action of the wick, which, as the candle burns down, projects farther above the spermaceti, causing a tall flame. Finally, by reason of charring and because of its own weight, it bends over, and the end burns off. The flame, following the wick, becomes shorter and, sometimes, broader. Since the wicks in the English standard candles are very uniform in construction, these drops succeeded each other after more or less regular time intervals, which were, in most cases, not far from three minutes. The curves show also a comparative freedom of these flames from minor fluctuations due to disturbing draughts of air. The ordinates of the English candle curves are not corrected for rate of burning. It will be seen from the tables that these corrections, while they have in each case the proper sign, are, in most cases, entirely insufficient to reduce the curves to a common mean ordinate.

Plate II. exhibits the curves of German standard paraffin candles as prescribed by the *Deutscher Verein von Gas- und Wasserfachmännern*, and called the "Vereinskerze."

Curves V. and V. (*a*) and VI. and VI. (*a*) were obtained from candles burned in exactly the same way as III. and III. (*a*) and IV. and IV. (*a*), the time interval between V. and V. (*a*) being one hour, and that between VI. and VI. (*a*) being 41 minutes. Curve V. (*b*) was taken with the remainder of the candle used for V., relighted on another night; so that it is entirely independent of V. and V. (*a*), excepting that the same candle was used.

The general level of these curves is rather closely adhered to. The numerous minor fluctuations show that the flame of this candle is quite subject to disturbances by draughts. Occasionally the flame increases to a size far beyond its normal, and smokes. These large fluctuations, extending over a considerable period of time, form a marked feature of the curves. It will be noticed that VI. did not reach its normal light-giving power until 15 minutes later than V.

When we come to compare the English with the German candle, we notice, first, that the variations of the English candle were much larger and that large variations were much more frequent. The German curves are free from the semi-periodic drops which characterize the English. Curve I. at 24 minutes reaches a value which is 1.235 of the mean ordinate of the English candle curves. Curve IV. at 49 minutes drops to a point which is only 0.77 of the mean ordinate. The total variation is, consequently, 46.5 per cent. Both of these curves were taken with the same candle, but on different nights. The highest point of the German curves is on V. (*a*) at 86 minutes. The ordinate reaches a value which is 1.155 of the mean. The lowest point is on VI. (*a*) at 72 minutes, and is 0.915 of the mean. The total variation was consequently 24 per cent, or only about half that shown by the English candle.

Moreover, we see from the table that the percentage deviation of curve III., 55–60 minutes, from the mean of the English candles is + 10.45 per cent. Curve IV., 35–40 minutes, shows a deviation of − 13.08 per cent. The total deviation for a period of five minutes is 23.53 per cent for the English candles. In the German candles the maximum positive and negative deviations for five-minute periods are + 11.52 per cent and − 10.43 per cent respectively. These give a total of 21.95 per cent, and exhibit a performance but little better than that shown by the English candle.

These figures make perfectly evident the futility of any attempt to get concordant photometric results from freely burning candles, unless, indeed, a very long series of observations is taken. In favor of the English candle it can be said that its flame is more stable and is much less subject to smaller fluctuations than is the German candle. When it comes to measuring the dimensions of the flame and reducing readings to a standard size of flame, there is little doubt that the English candle would be found much superior to the German. In the early part of the work with candles a rather rude measuring device was set up and heights of flame of the English candle were determined and compared with the corresponding galvanometer readings. The accordance between definite flame heights and definite galvanometer readings was found to be very close.

Investigations along this line have not been prosecuted farther for lack of another observer. A device has been made, however, by which it is hoped that the height, or the height and the width, of a candle flame can be measured with great quickness and accuracy, and it is the purpose of one of the present observers to attempt to ascertain, by means of this measuring apparatus and the bolometer, whether the radiating power of a candle can be expressed conveniently in terms of its simpler dimensions or not. That the light-giving power of a candle must vary directly with the cubical contents of the luminous portion of its flame, provided the surrounding conditions are unaltered, seems obvious. That the relative luminous intensities of candles of different materials can be determined with a fair degree of accuracy by measuring the cubical contents of the light-giving portion of their flames has recently been shown by Glan.[1] For photometric work, however, it is essential that the luminous intensity at any time should be deducible from simple measurements of the flame — such measurements as can be made the instant before or after a photometer setting is made.

Plate III. shows the behavior of the Hepner-Alteneck amyl-acetate lamp. The lamp used is one by Hartmann and Braun, of the type having the optical projection device for fixing the height of flame at 40 mm.

Curve VII. was taken when a window in the farthest corner of the room was raised about 2 cm. Curve VII. (*a*) was begun 25 minutes after the end of VII., the lamp having burned during the interval, and the flame having been readjusted in height. The window was closed. The marked difference between the two curves is due to the stoppage of this slight draught.

In taking curve X., the sensitiveness was adjusted at 32.5, or about double the standard sensitiveness for the other tests of the candles and of the Hefner lamp. Then a computation was made, from the law of inverse squares, of the distance at which it would be necessary to place the lamp in order to produce the ordinary deflection. The fact that the ordinates of this curve agree with those of the curves taken with a sensitiveness of 16.2 shows that the method of getting the sensitiveness by taking only first throws of the galvanometer needles, was an allowable one.

Curves XI., XI. (*a*), XI. (*b*), XI. (*c*) show the results of successive attempts to adjust the height of flame to just 40 mm., the wick having been lowered between each one. No zero and sensitiveness readings were taken between the beginning of XI. and the end of XI. (*c*).

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[1] P. Glan, Ueber ein Gesetz der Kerzenflammen, Wied. Ann., Vol. 51, p. 584.

If the times that the lamp had been burning, when curves VIII., XI., XII., and XIV. were begun, are taken from Table I., it will be seen that, in each case, it was 20 minutes or less. Each of these curves shows a gradual increase in the amount of radiation. This emphasizes the advisability of lighting the Hefner lamp a considerable time before it is to be used in photometric work.

It is to be understood that, in the case of each of these runs except the one when curve VIII. was taken, the greatest care was exercised to have the room as quiet as possible. Curve VII. was the first of the Hefner lamp curves taken, and the need of extraordinary care in excluding draughts of air was not appreciated.

The Hefner light is a much more promising standard than the candles, although its curves show that it is subject to incessant fluctuations and that considerable variations may last for a minute or more. It is clear, however, that from a series of observations, extending over a comparatively short time, a very good average may be expected.

The highest point on the Hefner curves is 1.093 of the mean, and the lowest is 0.867, giving a total deviation of 22.6 per cent. This large deviation is not so significant, however, as a similar one in the case of candles, since in dealing with the Hefner lamp we have to do with an adjustable flame, and these fluctuations may indicate only that a slight readjustment was needed.

The most important question in regard to the Hefner lamp is the accuracy with which it can be adjusted to its normal light intensity. Some idea may be formed of this by noting in Table II. the percentage deviations of the first five minutes of each of the curves from the mean ordinate of the first five minutes of all the curves. The maximum deviation will be seen to be 8 per cent, while the mean deviation is 2.3 per cent. To show the accuracy with which the flame can be adjusted under given unchanging conditions, curves XI., XI. (*a*), (*b*), and (*c*) are of especial value.

In Table III. will be found the mean ordinates for the first five minute of XI. and of all of XI. (*a*), (*b*), and (*c*), and the percentage deviation of each from the mean of all.

Table III.

Curve.	Mean ordinate.	Deviation from mean.	Percentage deviation.
XI. (5^m)	39.42	−0.14	−0.35
XI. (*a*)	39.37	−0.19	−0.48
XI. (*b*)	40.00	+0.44	+1.11
XI. (*c*)	39.46	−0.10	−0.25
Mean	39.56	Mean	0.57

If, now, the assumption be made that the light-giving efficiencies of the English and German candles and of the Hefner lamp are equal, it is possible to get their relative intensities by a comparison of the mean ordinates of their curves. Reducing the deflections as read on the telescope scale to angular measure and taking double the tangents of these angles, we have for the true deflections the following values:—

English candle = 41.06
German candle = 50.40
Hefner light = 38.66

From these values we get the following ratios:—

$$\frac{\text{German candle}}{\text{English candle}} = 1.2275$$

$$\frac{\text{Hefner light}}{\text{English candle}} = 0.9415$$

Violle's values for these ratios are 1.13 and 0.98 respectively.

The curves given by the Carcel lamp are numbered XV., XVI., XVI. (*a*), XVII., and XVII. (*a*). They are all reduced to a common rate of burning of 42 grams per hour. XV. was begun when the lamp had been burning 38 minutes. It shows a slight upward tendency which decreased as time went on. The rate of consumption was high,—48 grams per hour. XVI. was begun 35 minutes after the lighting of the lamp, and the rate of burning was 43 grams. XVI. (*a*) shows the radiation from the lamp after it had been burning nearly two hours. During XVI. the radiation was increasing throughout the test. After the lamp had been burning during the interval between XVI. and XVI. (*a*), the radiation was falling off very rapidly as the wick became more and more charred. The rate for XVI. (*a*) was higher than for XVI., having been altered by a change in height of the chimney.

XVII. was begun 21 minutes after the lamp was lighted, while XVII. (*a*) was not begun until the lamp had been burning three hours. XVII. shows a rise as the lamp became warmed, the curve becoming parallel to the base line when the lamp had been burning half an hour. XVII. (*a*), however, shows that the radiation was again increasing. During XVII. the rate was 46.8 grams per hour, while the rate during XVII. (*a*) was only 40 grams per hour.

These curves show clearly the great gain in steadiness of a flame resulting from the use of a proper chimney. The deviations were, for the most part, no larger than the swings of the galvanometer when the bolometer was unexposed to radiation. The variations extending over a considerable period of time are, however, by no means inconsequential. In curve XVII. there was a total deviation in the mean ordinates of the five-minute periods of 5.1 per cent, taking place in 10 minutes. Curve XVI. shows a deviation of less than 0.8 per cent in 35 minutes, and this is perhaps a more typical illustration of what may be expected from this lamp when burned under the best conditions. The highest and lowest points on any of the Carcel curves are 29.9 and 25.4 respectively, corresponding to deviations from the mean of the curves of + 5.7 per cent and − 12.5 per cent, or a total of 18.2 per cent. Table II. shows the maximum deviations, for periods of five minutes, to be + 5.3 per cent and − 8.4, giving a total of 13.7 per cent.

Table IV. serves to show the reproducibility of the standard at different times.

TABLE IV.

Curve.	Mean ordinate.	Deviation from mean.	Percentage deviation.
XV.	27.60	−0.50	+1.78
XVI.	29.37	+1.27	+4.52
XVI. (*a*)	28.63	+0.53	+1.89
XVII.	26.82	−1.28	−4.56
XVII. (*a*)	28.06	−0.04	−0.14
Mean	28.10	Mean	2.58

The mechanical parts of the lamp used were in good working order. Pure Colza oil was used for burning, and rates of combustion were determined by weighings made before and after the runs. The regulation wicks were employed, a fresh one being used for each run.

The curves given by the Methven screen are numbered XVIII., XIX., XX., XXI., and XXII. Curve XVIII. was taken with the Argand burner connected directly to the gas-pipes in the building. The pressure of the gas was controlled only by a large regulator on the main pipe leading into the building. The consumption of gas in the building was constant during the run. The curve shows many large, but quick, variations and certain decided waves. The other Methven curves were taken with a gas-holder of about 10 gallons' capacity interposed between the gas main and the Argand burner. The effect of this in smoothing the curves is very marked, and indicates that for photometric purposes the Methven screen is much improved by the interposition of a rather capacious reservoir. The use of such a reservoir tends to minimize the effect of changes of pressure and to absorb any waves in the gas due to water in the pipes or some similar cause.

The way in which these waves of variable pressure produce fluctuations in the amount of radiation is by causing changes in the quality of those portions of the flame which cover the slit. At times the top of the flame becomes forked, so that not all of the slit is covered by it. It happens perhaps more frequently that some of the non-luminous portion of the flame rises so as partially to cover the slit. In either case the result is seen in a deviation of the curve. It must be true, also, that the amount of luminous radiation suffers a much larger proportional change than the total amount of radiation; hence the deviations recorded on the curves are too small to represent correctly the fluctuations in luminous intensity. There is an additional reason for these fluctuations, which is to be found in the slight protection against draughts of air which the wide open chimney used on the London Argand burner affords. In this respect its action is in marked contrast with that of the smaller close chimney of the Carcel lamp, which, of course, can be used only with much richer gases than ordinary illuminating gas.

These Methven screen curves cannot be trusted to give a true idea of the reproducibility of the standard. This is because the ordinates of the curve represent not only the radiation from the flame, but also that from the chimney and from the screens interposed. Since the burner was not always replaced in exactly similar positions before the bolometer, the amount of this radiation was variable. A more serious source of error, however, arose from the fact that after several of the curves had been taken it was found advisable to change the original arrangements for screening. The plan finally followed was to interpose between the Methven screen and the bolometer-box a heavy piece of asbestos paper, having a hole of proper size cut in it. This was placed so far from the burner that it did not become heated perceptibly.

The difficulty arising from the intermingling with the radiation from the flame of a variable amount of non-luminous radiation from chimney and screens could have been entirely obviated by determining, in a way which did not suggest itself until after most of the runs had been taken, the effect of this variable radiation. After the burner had been lighted long enough for the chimney and screens to become heated to their ultimate temperature, the flame was suddenly extinguished and the screen covering the bolometer strip was raised. Then a curve of cooling of the hot chimney and screens was plotted, using times as abscissas and deflections as ordinates. This curve carried back to the time of extinguishing the light gave the amount of radiation received from all parts except the flame. The importance of this correction was not fully appreciated until the results of observations were worked over, after all the runs had been taken; consequently the method was not developed very completely. The records show that in the case of the Carcel lamp the amount of radiation from the chimney varied slightly. At the end of curve XV. the chimney radiation was such as to produce a deflection of 6.2 scale divisions, while at the end of XVII. (*a*) the chimney radiation was 5.8. For curve XVI. no observations of chimney radiation were made; but since the conditions under which the run was taken were very similar, the amount of such radiation could not have differed greatly from that found for the other curves. That the effect of applying this correction is not necessarily to tend to reduce the curves to a common mean ordinate is indicated by the fact that if we correct the end of XV. by subtracting from it 6.2, and correct the end of XVII. (*a*) by subtracting from it 5.8, the difference between these portions of the curves will be exaggerated.

The only determinations of the radiation from the chimney and the intervening screens in the case of the Methven standard were made at the end of curves XXI. and XXII. For curve XXI. this radiation caused a deflection of 2.8 divisions, and for XXII. the deflection was 2.7 divisions. These curves were taken on the same night, and only a short time intervened between them. The apparatus was not displaced in the meantime.

The above work was done during the fall and winter of 1893–4. We have, in conclusion, to express our earnest thanks to Professor E. L. Nichols, not only for having proposed the line of research, and having outlined the method to be employed, but also for many valuable suggestions given during the progress of the investigation.

CORNELL UNIVERSITY, April, 1894.

ON A RELATION BETWEEN SPECIFIC INDUCTIVE CAPACITY AND CHEMICAL CONSTITUTION OF DIELECTRICS.

By Charles B. Thwing, Ph.D.

THAT a relation between specific capacity and chemical constitution of dielectrics has not up to this time been found, is no doubt due to the small number of substances hitherto determined which were adapted to show such a relation. That I have been able to determine, with a good degree of accuracy, a very large number of substances, and so to find, as I believe I have done, such a relation, is due to the extreme simplicity of the method employed for determining capacities.

For the ground idea of this method, the determination of capacity by means of resonance, I am indebted to Professor Hertz, by whose untimely death I was deprived, almost at the beginning of the work, of that friendly counsel and encouragement which he was always so willing to bestow.

The principle may be stated in the following form: since the period of the electrical oscillations which occur at the discharge of an electrical system depends only upon the capacity and self-induction of the system, it follows that, if two systems, A_1, A_2, have the same self-induction, their capacities, C_1, C_2, will be equal when both systems are in resonance with a third system, A_3, whatever the self-induction and capacity of the latter may be.

A convenient method of employing this principle for the measurement of capacities requires the following apparatus: (1) a primary circuit with variable air-condenser and an induction coil of moderate dimensions; (2) a secondary circuit with variable air-condenser which may be replaced by the condenser whose capacity is to be measured; (3) a convenient means of measuring the amount of current developed in the secondary circuit as the relative capacities of the two circuits is varied.

For the latter purpose a dynamometer invented by Professor Hertz[1] was found to be, with some modifications, very well adapted. The instrument is shown in Fig. 1. A german silver wire, *ab*, 60 cm. long and 0.12 mm. in diameter, is divided at its middle point, and the two portions soldered to opposite sides of a short steel wire, *cd*, 0.86 mm. in diameter. To each end of the thick wire, *cd*, is soldered a thin steel wire the free ends of which are attached, the lower to a fixed support at *e*, the upper to a torsion head, *T*, by means of which the long wire, *ab*, is kept constantly taut. The passage of a current through *ab* causes it to expand, allowing the wire, *cd*, to turn upon its axis in the direction of the torsion. A mirror, *m*, is attached to *cd*, the deflections of which are observed by means of a lamp and scale placed at a distance of about 2 m. from *m*. A lens of about 2 m. focal length, placed just in front of *m*, gives a sharp image of the flame upon the scale.

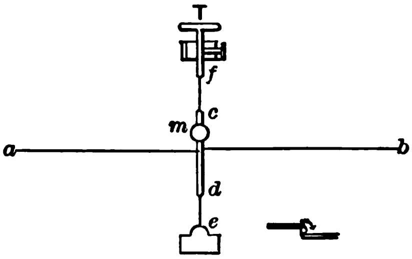

Fig. 1.

With interrupted currents the alternate heating and cooling of the wire produces small oscillations in the mirror, which, while blurring the image to such an extent as to exclude the use of a reading telescope, are not sufficiently great to prevent accurate readings with the lamp and scale. The current obtained by connecting one Planté element in the dynamometer circuit gave a deflection of 250 scale divisions. The deflection ordinarily obtained in making determinations varied from 100 to 250.

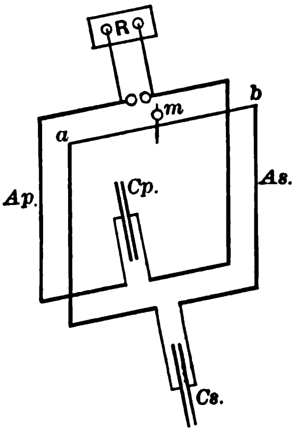

Fig. 2.

The arrangement of the apparatus as a whole is shown in Fig. 2. The primary circuit, A_p, is a quadrangle of copper wire 60 cm. square, the wire being 1 mm.

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[1] Zeitschrift für Instrumentenkunde, III. p. 17, 1883.

in diameter. The secondary circuit, A_s, has the same dimensions and is placed 15 cm. distant from A_p with its sides parallel to it. The side of A_s which lies opposite the spark gap in A_p is replaced by the dynamometer.

The primary condenser, C_p, Fig. 3, consists of two circular plates of zinc, each 40 cm. in diameter, one of which is attached to a fixed support, the other to a support which moves smoothly upon two stout glass tubes, R, R, and is provided with a pointer, z, and millimeter scale, m. As employed in this research it was not essential that the plates be exactly parallel; it was, however, necessary that they remain always in the same relative position for the same scale reading.

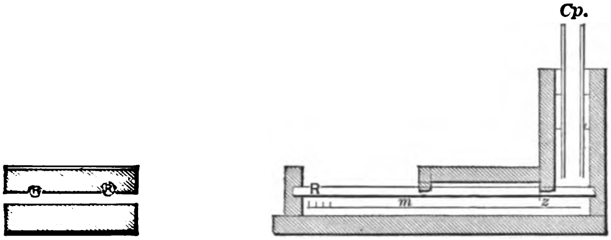

Fig. 3.

The secondary condenser, C_s, was a Kohlrausch condenser, with plates of 15 cm. diameter, and was provided with adjusting screws and a scale and vernier reading to tenths of a millimeter.

A table was computed for this condenser for all values of a from 0.1 mm. to 30 mm. by Kirchhoff's[1] formula:

$$C = \frac{R^2}{4a} + \frac{R}{2\pi}\left(\log\frac{8\pi R}{a} - 1\right),$$

where R is the radius and a the distance apart, of the plates. The values run from 149.59 at 0.1 mm. to 8.49 at 30 mm. To save time, the capacity of C_s corresponding to each position of C_p was determined once for all. With C_p set, for example, at 73 mm., C_s was adjusted to show a maximum scale reading, the distance a, read and recorded, and the observation repeated. Table I. shows a series of readings taken at random, and will serve as an indication of the accuracy of the observations in general.

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[1] Kirchhoff, Electricität, p. 109.

TABLE I.

a_s	80	84	89	84	80	89	90	Mean 85
C_s	23.07	22.35	21.19	22.35	23.07	21.19	20.95	22.12

In order to obtain these values, which enter into all following observations, with the greatest possible accuracy, the individual values were plotted in a curve, and a mean thus obtained of the series.

Table II., and the corresponding curve in Fig. 4, shows the sensitiveness with which a maximum deflection is indicated. The first line gives the distances, the second the deflections with condenser III., containing turpentine as dielectric, in the secondary circuit.

TABLE II.

Distance.	35	40	45	50	55	60	62	64
Deflection.	80	102	132	150	165	182	195	200
Distance.	66	70	75	80	90	100	110	130
Deflection.	195	181	162	141	105	86	70	50

In the case of substances with a perceptible conductivity, the deflection was smaller. No determination was attempted when the deflection fell below 30 scale divisions. It is, however, to be borne in mind that the conductivity of the dielectric is independent of its capacity; it diminishes, nevertheless, the exactness of the determination by decreasing the magnitude of the quantity we are to measure, — the deflection of the dynamometer.

Owing to the proximity of the secondary circuit, the primary circuit is noticeably affected by changes in the secondary. When the current breaker is well adjusted, the approach of the secondary circuit to the condition of resonance is often indicated by a change in the note emitted by the current breaker. The vibrations become more regular as resonance is approached, and again irregular as soon as the point of maximum effect is passed. An instrument that should respond with sufficient certainty to this reaction of the secondary circuit upon the primary might be made a very delicate means of detecting resonance.

It remains to describe the condensers used for determining specific capacities. Solids were examined in plate form clamped between two circular plates of metal varying in diameter from 3 to 12 cm., the capacities being computed by Kirchhoff's formula given above. The specific inductive capacity, *K*, of the substance is, of course, equal to the ratio between the capacity of the condenser with the given dielectric between the plates and the capacity of the same condenser in air as computed by the formula.

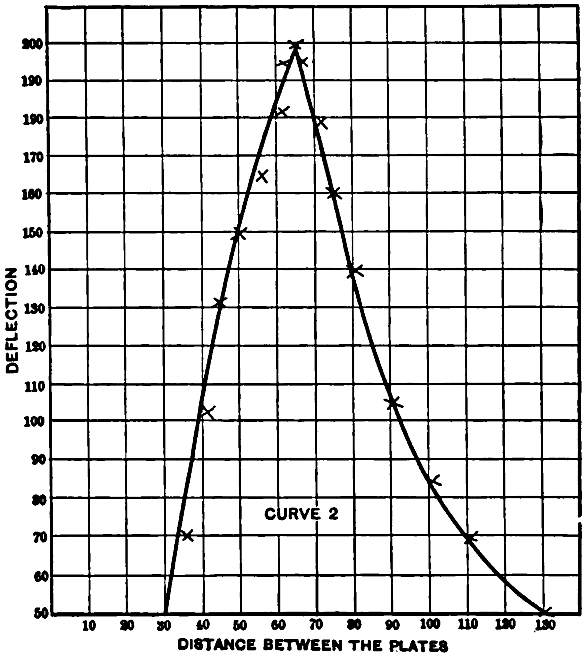

Fig. 4.

The first determinations of liquids were made with a condenser consisting of two balls supported at the middle of a glass cylinder upon wires admitted through the sides of the cylinder. The disadvantage of this plan was the large amount of liquid required. It was therefore abandoned for the form shown in Fig. 5. The liquid is here contained between that portion of two concentric cylinders in which the field of force is uniform, the space at the ends of the tube being filled with ebonite plugs which are screwed into the cylinders to a depth of 5 mm. The stopper, *s*, which fits flush with the inside of the cylinder, is hollow and has an opening in the bottom 1 mm. in diameter to allow the surplus liquid to escape. In this form the condenser offers no exposed liquid surface for the conduction of an air discharge between the two cylinders.

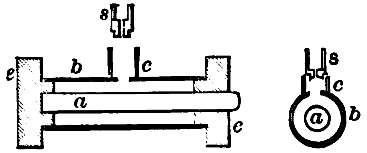

Fig. 5.

The capacity C_e of the ends is determined by measuring the capacity of the whole condenser in air and deducting the computed capacity of the central portion. The latter is computed by Maxwell's[1] formula:

$$C=\frac{1}{2}\frac{l}{\log\frac{b}{a}},$$

where *l* is the length of the cylinders, *b* the radius of the outer, and *a* of the inner cylinder.

We then have:

$$K=\frac{C'-C_e}{C}$$

For the determination of substances having widely varying specific capacities three condensers of the dimensions shown in Table III. were employed. It will be seen that most substances could be measured by two of these condensers. The amount of liquid required, as shown in column 3, was never more than 3.5 cc., — a very important item when a large number of substances are to be examined.

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[1] Maxwell, Electricity and Magnetism, Vol. I. p. 177.

TABLE III.

No.	l	Volume.	b	a	C	C_e	For measuring capacities.
I.	1	1.55	7.15	1.25	0.287	8.0	From $K = 15$ to $K = 100$
II.	2	2.45	7.15	3.48	1.388	10.8	From $K = 5$ to $K = 25$
III.	4	3.37	7.15	5.04	5.5998	11.5	From $K = 1$ to $K = 15$

Owing to the difficulty of obtaining many solid substances in the form of plates, I was led to adapt the cylindrical condensers to the measurement of solids as well as liquids. The condenser was first weighed, then filled with the solid substance in the form of powder, and again weighed. The space between the solid particles was then filled with a liquid in which the solid is insoluble, usually alcohol or ether. The volume of the substance is computed from its known specific weight, and this volume subtracted from the volume of the condenser gives the volume of the liquid. The specific capacity of the mixture is equal to the volume of the liquid times its capacity plus the volume of the solid times its capacity, divided by the total volume of the mixture; or,

$$K_m = \frac{V_l K_l + V_s K_s}{V_m}$$

whence,

$$K_s = \frac{V_m K_m - V_l K_l}{V_s}.$$

The Results.

The results obtained are shown in the following tables. Those given in the first half of Table IV. were obtained with the solid substance in the form of plates; those in the second half were obtained by means of the cylindrical condensers.

The first value for ice is entitled to less weight than the second, since the temperature of the room was 0°, so that the formation of a film of water on the plates was not impossible. In the second case the condenser was immersed in an oil bath which was kept for half an hour at a temperature of − 5°.

The determinations were made in all cases where the temperature is not given, at a temperature of 15° C.

The value given for sugar in liquid form was obtained by extrapolation from three aqueous and two alcoholic solutions. Honey, a substance of indefinite chemical constitution, but containing always some cane sugar, was found to have a specific capacity of 35. Tartaric acid was determined from an alcoholic solution.

Table IV.

SOLIDS.

Substance.	Plates.		K	Substance.	Mixture in cylinder.	K
	a	R				
Paraffine . .	0.42	4.05	2.14	Ice, -5°		2.85
	0.66	4.05	2.18	Cetylalcohol . . .		6.42
Ebonite . . .	0.42	4.05	2.71	Acetophenone . .		16.24
Wood, oak . .	2.64	6.00	2.93	Glacial acetic acid .		2.79
" pine .	1.58	6.00	2.95	Sand	Ether	7.43
Paper . . .	0.6	4.05	2.82	Potassium chlorate .	"	6.18
Porcelain . .	0.8	4.05	4.20	" carbonate	"	5.62
Glass (mirror) .	0.1	4.05	5.84	" sulphate .	"	6.45
Sandstone . .	2.8	4.05	6.20	Copper sulphate . .	"	5.46
Marble (white)	1.93	4.05	6.13	" oxide . . .	Alcohol	18.10
" (black)	1.86	4.05	6.15	Iron oxide . . .	Ether	14.20
Iceland spar .	1.4	2.0	7.40	Barium nitrate . .	"	9.15
	1.4	2.5	7.34	" sulphate .	"	11.40
Rock salt . .	0.68	2.0	5.81	Lead carbonate . .	"	18.58
Gypsum . . .	0.54	2.0	5.61	" oxide . . .	Alcohol	25.90
Ice, -2° . .	1.50	4.05	3.36	" sulphide . .	"	17.92
Cane sugar . .	1.0	2.0	4.19	Cane sugar . . .	Ether	4.13

It seemed to me a matter of especial interest to examine a series of mixtures of alcohol and water in various proportions, as well as similar mixtures of other substances which mix readily with water in all proportions. The results are shown in Tables VI. to XI., and in the curves with corresponding numbers shown in Figs. 6 and 7. For mixtures of ethyl and methyl alcohol the value lies always under the mean value for the two bodies in a free state. For all the aqueous mixtures examined, each of the

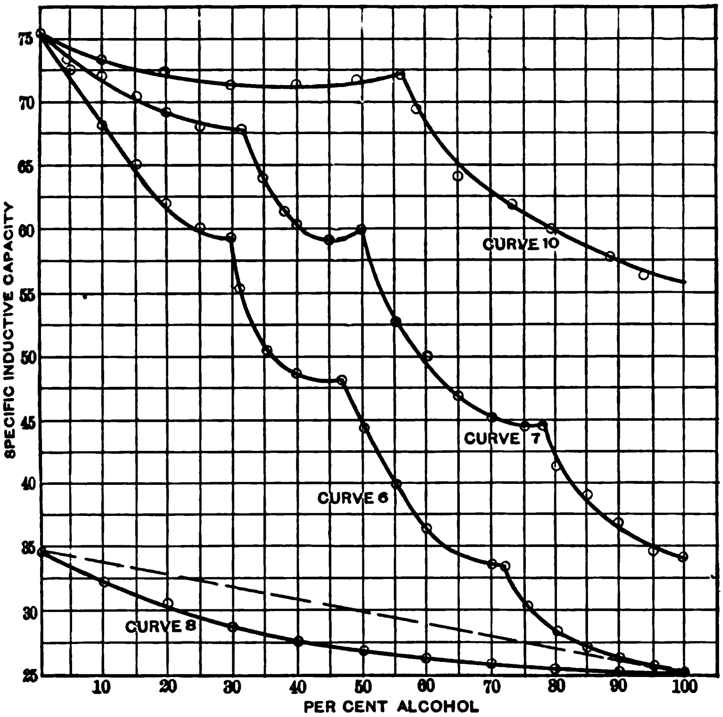

SPECIFIC INDUCTIVE CAPACITY
75
70
65
60
55
50
45
40
35
30
25
10
20
30
40
50
60
70
80
90
100
PER CENT ALCOHOL
CURVE 10
CURVE 7
CURVE 6
CURVE 8

Fig. 6.

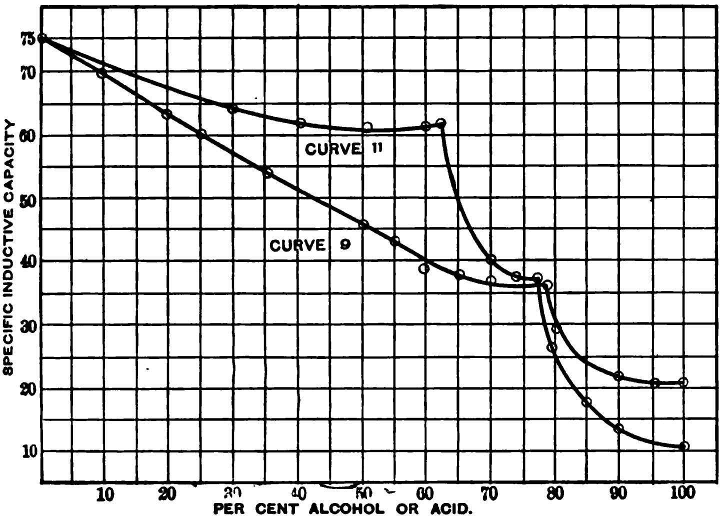

SPECIFIC INDUCTIVE CAPACITY
75
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
100
PER CENT ALCOHOL OR ACID.
CURVE 11
CURVE 9

Fig. 7.

TABLE V.

LIQUIDS.

Substance.	*K*	Substance.	*K*
Water	75.50	Aldehyde	18.55
Glycerine	56.20	Salicylaldehyde	19.21
Cane sugar (in water)	52.00	Propylaldehyde	14.41
" " (in alcohol)	55.00	Benzaldehyde	14.48
Methylalcohol	34.05	Valeraldehyde	11.76
Ethylalcohol	25.02	Cuminaldehyde	10.68
Propylalcohol	20.45	Chloral	5.47
Isopropylalcohol	19.82	Acetone	21.85
Amylalcohol	14.62	Methylethylketone	18.44
Allylalcohol	21.60	Acetylchloride	25.30
Tartaric acid	35.90	Methylpropylketone	16.75
Formic acid	62.00	Acetophenone	16.24
Lactic acid	20.90	Dipropylketone	12.44
Acetic acid	10.30	Methylhexylketone	10.42
Propionic acid	5.50	Nitrobenzole	32.19
Butyric acid	3.16	Nitromethane	56.36
Valeric acid	3.06	Nitrotoluole	26.58
Creosol	11.75	Ethylnitrate	17.72
Toluole	2.37	Ethylether	4.27
Turpentine	2.28	Chloroform	3.95
Carbon disulphide	2.50	Bromoform	7.42

curves shows one or more singular points, which fall, without exception, upon such percentages as correspond to a chemical combination of the substance with water.

In the case of propylalcohol and of acetic acid such hydrates are already known. The remaining substances give evidence of chemical union, either by absorbing water vigorously from the air or by contraction or the evolution of heat on mixing.

It is to be remarked, with reference to acetic acid, that, owing to the increase in conductivity on the addition of water, no exact determinations could be made of mixtures containing less than 30 per cent acid.

TABLE VI.

WATER AND ETHYLALCOHOL.

Per cent alcohol.	*K*	Corresponding hydrate.	Per cent alcohol.	*K*	Corresponding hydrate.
0.0	75.50		55.0	39.93	
5.0	72.30		60.0	36.31	
10.0	67.95		65.0	34.60	
15.0	65.36		70.0	33.66	
20.0	61.79		72.0	33.86	$C_2H_6O + H_2O$
25.0	60.21		75.0	30.30	
30.0	59.55	$C_2H_6O + 6\,H_2O$	80.0	28.15	
32.0	55.20		85.0	26.58	
35.0	50.52		90.0	25.71	
40.0	48.40		95.0	25.27	
46.0	48.40	$C_2H_6O + 3\,H_2O$	99.8	25.02	
50.0	44.11				

TABLE VII.

WATER AND METHYLALCOHOL.

Per cent methyl-alcohol.	*K*	Corresponding hydrate.	Per cent methyl-alcohol.	*K*	Corresponding hydrate.
0.0	75.50		50.0	57.14	
5.0	73.24		55.0	52.51	
10.0	72.30		60.0	50.00	
15.0	70.73		65.0	46.76	
20.0	69.20		70.0	45.16	
25.0	67.94		75.0	44.11	
30.8	67.94	$CH_4O + 4\,H_2O$	78.0	44.62	$2\,CH_4O + H_2O$
35.0	64.14		80.0	40.98	
37.0	61.50		85.0	38.05	
40.0	60.22		90.0	36.30	
45.0	59.41		95.0	34.05	
47.5	60.22	$CH_4O + 2\,H_2O$	99.7	34.05	

Table VIII.

METHYLALCOHOL AND ETHYLALCOHOL.

Per cent ethylalcohol.	*K*	**Per cent ethylalcohol.**	*K*
0.0	34.05	60.0	26.20
10.0	31.91	70.0	25.71
20.0	30.21	80.0	25.34
30.0	28.54	90.0	25.36
40.0	27.59	100.0	25.02
50.0	26.78		

Table IX.

WATER AND PROPYLALCOHOL.

Per cent propyl-alcohol.	*K*	**Hydrate.**	**Per cent propyl-alcohol.**	*K*	**Hydrate.**
0.0	75.50		65.0	37.52	
10.0	69.50		70.0	36.87	
20.0	62.80		77.0	36.00	$C_3H_8O + H_2O$
25.0	60.08		80.0	28.92	
35.0	53.35		90.0	21.60	
50.0	44.63		95.0	20.73	
55.0	42.00		99.8	20.45	
60.0	38.75				

Table X.

WATER AND GLYCERINE.

Per cent glycerine.	**K**	**Hydrate.**	**Per cent glycerine.**	*K*	**Hydrate.**
0.0	75.50		58.0	69.20	
10.0	73.24		65.0	64.14	
20.0	72.30		73.0	61.79	
30.0	71.36		80.0	59.40	
40.0	71.36		89.0	57.80	
48.5	71.46		94.0	56.54	
56.0	72.30	$C_3H_8O_3 + H_2O$			

Table XI.

WATER AND ACETIC ACID.

Per cent acetic acid.	*K*	Hydrate.	Per cent acetic acid.	*K*	Hydrate.
0.0	75.50		70.0	39.48	$C_2H_4O_2 + H_2O$
30.0	64.20		74.0	36.62	
40.0	61.79		77.0	36.87	
50.0	60.80		80.0	22.90	
60.0	60.80		100.0	10.30	
62.5	61.25	$C_2H_4O_2 + 2\,H_2O$			

Table XII., and the corresponding curve in Fig. 8, shows the influence of temperature upon the specific capacity of water, concerning which I will speak again farther on.

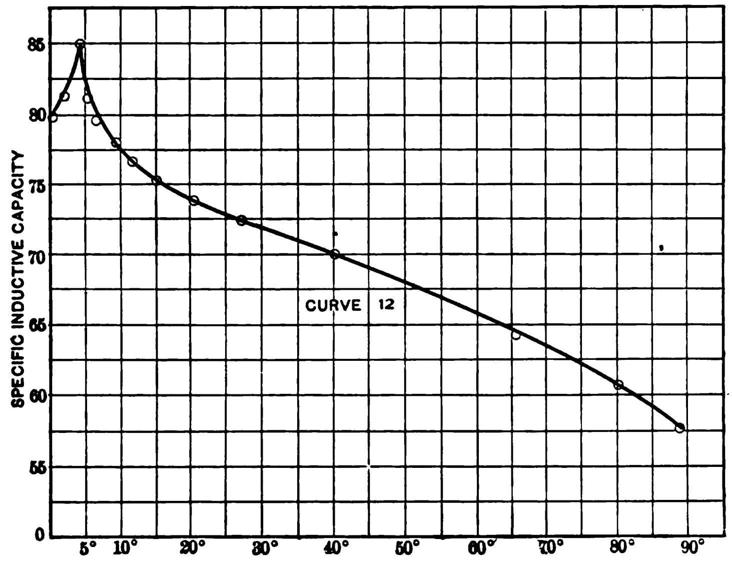


Fig. 8.

TABLE XII.

WATER.

Temperature.	K	Temperature.	K
0°	79.46	15°	75.50
2°	80.84	20°	73.92
4°	85.20	27°	72.70
6°	80.84	40°	69.80
7°	79.40	65°	64.32
9°	77.95	80°	60.50
12°	76.20	88°	57.90

The evident connection between the dielectric constants of the alcohol series led me to seek a relation which might be expressed as a function of the chemical constitution of the dielectric. The variations produced by temperature, and the high value found for sugar and glycerine, as well as the difference between the two propylalcohols, made it probable that the specific capacity varies directly with the density. It was further clearly noticeable that the members of the series containing a large proportion of the atom group hydroxyl had relatively large valves.

By way of trial I assumed the following formula: —

$$K=\frac{D}{M}(a_1K_1+a_2K_2+\cdots),$$

where D is the density, and M the molecular weight of the substance, K_1, K_2, etc., constants for each element or atom group, and a_1, a_2, etc., the number of atoms of the element in the molecule. I determined first the values for hydrogen, K_H, and carbon, K_C, from such substances as contained only those elements. The values were so nearly in the ratio of the atomic weights of the substances that that ratio was adopted. Calling, then, $K_H=2.6$, $K_C=31.2$, that value for hydroxyl ($K_{(OH)}=1356$) was taken, which satisfies the equation for water. The closeness with which the equation was satisfied for all the other liquids containing the hydroxyl group was so remarkable that I proceeded to examine the aldehydes, acetones, etc. The computed and observed values are given in parallel columns in Table XIII.

TABLE XIII.

$$K=\frac{D}{M}(a_1K_1+a_2K_2+\cdots)$$

$K_H = 2.6$	$K_{(OH)} = 1356$	$K_{(CH_2)} = 41.6$
$K_C = 2.6 \times 12$	$K_{(CO)} = 1520$	$K_{(CH_3)} = 46.8$
$K_O = 2.6 \times 16$	$K_{(COH)} = 970$	$K_S = 2.6 \times 32/2$
$K_X = 2.6 \times M_X$	$K_{(NO_2)} = 3090$	

Substance.	Formula.	*D*	*M*	*K* computed.	*K* observed.	Observer.
Water	H.OH	1.0	18.0	75.50	75.50	Thwing
					75.70	Rosa
					76.00	Cohn & Arons
					73.50	Cohn
					83.70	Tereschin
					69.00	Yule
Glycerine	$C_3H_5(OH)_3$	1.26	92.0	57.17	56.20	Thwing
Cane sugar	$C_{12}H_{14}(OH)_8O_3$	1.588	342.0	53.67	53.50	"
Methylalcohol	$CH_3.OH$	0.796	32.0	34.38	34.05	"
					32.60	Tereschin
Ethylalcohol	$C_2H_5.OH$	0.8	46.0	25.00	25.02	Thwing
					26.20	Mean: 5 observers
Propylalcohol	$CH_3(CH_2)_2.OH$	0.82	60.0	20.32	20.45	Thwing
-iso		0.7879	60.0	19.51	19.82	"
Amylalcohol	$C_4H_9(CH_2)(OH)$	0.8296	88.0	14.48	14.62	"
					15.00	Cohn & Arons
Allylalcohol	$C_3H_5.OH$	0.85	58.0	21.54	21.60	Thwing
Cetylalcohol	$C_{16}H_{33}.OH$	0.818	242.0	6.56	6.42	"
Tartaric acid	$C_2H_2(OH)_2(CO_2H)_2$	1.75	150.0	35.02	35.90	"
Lactic acid	$CH.OH.CH_3.CO_2H$	1.215	90.0	20.97	20.90	"
Creosol	$C_6H_3.OH.OCH_3.CH_3$	1.0894	138.0	12.63	11.75	"
Formic acid	OH.COH	1.223	46.0	61.88	62.00	"
Aldehyde	$CH_3.COH$	0.807	44.0	18.55	18.55	"
Salicylaldehyde	$C_6H_4.OH.COH$	1.172	132.0	20.46	19.21	"
Propylaldehyde	$C_2H_5.COH$	0.866	58.0	14.52	14.41	"
Benzaldehyde	$C_6H_5.COH$	1.063	106.0	11.70	14.48	"
Valeraldehyde	$CH_3(CH_2)_3COH$	0.884	86.0	11.92	11.76	"
Cuminaldehyde	$C_6H_4(C_3H_7)COH$	0.9832	148.0	8.37	10.68	"
Chloral	$CCl_3.COH$	1.549	147.5	13.43	5.47	"
Acetone	$CH_3.CO.CH_3$	0.792	58.0	21.85	21.85	"
Methylethylketone	$CH_3CO.C_2H_5$	0.8125	72.0	18.42	18.44	"
Acetylchloride	$CH_3.CO.Cl$	1.138	78.5	24.06	25.30	"
Methylpropylketone	$CH_3.CO.C_3H_7$	0.828	86.0	16.06	16.75	"

TABLE XIII. (*continued*).

Substance.	Formula.	*D*	*M*	*K* Computed.	*K* Observed.	Observer.
Acetophenone	$CH_3.CO.C_6H_5$	1.032	120.0	15.08	16.24	Thwing
Dipropylketone	$C_3H_7.CO.C_3H_7$	0.82	114.0	12.52	12.44	"
Methylhexylketone	$CH_3.CO.C_6H_3$	0.818	128.0	11.41	10.42	"
Nitrobenzole	$C_6H_5NO_2$	1.20	123.0	32.19	32.19	"
Nitromethane	$CH_3.NO_2$	1.096	61.0	56.35	56.36	"
Nitrotoluole	$C_6H_4.CH_3.NO_2$	1.136	137.0	27.38	26.58	"
Ethylnitrate	$C_2H_5.O.NO_2$	1.132	91.0	35.30	17.72	"
Valeric acid	$CH_3(CH_2)_3CO_2H$	0.9577	102.0	3.15	3.06	"
Butyric acid	$CH_3(CH_2)_2CO_2H$	0.9587	88.0	3.20	3.16	"
					3.00	Tereschin
Toluole	$C_6H_5.CH_3$	0.886	92.0	2.37	2.37	Thwing
					2.36	Palaz
					2.30	Thomasewski
					2.37	Landolt & Jahn
Metaxylole	$C_6H_4(CH_3)_2$	0.878	106.0	2.41	2.38	Negreano
Paraxylole	$C_6H_4(CH_3)_2$	0.862	106.0	2.37	2.38	Thomasewski
					2.23	Landolt & Jahn
Carbon disulphide	CS_2	1.292	76.0	2.47	2.50	Thwing
					2.61	Palaz
					2.60	Quincke
					2.30	Weber
Sulphur	S	1.98	32.0	2.58	2.69	Mean: 12 observers
Gypsum	$CaSO_4$	2.30	232.0	5.60	5.61	Thwing
Copper sulphate	$CuSO_4$	2.274	249.5	5.51	5.46	"
Barium sulphate	$BaSO_4$	4.47	233.0	11.20	11.40	"
Lead sulphate	$PbSO_4$	6.38	302.0	15.72	15.80	"
" sulphide	PbS	7.30	238.4	17.91	17.92	"
Potassium sulphate	K_2SO_4	2.65	174.0	6.25	6.45	"

In the case of solids and of such liquids as contain no atom groups the atomic capacities seem to be in the ratio of the atomic weights, so that the formula takes the simple form $K=\kappa D$, where $\kappa = K_H$.

The values given in Table XIV. are reckoned by the formula $K=2.6D$. Sulphur requires the value $K_s = M\kappa/2$, instead of $M\kappa$, and is therefore entered with its compounds in Table XIII.

So close an agreement in the case of such a large number of substances can hardly be the result of chance.

TABLE XIV.

$K = 2.6\,D$

Substance.	Formula.	D	K		Observer.
			Computed.	Observed.	
Benzole	C_6H_6	0.898	2.32	2.31	Mean: 9 observers
Octane	C_8H_{18}	0.718	1.87	1.93	Landolt & Jahn
Decane	$C_{10}H_{22}$	0.746	1.94	1.97	" " "
Petroleum	—	0.8	2.08	2.06	Mean: 10 observers
Turpentine	$C_{10}H_{16}$	0.856 0.87	2.23 2.26	2.23	" 12 observers
Citron oil	—	0.853	2.22	2.25	Thomasewski
Vaseline oil	—	0.863	2.24	2.17	Hopkinson
Chloroform	$CHCl_3$	1.526	3.97	3.95	Thwing
Bromoform	$CHBr_3$	2.83	7.36	7.42	"
Paraffine	—	0.87 to 0.91	2.26 2.36	2.18	"
				2.32	Boltzmann
				1.99	Gordon
				2.28	Hopkinson
				2.17	Winkelmann
				2.31	Donle
Ebonite	—	0.97	2.52	2.53	Mean: 12 observers
Porcelain	—	1.739	4.52	4.20	Thwing
Glass (mirror)	—	2.4	6.24	5.84	"
Sandstone	—	2.36	6.18	6.20	"
Sand	—	2.8	7.28	7.43	"
Marble	$CaO.CO_2$	2.65	6.89	6.14	"
Iceland spar	$CaO.CO_3$	2.75	7.20	7.35	"
Rock salt	$NaCl$	2.16	5.61	5.81	"
Cane sugar	$C_{12}H_{22}O_{11}$	1.58	4.12	4.16	"
Ice	H_2O	0.92	2.50	2.85	"
Glacial acetic acid	$C_2H_4O_2$	1.06	2.76	2.79	"
Potassium chlorate	$KClO_3$	2.33	6.06	6.18	"
" carbonate	K_2CO_3	2.29	5.95	5.62	"
Copper oxide	CuO	2.65	6.25	6.45	"
Iron oxide	FeO	5.12	13.31	14.20	"
Barium nitrate	$Ba(NO_3)_2$	3.23	8.39	9.15	"
Lead carbonate	$PbCO_3$	6.57	17.08	18.58	"
" oxide	PbO	9.29	25.15	25.90	"

Water, acetic acid, and sugar behave in the solid form exactly as if the atom groups to which they seem to owe their high specific capacity in the liquid form were as such no longer present.

If we assume this to be the case, the remarkable variation of water with the temperature as well as the anomalous behavior of alcoholic mixtures could be explained on the supposition that a complete association of the elements to hydroxyl occurs only at a definite temperature and under conditions of absolute purity. The thought at once suggests itself that where the singular points occur in the mixture curves, a complete chemical union takes place, and we are dealing, not with a mixture, but with a pure substance.

If we bear in mind that hydroxyl is nearly one and one-half times as dense as water, and does not freeze at a temperature of −30°, our hypothesis will explain the density maximum of water at 4°, and the fact that water may be cooled to a point several degrees below the freezing-point without its freezing, the effect of the jar, which finally causes the water to freeze, being to set in motion the dissociation of the hydroxyl.

The complete establishment of the hypothesis here put forward requires, of course, a larger volume of experimental facts than are yet at hand. The extremely simple method, however, — which I trust will be made still more exact by others, if not in further experiments of my own, — offers a ready means of obtaining the necessary facts on which to base a theory of dielectrics. The method is but one of many applications which might be made of the beautiful discovery of Hertz to the solution of electrical problems.

MINOR CONTRIBUTIONS.

The Physical Laboratory of Adelbert College.

By Frank P. Whitman.

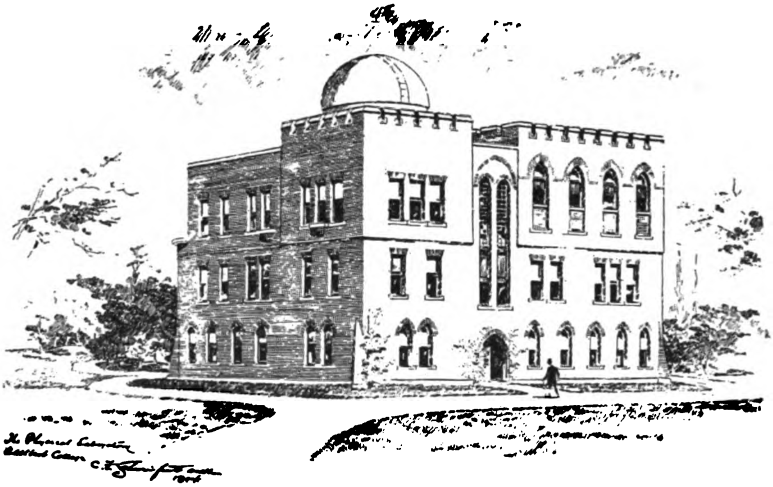
THE Physical Laboratory of Adelbert College, now in process of construction, is a building measuring about eighty feet by sixty, and three stories in height. It has no basement, the floor of the first story being just above the level of the ground. It is built of hard-burned brick set in black mortar. The construction is of the so-called "mill" or slow-burning type, but with unusually heavy timbers and floors. The building is to be heated and ventilated by a blower, forcing air through brick conduits. The steam to drive the blower, heat the air, and supply power, will be taken from another building. By this method of heating, and the construction, the presence of iron in the building is almost completely avoided.

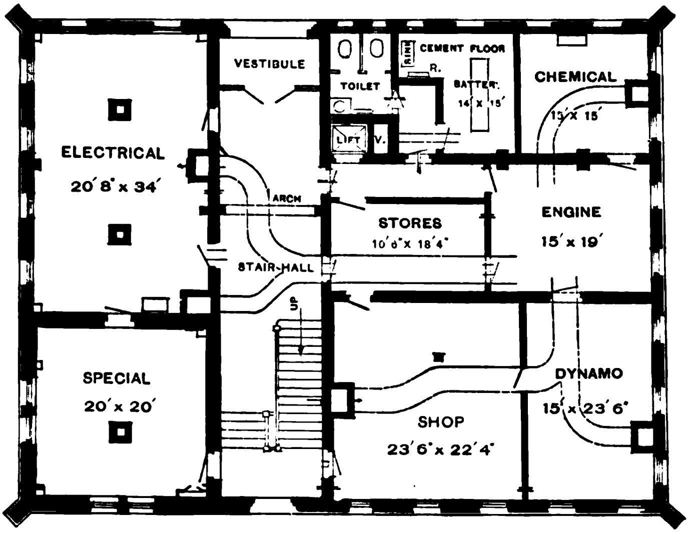
The stairway hall divides the building into two parts, the one on the north containing the large rooms necessary for the elementary classes, the southern portion divided into smaller parts for more advanced work. In this part of the building a tower twenty feet square with heavy walls adds to the general stability of the structure, and is arranged to support, if required, a telescope and dome. It is not, however, certain that this feature will be added at present.

The Main Laboratory, on the second floor, is made long and narrow to secure good lighting. It will accommodate forty students or more. Tables fastened on wall-brackets will run around three sides of the room. A large stone slab (marked "Table" on the plan), flush with the floor, resting on a pier, will afford additional support for apparatus requiring steadiness.

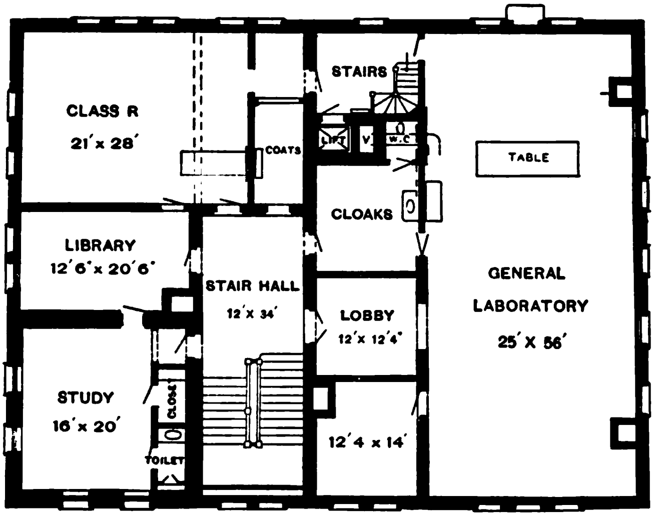
A small stairway ascends from this room to the cabinet of lecture apparatus above, making it easy to transfer instruments when required from floor to floor. Most of the elementary apparatus will be stored in the Laboratory and the small rooms adjoining.

The Lecture Room is seventeen feet high, and will accommodate about two hundred. It is lighted by four windows and a large skylight, which will be darkened, when necessary, by light opaque screens, running in grooves

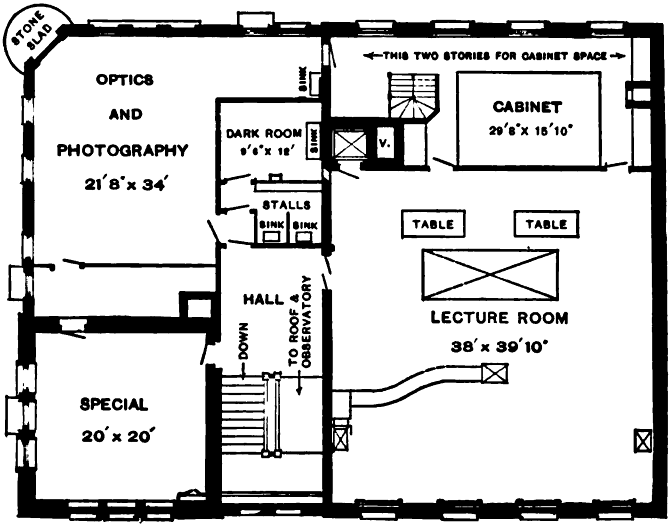




Ground Story.



Second Story.



Third Story.

like a window sash, and controlled by mechanism at the lecture table. The apparatus cabinet, just behind the Lecture Room, is divided into two stories, each eight feet high, connected by a short stairway. All shelves are thus brought into reach from the floor.

The corner looking directly south (the building faces southwest) is cut off in this story to make room for a large window with shelf for heliostat.

The Laboratory will be substantially constructed throughout, and well though plainly finished. The architect is C. F. Schweinfurth of Cleveland. The building, with necessary furnishings, is the gift of Mr. Samuel Mather.

A LABORATORY EXPERIMENT IN SIMPLE HARMONIC MOTION.

BY JOHN O. REED.

ALTHOUGH the interest in the study of simple harmonic motion is a constantly increasing one, yet simple and practical laboratory experiments in this subject are by no means common. The following is offered as likely to prove useful and suggestive: —

An elastic spiral, if suspended by one end and carrying a heavy ball attached to the other, will, if pulled down and released, execute approximately simple harmonic vibrations whose period may be accurately computed. The experiment consists in comparing the computed period of the oscillations of such a system with that determined by direct observation. If we neglect the weight of the spiral and assume Hooke's law to hold throughout the motion, we may derive our working formula as follows:[1] Letting b be the extension of the spring due to the weight W, c its extension at the beginning of the motion, x the extension at any time t, S the corresponding tension of the spring, then, by Hooke's law, we have

$$S = \frac{W}{b}x.$$

The differential equation of motion is

$$M\frac{d^2x}{dt^2} = W - S,$$

or

$$\frac{d^2x}{dt^2} + \frac{g}{b}(x - b) = 0.$$

The solution of this, after determining the constants, is

$$x = b + (c - b)\cos\sqrt{\frac{g}{b}}\,t.$$

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[1] Williamson's Dynamics, p. 157.

This represents a simple harmonic motion, the period of which is clearly

$$T = 2\pi\sqrt{\frac{b}{g}}.$$

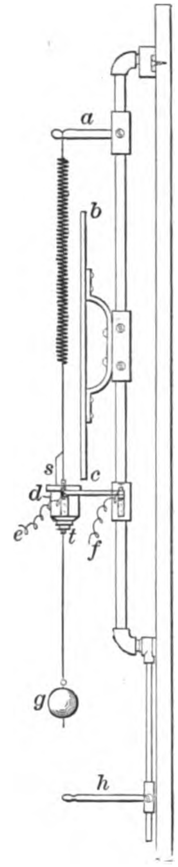

Fig. 1.

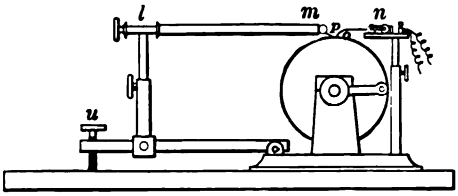

Fig. 2.

Hence for laboratory purposes we need to measure simply the extension b due to the weight W in order to compute the period of the oscillation, if we suppose the value of g to be known. For the determination of the period T by observation, the following method was adopted: A piece of $\frac{5}{8}$ inch gas-pipe, one meter long, has at its upper end a movable clamp a, Fig. 1, which supports the spiral spring in front of a mirror scale bc, etched in millimeters for reading off directly the length of the spring when free, and when stretched by the ball g. The difference between these readings gives the value b in the formula. The wire connecting the ball to the spring passes through a cylindrical mercury cup d by means of a small glass tube t, around which the mercury in d forms a ring about one centimeter in width and six centimeters deep. Electrical contact is made by means of the light steel wire s dipping into the mercury whenever the spring is extended beyond the stretched length due to the ball g. A tuning-fork, Fig. 2, a cylinder covered with smoked paper and provided with an electric scriber o, actuated by the current which is interrupted at the contact in d, complete the outfit. To use the apparatus, the spring is drawn down and tied by a fine cotton thread passed through the ring in the under side of the ball g and around the arm h, which must be so adjusted as to allow the wire to clear the sides of the glass tube through which it passes. When the ball has come to rest, the fork is set in vibration, the thread burned off, and the cylinder turned steadily while the ball oscillates. In practice it is found convenient to insert a key in the battery circuit by means of which the scriber may be allowed to record only such vibrations as are relatively free from the slight damping effect of the air and the little plunger s. The period of the tuning-fork, armed with its light tracing-point, having been first carefully determined, the fork is thereafter used as the measurer of time, and the number n of waves between two successive movements of the scriber o, caused by the wire s plunging into the mercury, multiplied by the period of the fork, gives the period T of the spring in seconds.

As may be seen from the formula

$$T = 2\pi\sqrt{\frac{b}{g}},$$

an error of one millimeter in the reading of b would, if b were five centimeters, produce an error of 0.0045 of a second in T. Only gross carelessness could produce such an error as this. In fact, with nothing but the mirror scale the value of b may be readily determined to one-tenth of one millimeter, corresponding to an error of 0.00045 of a second. With cathetometer readings, the accuracy of this part of the experiment might be considerably increased. With a tuning-fork making 256 vibrations per second, it is easy to determine time by means of the smoked paper to at least 0.0005 of a second, although very satisfactory results may be obtained from the use of a fork of twice this period.

Some idea of the accuracy of the method may be formed from the following results taken from students' laboratory note-books: —

SPRING NO. 1. — STEEL WIRE.

Period of fork $\frac{1}{65.5}$ sec.

$b = 6.81$ cm. $n = 34.26$.

T.

Computed, 0.5238 second.
Observed, 0.5230 "
Difference, 0.0008 "

SPRING NO. 2. — BRASS WIRE.

$b = 6.12$ cm. $n = 32.5$.

T.

Computed, 0.4964 second.
Observed, 0.4962 "
Difference, 0.0002 "

SPRING NO. 3. — BRASS WIRE.

$b = 5.02$ cm. $n = 29.53$.

T.

Computed, 0.4492 second.
Observed, 0.4508 "
Difference, 0.0016 "

The apparatus may also be used for the determination of *g*, although in this case the effect of slight errors of observation is obviously much greater than before. By furnishing the wire with a light tracing-point instead of the small wire *s*, and mounting the cylinder vertically, the same apparatus may be used to produce excellent specimens of damping curves, from which the retardation due to friction may be studied.

Physical Laboratory of the University of Michigan.

Some Experiments in Electric Photography.

By Fernando Sanford.

DURING the past three years, I have been occasionally engaged in some experiments upon electric photography which may be of interest to the readers of the *Review*, notwithstanding the fact that Professor F. J. Smith, in his recent announcement of his "Inductoscript," has published similar results to those which I have obtained.

In 1891 I developed a negative image upon a piece of bromide paper which had been placed between, and in contact with, two pieces of metal which were connected with the poles of two chromic acid cells. These negatives showed only the places of contact of the sensitized side of the paper and the one piece of metal. During the same year, I developed characters written upon a sensitized plate with the end of a wire, the other end of which was connected to one terminal of an induction coil. In September, 1892, I read the article upon Breath Figures, by Mr. W. B. Crofts, in the *Philosophical Magazine* of August, 1892, and it occurred to me at once that these figures might be developed upon a photographic plate. I at once tried the experiment of laying the coin upon the sensitized side of the plate, and connecting it with the terminal of a small induction coil, capable of giving a spark of three or four millimeters, while a piece of tin foil upon the opposite side of the plate was connected with the other terminal of the coil. From the very short notice which I have read of Professor Smith's Inductoscript, this is exactly the method which he is now using, except that he uses a higher potential and exposes the plate only a second, while I exposed the negatives which I made from half an hour to an hour.

Several negatives were made in this way, the accompanying photograph, No. 1, being from one of them. With one exception, they all show a fringe around them, due to the escape of the charge from the edge of the coin, which accounts for the formation of the dark ring observed around the breath figures made upon glass.

Last September I undertook to photograph in the same way objects insulated from the photographic plate, and have since made negatives of coins separated from the plate by paraffine, shellac, mica, and gutta percha. The accompanying photograph, No. 2, was made on Oct. 7, the coin being insulated from the photographic plate by a sheet of mica about 0.04 mm. thick. The mica was laid directly upon the film side of the plate, and the coin was placed upon it and connected to one terminal of the small induction coil already mentioned. A circular piece of tin foil of the circumference of the coin was placed upon the glass side of the plate directly opposite the coin, and was connected to the other terminal of the induction coil. The little condenser thus made was clamped between two boards, and was covered up in a dark room. Two small discharging knobs were also attached to the terminals of the induction coil, and were separated by a space of less than a millimeter, so that when a single cell was connected with the primary coil, the spark between the knobs seemed continuous.

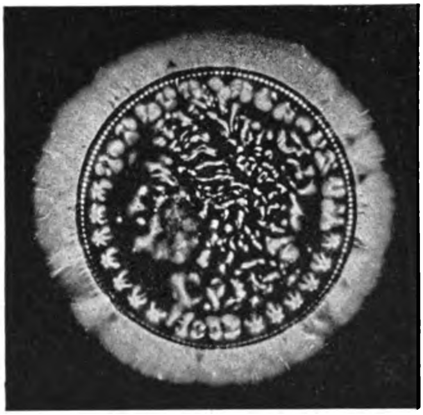

Fig. 1.

The plate was exposed to the action of the waves set up in this condenser for one hour, when it was taken out and the negative image developed upon it by the usual process. This photograph was made before I had heard of Professor Smith's process.

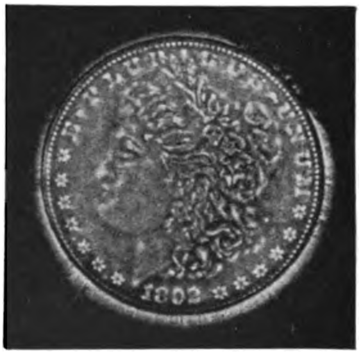

Fig. 2.

Several interesting facts are shown by these photographs, and it has seemed to me that the process might furnish a valuable method of studying some of the phenomena of condensers. The fact that the electric waves sent off by the condenser plates are propagated in a direction perpendicular to the plane of emergence is plainly shown by some of the negatives in my possession. The tendency for the charge to escape from the surfaces of greatest curvature is plainly shown, as in photograph No. 1. The surface of a coin which had been stamped with fine parallel lines gave no image upon the plate, on account of the dispersion of the waves from its surface. In the case of a medal with a head of Washington in high relief, the whole head was left blank, but was outlined by a fringe like that around the edge of photograph No. 1.

I did not succeed in making a good negative through a thickness of insulator as great as one millimeter, but blackened spots of the size of the coin were made through several millimeters of gutta percha, even with the low potential mentioned above. Unquestionably, with a greater sparking distance, the effect could be observed throughout a condenser of much greater thickness.

STANFORD UNIVERSITY, CAL.,
Jan. 6, 1893.

THE ELECTRICAL CONDUCTIVITY OF COPPER AS AFFECTED BY THE SURROUNDING MEDIUM.

(1) *A Reply to Professor Carhart.*

BY FERNANDO SANFORD.

IN the March-April number of the *Physical Review*, Professor Henry S. Carhart publishes the results of an investigation which he says was carried on under his supervision for the purpose of confirming or refuting some conclusions reached by me in regard to the dependence of the electrical conductivity of a copper wire upon the nature of the surrounding medium.[1] In this investigation reported by Professor Carhart, the conductivity of a copper wire was measured in air, in alcohol, and in kerosene, and no difference in its conductivity was observed; hence he regards my conclusions as refuted, especially as he claims for his measurements a much higher degree of accuracy than was possible with the apparatus which I used.

Without discussing the question whether a set of measurements made with a much finer wire than was used by me and in two dielectrics besides air would furnish a sufficient basis for denying the accuracy of my observations made, as my published papers will show, with much greater care and with fifteen different dielectrics besides air, I wish in this article merely to discuss Professor Carhart's claims to a more accurate method of observation, and to call attention to some serious discrepancies between his published data and the graphical representation of them which he has given in his article.

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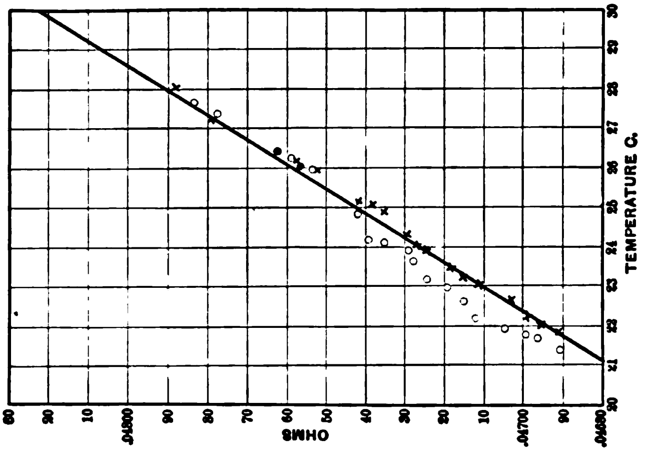
[1] Leland Stanford Junior University Publications, Studies in Electricity, No. 1; Phil. Mag., January, 1893, Vol. 35, No. 212, p. 65.



OHMS
.04680
90
.04700
10
20
30
40
50
60
70
80
90
.04800
10
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TEMPERATURE C.
20
21
22
23
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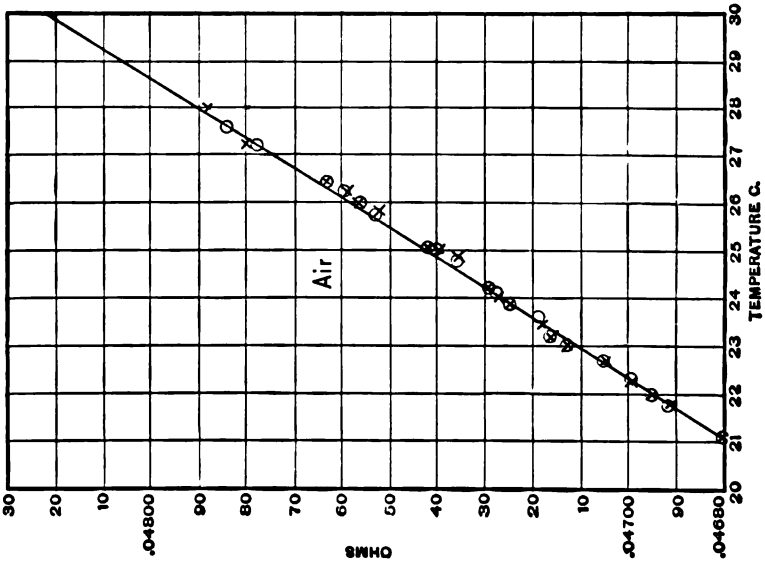
Fig. 1.



Air
OHMS
.04680
90
.04700
10
20
30
40
50
60
70
80
90
.04800
10
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30
TEMPERATURE C.
20
21
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27
28
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Fig. 2.

The general arrangement of the resistance to be measured was the same in Professor Carhart's investigation as in mine. A copper wire was stretched lengthwise through the middle of a copper tube, making electric contact with one end of the tube and passing out through an insulating stopper or plug in the other end, so that the current could be sent one way through the tube and return through the wire. The tube used by Professor Carhart was 86.3 cm. long. The tube used by me was 120 cm. long. The diameter of the wire used by Professor Carhart was 0.7 mm. The wire used by me in the experiments referred to by Professor Carhart was 1 mm. in diameter. The resistance of Professor Carhart's tube and wire at 21 c. was 0.0468 ohm, while the resistance of the tube and wire used by me was, at the same temperature, 0.338 ohm. It will thus be seen that the tube and wire used by Professor Carhart had a resistance of 140 per cent, and a surface (considering the wire alone) of only 50 per cent the tube and wire used by me.

The temperature was in both cases measured by a thermometer inserted into a side tube with its bulb in contact with the wire. In Professor Carhart's measurements, the thermometer was graduated to half degrees, while in my experiments a standard thermometer graduated to tenths was used. A change of temperature of one-tenth of a degree made, in Professor Carhart's wire, a change in resistance of 0.000016 ohm. The same temperature change made a change in resistance of 0.000012 ohm in the wire used by me.

The galvanometer described by me in the article referred to by Professor Carhart would not give a noticeable deflection upon reversing the current for a change of resistance of less than 0.00002 ohm, but by observing the deflections in opposite directions upon changing the resistance by 0.0001 ohm, the resistance was estimated to 0.00001 ohm. While a high degree of accuracy was not claimed for this method of estimating the value of the figure in the fifth decimal place, it would not compare unfavorably with the accuracy of estimating temperature to tenths of a degree with a thermometer graduated to half degrees. The complete notes published with my first article show that a variation of two in the fifth decimal place from the straight line representing the temperature curve of the wire was very rare, and that in no case could there be an uncertainty of one in the fifth decimal place as to the true position of the curve.

Professor Carhart seems to have made a special effort to secure accurate resistance measurements, and claims for his galvanometer and bridge combination a sensitiveness of at least two and one-half times as great as mine. To this part of his claim I have no objection to offer. Whether the resistance could be accurately measured to the fifth or the seventh decimal place is of no consequence. The fact remains that the smallest temperature variation measured by Professor Carhart's thermometer would correspond to a difference of resistance of 0.00008 ohm, and granting that he could estimate the temperature with a fair degree of accuracy to one-tenth of a degree, this would correspond to a difference of resistance of 0.000016 ohm. I accordingly fail to see wherein the apparatus employed by Professor Carhart had an advantage in sensitiveness over that employed by me. On the other hand, if the methods of measurement employed in the two investigations are considered, it will be seen that my method has the advantage of much greater accuracy than that described by Professor Carhart. Manifestly, the most uncertain factor in the whole measurement is the temperature of the wire. This is especially true when the tube contains air or another gas. What the thermometer really indicates is its own temperature, taken from the air very near the wire. If the temperature of the air is rapidly changing, this certainly is not the temperature of the wire. The thin wire would certainly take the temperature of the air more quickly than the mercury in the thermometer bulb, protected, as it is, by its non-conducting covering of glass. To guard against this possible difference in the temperature of the wire and the thermometer, my measurements were all made at the room temperature, and in a room where the total temperature change for twenty-four hours rarely amounted to 10°, and frequently did not reach 5°. They were also made at all hours of the day, and frequently late in the evening, with a period of several days (in one case a month) devoted to each set of measurements, so that approximately the same number would be made with a rising as with a falling temperature.

Professor Carhart's measurements were made by heating the tube up to about 30° with warm water, and then making a series of measurements as it cooled off. The published data would indicate that all the measurements were made with a falling temperature. With the tube filled with a liquid and cooled slowly enough, this method might be made to give sufficiently accurate temperature measurements, but with air in the tube, a much greater time of cooling would be necessary to give the same accuracy. Professor Carhart's published data show that he has not overcome this difficulty. The data given in Tables I. and II., page 326, are all the data he has seen fit to publish regarding his resistance measurements in air, and the two curves platted from them show no close relation to each other, either in direction or position, notwithstanding the fact that Professor Carhart says in referring to them, page 325, "There is no difference between them."

In order to show the high degree of accuracy of Professor Carhart's method somewhat better than it is shown in the curve given by him, I have platted accurately upon millimeter paper the points representing the resistances and the corresponding temperatures, as given in Tables I. and II., and have drawn through them a line representing the position of Professor Carhart's curve, Fig. 2, page 329. I have then copied these points upon tracing cloth laid over the ruled paper, and have photographed the figure thus given, reducing it to the scale of Professor Carhart's published diagram. The figure thus photographed is shown as Fig. 1 of the present article, while Fig. 2 is a reproduction of the diagram published by Professor Carhart as representing the same data. The circles, as in Professor Carhart's diagram, show the points taken from Table I., and the crosses the points taken from Table II. It will be seen that in Professor Carhart's diagram there are thirteen pairs of coinciding points out of the forty points given, while in reality there are but two such coinciding pairs given in the two tables. Beginning at the lowest temperature, the first seven pairs of points are made to coincide in Professor Carhart's diagram. Most of these points thus brought into coincidence were made at temperatures of about half a degree apart. The points given in Table I. frequently fall 0.00007 or 0.00008 ohm above the curve drawn, and one point, made at 22°.3, falls 0.00014 ohm above the curve.

And this is the air curve which Professor Carhart has found to coincide with all his other curves, thus refuting my conclusions.

STANFORD UNIVERSITY, CAL.
March 31, 1894.

(2) *Reply to Professor Sanford.*

BY HENRY S. CARHART.

The only part of Professor Sanford's criticism to which I wish to make particular reply, is that in which he plats the data of my Table I., and creates the impression that I have warped the data to suit my statements. The fact remains that Fig. 2 of my paper, reproduced by Professor Sanford, is entirely correct, or as nearly so as one should expect in so small a diagram. The error is in the table. The diagrams were plotted from the original data on engraved millimeter paper, and were transferred from that to the drawings for the engraver. But, I regret to say, the table has an error in it, made by Mr. Keeler in copying.

If Professor Sanford had run down the column from the top, he would have found the points right for the first six temperatures. Then the discrepancy begins. But if he had examined the diagram carefully, he would have found platted two interpolated temperatures not found in the table. The others then follow in order, and not half a degree, as he asserts, out of the way. The column of temperatures was copied with the omission of two values where the discrepancy begins, so that the resistances are two points out of step with the temperatures. Fortunately, Mr. Keeler has the original data. I reproduce the resistances and temperatures of Table I., together with the correct temperatures from which the small circles of Fig. 2 were platted. From this it will be seen that a blunder was made in copying the table, but the diagram is correct, and there was no warping of facts to suit any one's purpose.

R	*t*	Correct *t*
0.04784	27°.6	27°.6
78	.3	.3
63	26°.4	26°.4
59	.25	.25
57	.0	.0
54	25°.9	25°.9
42	21°.8	{ 25°.1 }
40	.2	{ 25°.0 }
36	.1	24°.8
29	23°.9	.2
28	.65	.1
25	.2	23°.9
19	.0	.65
16	22°.7	.2
13	.3	.0
05	.0	22°.7
0.04699	21°.8	.3
96	.6	.0
91	.3	21°.8
80	.1	.2

The numbers omitted I have enclosed in brackets. Hence Fig. 2 was correctly platted, and it remains the correct air curve, which I have found to coincide with all the other curves. If they refute Professor Sanford's conclusions, it is the refutation of facts.

As to the use of a thermometer reading to half degrees instead of tenths, the choice was made deliberately. If an error of a tenth, or even a half, degree were made on individual readings, the only effect would be to make the points scatter more on either side of the best line representing them; for on an average, as many readings would be too high as too low. But notwithstanding Professor Sanford's use of a tenth-degree thermometer, our points lie more nearly on a straight line than do his.

The observations were all made on a falling temperature, but the tube was heated by a large mass of water in a trough immediately beside it, and was covered over with waste or other poor conductor to prevent loss of heat by radiation and convection. It is not at all probable that the thermometer lagged appreciably behind the wire with a temperature falling only three or four degrees an hour; and if it did, it would be strange indeed if in every case this lag should be just sufficient to eliminate the difference between the air and alcohol or other curves, which Professor Sanford ascribes to the influence of the dielectric.

(3) *Final Reply to Professor Carhart.*

By Fernando Sanford.

I have read Professor Carhart's explanation of the discrepancy between his diagram and his published data. It is entirely satisfactory. If I created the impression in Professor Carhart's mind that he had warped his data, I did it by simply stating the fact that his data and the diagram did not correspond. I offered no explanation for the discrepancy. With the corrected data which Professor Carhart gives, the printed diagram corresponds closely enough for the purpose. If, as it now appears from the data, the two tables were made at the same time by merely reversing the current and taking two readings at the same temperature, I was wrong in taking one curve as a measure of the accuracy of the other. The coincidence of the points merely shows that the same galvanometer deflection and temperature were read twice alike.

The question of the refutation of my observations is not under consideration. How Professor Carhart can suppose that an observation which his students did not make can refute my statements about an observation which I did make, I am unable to understand. Granting that the wire used by Professor Carhart's students showed no difference in resistance in the dielectrics in which it was tested, it refutes nothing which I have said. Since the publication of my original paper, I have accumulated a much larger amount of data upon the same subject which I shall publish later. My only reason for replying to Professor Carhart's article was his assumption of superior accuracy in his work. I think I have shown that notwithstanding his special effort at refinement in resistance measurement and his deliberate choice of a thermometer graduated to half degrees, his apparatus was not capable of giving as accurate results as was the apparatus which I used, and his precautions against error were not nearly so great as were mine.

NOTE.

August Kundt. — The news of the death of Professor Kundt, Director of the Physical Laboratory in Berlin, who passed from this life at his country place in Israelsdorf, near Luebeck, on the 21st of May, 1894, comes to very many physicists in America as a message of personal bereavement. During his sixteen years as director of the laboratory in Strasburg, and during the eight years following upon that when he was at the head of Physical Institute of the University of Berlin, Kundt numbered among his pupils many Americans upon all of whom, it is safe to say, he made an impression never to be effaced.

August Adolf Eduard Eberhard Kundt was born on the 18th of November, 1839. In 1860 he was matriculated at the University of Leipsig as a student of mathematics and the sciences. In his third semester he entered at Berlin, where he completed his university career. In Berlin he became the pupil of Magnus, who gave him a place in his private laboratory. Kundt received the degree of Doctor of Philosophy at Berlin in 1864, the title of his thesis being *De Lumine Depolarisato.* He began his career as a gymnasial teacher in that city, and in 1867 he registered himself as privatdocent in the University. In the following year he was called to the Polytechnic Institute of Zürich as Professor of Physics, and in 1870 he became the successor of Clausius in the University of Wuerzburg. Upon the reestablishment of the University of Strasburg, at the close of the Franco-German war, he was made director of the Physical Laboratory, which position he held until 1888. In that year Von Helmholtz resigned the directorship of the Physical Laboratory in Berlin, and Kundt, under whose administration the Department of Physics in Strasburg had reached a position second to none in the empire, was called to be his successor. This position he held till the end of his life. To the further development of physics in Berlin, he gave the efforts of his ripest years. To it, indeed, it may be said, he sacrificed his life, for there can be no doubt that the increasing burden thus laid upon him hastened his death.

Kundt was above all an experimental physicist. He is best known to the scientific world through his investigations upon the velocity of sound upon the specific heat of mercury, a research which he carried out in collaboration with Warburg upon the polarization of light and upon the index of refraction of metals. The first of these is perhaps his most widely known work, his ingenious method of measuring wave-lengths by means of dust figures having become classical. It is noteworthy that this research was completed while he was still an undergraduate in the University of Berlin. Among those who were fortunate enough to be his pupils, Kundt will be remembered not only for his fertility in experimental research, but also for his contagious enthusiasm for all that related to physics, and his extraordinary skill as a demonstrator. One who had the privilege to sit under him during the last course of lectures which he gave says: "There was something inspiring and magnetic about his lectures which I have never noticed to the same extent with any one else. He seemed to address each hearer personally. In more direct personal intercourse, his power of arousing enthusiasm was still more marked. Best of all, he had a way of making one feel that his interest was a personal one and that he was a friend." These were the qualities which enabled Kundt to take up the directorship of the great laboratory in Berlin, which his illustrious predecessor, von Helmholtz, had laid down, extending its influence and stimulating the inmates to a pitch of performance unparalleled in the history of such institutions.

E. L. N.

NEW BOOKS.

Neudrucke von Schriften und Karten ueber Meteorologie und Erdmagnetismus. Von Prof. Dr. G. Hellmann.

No. 2. *Blaise Pascal. Récit de la Grande Expérience de l'Équilibre des Liqueurs. Paris,* 1648. Facsimiledruck mit einer Einleitung. 4to. pp. 10, 20. Price, 3 marks (75 c.). Berlin, A. Ascher & Co., 1893.

In the volume before us, which forms No. 2 of the excellent and beautifully executed series of reprints in facsimile of epoch-making books and charts edited by that eminent meteorologist and bibliographer, Professor Hellmann, we have a book that will surely interest every physicist. It is the reprint of a work of the greatest rarity, as but three copies — two in Paris and one in Breslau (from which the reproduction was made) — could be found. It contains Pascal's *first* announcement to the world of the successful outcome of the all-important experiments carried out by his brother-in-law, Perier, at Pascal's suggestion. These experiments, as is well known, were the first to give experimental proof of the existence of atmospheric pressure, and hence of the real cause for the rise of mercury in an inverted tube exhausted of air — the memorable discovery of Torricelli (the experiment was actually performed by one of his pupils) in 1643. This great discovery completely overthrew the doctrine of nature's "*horror vacui*," which had been prevailing up to that time, and in which Pascal himself had believed (as appears from the *Récit*).

The first eight pages of the facsimile reprint form the letter of instructions sent to Perier by Pascal, November, 1647. On September 22, 1648, Pascal received the glad tidings of the successful outcome of the experiments. This letter of Perier is given in pp. 17–20. As no small share of the credit for the discovery must be given to Perier, on account of the scrupulous care with which he set about his task, it may not be amiss to briefly outline the method employed. Perier filled two tubes with mercury and found that they read the same at Clermont. Emptying one of them, he ascended with it the Puy de Dome (3550 feet above Clermont) and again took a barometric reading. It was found less than before. After repeating the experiment at a station half-way down, and arriving at Clermont, the two tubes were again read and found in perfect accord. The true explanation of the Torricellian experiment thus became at once evident. So careful was Perier that he even surrounded himself with witnesses. Immediately upon receiving Perier's letter, Pascal hastily drew up a tract, concluding with the "consequences," which, as it appears, were added to after the distribution of the tract, and the additions printed on a separate slip, which was found pasted in the Breslau copy. And it is this interesting "messenger" which has now been made accessible for the first time to every one. Professor Hellmann and his co-operators, the German Meteorological Society and its Berlin branch, have thus merited the warmest praise from every scientist.

Pascal's tract was almost bodily incorporated in the *Traitez de l'Équilibre des Liqueurs*, etc., published by Perier in 1663, one year after Pascal's death, and this work is generally accredited with the first account of the discovery.

As to the great question whether Pascal was the real originator of these experiments, or *Descartes*, who asserted in two letters of 1649 to Carcavi that *he* had suggested them to Pascal (which statements Pascal never denied), Professor Hellmann states in the introduction that, after carefully examining all the evidence, he has to agree with Mr. Mourisson, who in his recent book[1] has decided in favor of Descartes. Be that as it may, the fact remains that we owe to *Pascal* the first promulgation of the "great secret" to the world.

It may not be amiss to say a word or two with respect to this interesting series of reprints. The title itself indicates that they are to have special reference to Meteorology and Terrestrial Magnetism. As, however, both these subjects are, in the broadest sense, but Applied Physics to Terrestrial Phenomena (or should be), these facsimile reprints cannot fail to create a wider interest. Moreover, the physicist and the mathematician seem to be waking up to the fact that here are two fields, comparatively unexplored, that present some of the most fascinating problems, and offer some of the prettiest applications of physical laws and mathematical formulæ with which nature has ever found pleasure in taxing and baffling the human mind.

No. 1 of the series bears the following title: *L. Reynman. Wetterbuechlin. Von wahrer Erkenntniss des Wetters.* 1510. Facsimiledruck mit einer Einleitung. 4to. pp. 42, 14. Price, 6 marks ($1.50). Berlin, A. Ascher & Co. 1893.

This, as will be seen from the title, will be most interesting to the professional meteorologist, though the bibliographer, the student of folk-lore, and others may find it no less so. It is the oldest meteorological work in the German language, being first published in 1505—of this edition all trace has been lost. It next appeared in 1510, and of this but one copy is extant, and in possession of Professor Hellmann. From it the elegant and attractive reproduction was made.

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[1] Pascal, Physicien et Philosophe, etc. Paris, 1888.

So successful was Reynman's little work that it ran through seventeen editions in thirty-four years, of all of which scarcely more than three dozen copies can now be found. It was incorporated in a number of works, and almost literally translated in the *Boke of Knowledge of Thynges Vnknown, appertaining to Astronomy, with certain necessary Rules*, published in London, 1585. The chief reason for this remarkable "run" lay in the fact that the author's aim apparently was to base *his* weather proverbs (many in rhyme) and weather signs upon *natural* laws and *natural* phenomena. This differentiated it at once from the current literature of the day, in which astrology played so prominent a part. In the introduction to the reprint, Professor Hellmann, in his scholarly way, gives the book a careful and critical discussion.

These "Neudrucke," as the circular announces, will all appear in elegant and substantial form, — small quarto (20½ × 25½ cm.), — and each year two at the most will be issued. Each number can be bought separately. Among those promised are the following: —

Luke Howard, On the Modifications of Clouds, 1803. In this valuable work the first attempt at classification of cloud-forms is found. It is still a standard work. In the reprints the cloud-forms will be reproduced in facsimile. This number will shortly appear.

Edmund Halley's famous *Isogonic Chart, or Lines of Equal Magnetic Declination for the Epoch* 1700. As will be remembered, Halley is the originator of the method of representing the distribution of terrestrial phenomena by drawing lines through all those places on the earth's surface where the particular phenomenon (be it magnetic declination, temperature, etc.) has the same numerical value. His idea was taken up and applied to the magnetic inclination, it is usually believed, first by Wilcke in 1768. It does not appear to be known, however, and perhaps is pointed out here for the first time, that he was preceded by Whiston in 1721,[1] though, to be sure, the latter drew *his* isoclinic lines for but a very small portion of the earth; viz. Southern England and Northwestern France. Humboldt next took up the idea, and applied it to the intensity of terrestrial magnetic force, and also in 1817 to the distribution of temperature on the earth's surface, and thus gave birth to the *isotherms*. These latter it is also intended to reproduce in facsimile.

It is to be hoped that the German Meteorological Society, and its happily chosen editor, will meet with sufficient encouragement to be induced to continue in the good work.

L. A. BAUER.

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[1] Will Whiston: Longitude and Latitude found by the Inclinatory or Dipping Needle, to which is appended Robert Norman's New Attractive, London, 1721. The said diagram will be found opposite p. xxvii. A copy of this work was found in the Royal Library of Berlin.

Vorlesungen über mathematische Physik. Vol. IV. Theorie der Wärme. By GUSTAV KIRCHHOFF. Edited by MAX PLANCK. Large 8vo, pp. x, 210. Leipzig, B. G. Teubner, 1894.

The fourth and last volume of this series is based upon lecture notes prepared by the author between the years 1876 and 1884, and has been edited, as was the third volume, by Kirchhoff's successor in the chair of Theoretical Physics at Berlin. Professor Planck has expressly refrained from introducing any alterations in treatment, except so far as such changes were found necessary in order to bring the notes into suitable form for publication. What few additions and explanations seemed desirable have therefore been placed in the form of foot-notes. The reader may thus feel assured that the method of presentation is that actually followed by Kirchhoff, and although it may at times appear that the editor has been over-cautious in keeping his own methods in the background, yet his work has been so carefully done that only in a few cases does the treatment appear really incomplete.

The general plan of the lectures is in accordance with a view expressed by Kirchhoff in the preface to his lectures on mechanics; namely, that theoretical physics should be looked upon as a *descriptive* science; that its object is to describe phenomena, and to express the observed relations between physical quantities as completely and as simply as is possible. The treatment throughout the book is consistent with this view. The phenomena first considered are those which require the fewest assumptions, and hypotheses as to the nature of heat and the molecular structure of matter are introduced only when they become necessary and useful. Especial attention is called to the fact that the fundamental laws of heat conduction and thermodynamics are independent of any kinetic hypothesis.

Since the study of conduction requires no assumption as to the nature of heat and the adoption of no absolute scale of temperatures, this branch of the subject is the one which is first discussed. The differential equations are at once deduced for the most general case, but the numerous examples which follow are in general simplified by the assumption that the bodies considered are both homogeneous and isotropic. Cases in which the conductivity is a function of the temperature, as well as cases in which the phenomena may be complicated by physical or chemical changes in the medium, are mentioned, but not discussed. The treatment in the four chapters devoted to conduction, while clear, exhibits no strikingly novel features, and possesses one fault that is very often met with in the discussion of partial differential equations, namely, that the reader is apt to gain no insight into the process of reasoning by which the author has reached his conclusions. While compelled to admit that the solutions are correct, and that the methods used in obtaining them are ingenious, the student will have learned little at the end of these four chapters which will help him in solving similar equations for himself.

In the fifth chapter the subject of Thermodynamics is begun, and the author plunges at once into the consideration of Entropy and the Second Law. Carnot's cycle is first discussed, and by using the principle that heat cannot of itself pass from a colder to a hotter body, it is shown that the ratio of the heat taken from one reservoir to that given up to the other is a function of the two temperatures alone, and independent of the working fluid. The temperatures of the two reservoirs being θ_1 and θ_2, we have, therefore,

$$\frac{Q_1}{Q_2} = f(\theta_1\theta_2).$$

The next step consists in showing that

$$f(\theta_1\theta_2) = \frac{\phi(\theta_1)}{\phi(\theta_2)},$$

where ϕ is some function whose form is as yet unknown. $\phi(\theta_1)$ is now defined as the absolute temperature T_1, and the relation

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$$

follows at once for a simple reversible cycle. It will thus be seen that the thermodynamic definition of temperature is introduced at once, and is in fact the *only* definition given. It is not until several chapters later that a consideration of the behavior of the fixed gases enables the relation between the absolute and the ordinary temperature scales to be deduced. Although this procedure possesses disadvantages, it appears to me on the whole commendable.

Proceeding with the development of the Second Law, the author now shows that for *any* cycle the algebraic sum $\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \cdots$ must be either equal to zero or negative. The statement of the Second Law is then put in the form

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \cdots \leqq 0,$$

without further comment. The lecture closes with an explanation of the term *entropy*, the proof that

$$\int \frac{\delta Q}{T} = 0$$

for a reversible cycle, and the final statement that the term *entropy* can with propriety be employed only when dealing with reversible processes.

I fear that this last statement will cause the thoughtful student some annoyance, since it appears to restrict the applications of the Second Law to quite narrow limits. In the following lecture these restrictions are partly removed by the statement that the term *entropy* may also be applied in the case of irreversible transformations, provided that the changes considered can also be brought about by reversible processes. The final step in the generalization is made by a foot-note of the editor, calling attention to the fact that no transformation has yet been observed which is *not* capable of being produced by such a process. The reader would have been saved some annoyance if these important statements had not been so widely separated.

The sixth chapter contains the application of the Second Law to isothermal changes, and to expansion in general. From Boyle's Law, and the fact that the specific heat for constant pressure is independent of the pressure, the laws of the perfect gas are developed as a special case. The three following chapters are devoted to applications of the law of Entropy to numerous purely physical problems, among which may be mentioned the dependence of melting and boiling points upon the pressure, Van der Waal's formula for the "imperfect" gases, critical points, and the absorption and development of heat in solution. It is to be regretted that the author has refrained from entering the domain of physical chemistry, so that applications of the Second Law to chemical transformations are wholly lacking.

Two chapters on the motion of fluids, illustrated by several interesting special cases, now follow, after which the discussion of the kinetic theory of gases is begun. The treatment of this subject, which occupies nearly half the book, is roughly in accordance with its historical development. Presented first in its simplest form, the theory is gradually developed by the introduction of new hypotheses, as these are shown to be necessary, and connected exposition of the subject is thus presented such as is rarely met with. Although by no means a complete treatise, this portion of the book presents so excellently the essential features of the theory that it will undoubtedly prove of great value.

In general, it may be said that if the volume before us is looked upon as a text-book, it will be found lacking in those concrete illustrations and incidental explanations which are so helpful to the student, but which rarely find their way into a lecturer's note-book. The logical and consistent method of treatment will, however, commend the book to all, and among those books which should be read in order that one may acquire a broad view of the theories of heat, Kirchhoff's lectures will take a prominent place.

ERNEST MERRITT.

The Mean Density of the Earth. By J. H. POYNTING, Sc.D., F.R.S. 8vo, pp. 176. London, C. Griffin & Co., 1894.

The author begins with a few statements regarding attractions, and shows by an example how the determination of the earth's mean density is connected with the law of gravitation. A valuable historical account follows, in which different methods are described, beginning with Bouguer in 1737, and ending with Laska in 1889. This is a very acceptable *résumé* for students of the subject, giving as it does the essential points of each method without becoming tiresome by going very much into details. Bouguer's formula on p. 11, for the diminution of gravity with elevation, seems to be erroneously written, and the error reappears on the following page. Bouguer's statement was that the decrease in gravity from the sea-level to the height h compared with the value at the sea-level is

$$\frac{2\,h\Delta - \frac{3}{2}\,h\delta}{r\Delta},$$

where r is the earth's radius, and δ and Δ are the densities of the intervening matter and the earth respectively. This reduces to

$$\frac{dg}{g} = \frac{2\,h}{r}\left(1 - \frac{3}{2}\,\frac{\delta}{\Delta}\right),$$

where g represents the value of gravity at the sea-level. The error is doubtless an oversight, but it should be pointed out for the benefit of readers who may wish to use the formula and who may not care to verify it, or may not have access to the original work. Of course the term $\frac{2\,h}{r}$ is the *decrease* on account of distance from the earth's center, and $\frac{3}{2}\frac{h}{r}\frac{\delta}{\Delta}$ is the *increase* on account of the matter lying between the sea-level and the upper station considered as a plain of infinite extent. Bouguer ultimately took $2\,\delta = \Delta$, from which the last term reduces to $\frac{3}{4}\,\frac{h}{r}$, and this combined with the first term gives $\frac{5}{4}\,\frac{h}{r}$ for the correction for both height and matter, which is the quantity now generally given as the total effect in the application of the so-called Young's rule. The quantity $\frac{3\,h}{r}\,\frac{\delta}{\Delta}$, given at the top of page 12, is really the effect of a spherical shell, and was so used by Airy in the Harton Pit experiment.

The mountain work done in England, both by Maskelyne, and later in connection with the Ordnance Survey, is next noticed. The great ingenuity and elegance displayed in the application of Hutton's formula is one of the cardinal virtues of the mountain method, and it seems to us that this should have been made more prominent by the author. The attraction of the mountain is given by a formula of triple integration where the successive integrations are in the direction of azimuth, horizontal distance, and height. Performing the operations indicated, the resultant force is

$$A = \delta h(\sin a_2 - \sin a_1) \text{ Nap. log } \frac{r_2}{r_1}$$

(Clarke's *Geodesy*, p. 295), a being the azimuth, and r the radii. From which it appears that if in making the compartments by means of azimuthal and radial lines, we take the sines of the azimuths in an arithmetical progression, and the radii of the circles in a geometrical one, the attractions vary as the heights, which simplifies enormously the computations. This is really the whole spirit of the method, and should be brought out prominently in any account of the work. Carlini's experiments on Mt. Cenis and Mendenhall's on Fujiyama falls under the same general head, — that of differences in the force of gravity, — and they depend on an estimation of the mountain density as given by specimens of the surface rock.

Another class of experiments is now described; viz. those where the pendulum is taken towards the center of the earth instead of being carried away from it.

Airy first conceived the idea of using a pendulum in a mine, and the experiment has been repeated in Austria by von Sterneck. The advantage of this is that a much more intimate knowledge of the material composing the intervening strata can be obtained than in the case of a mountain. The results, however, have not been very accordant. Indeed, a re-reduction of Airy's work by Haughton gives a value different from his by 20 per cent of the whole. Von Sterneck carried on the work at the surface and in the mine simultaneously, the time of oscillation of the two pendulums being determined by the same timepiece, the beats being transmitted telegraphically between the two stations.

The latter part of the historical account is devoted to what may be termed laboratory determinations. Instead of comparing the attraction of the earth with that of a mountain, or with that of a spherical shell of matter, as in the previous work, the attempt is now made to measure directly the attraction between known masses at a known distance apart.

Three different methods have been employed: Cavendish used a torsion balance, two lead balls being suspended by a copper wire, and deflected by large spheres of the same metal brought alternately to opposite sides of the attracted masses. This experiment was repeated by Reich and Baily. Cornu and Baille varied the method by using large globes of mercury for the attracting masses, and instead of shifting these masses bodily from one side to the other of the attracted ones, the mercury was pumped into an equal globe in the desired position. After the torsion balance comes an experiment by Jolly with the common balance, in which a mass was suspended at different distances, and its increase in weight noted for the lower position. Then a determination by Wilsing, where a pendulum balance was used, — a special feature of the work being that, by making the oscillations with varying masses, the value of the couple may be found without knowing the moment of inertia.

This completes the historical account. One or two experiments are referred to as being still in progress. A summary of results varying from 4.5 to 6.5 is given, from which it is seen that the mountain work gives the lowest values, the mine work the highest, and the laboratory experiments intermediate ones. Moreover, the laboratory results are more consistent than the others.

We now come to the principal part of the essay, being a determination by the author, following the common balance method. Two masses, each of about 22 kg. weight, are suspended from the arms of a finely constructed balance, and under each one of these is alternately brought a mass of about 150 kg. The quantity to be measured — the difference of weight — is doubled by this alternation. An ingenious method of eliminating the attraction of the large balls on the beam of the balance was employed by suspending the small balls at different distances, and equal pressures on all parts of the table before and after reversal were secured by placing on the table a mass equal to half that of the attracting mass, but at double its distance from the axis. This gave equilibrium at the same time that it did not very much alter the attractions. Some preliminary experiments, in which the attracted mass was 453 grams and the attracting mass 154,000 grams, gave unsatisfactory results on account of the small amount of the attraction (0.00001 gram), which led to the employment of larger balances and larger attracting masses. The attractions were then compared with the weights of riders. These were determined at the International Bureau of Weights and Measures at Paris by measuring their difference of weight taken two and two in all possible combinations, and then comparing the weight of all with a nearly equal known weight. The great accuracy of this method is at once apparent, as the separate weights are finally obtained through precise measure of small differences. The uncertainty of the results was not greater than one millionth of a gram. Poynting's work seems to have been done with great care, and is without doubt a valuable contribution to the literature of the subject. Space will not allow us to review the work in detail, — the essential and characteristic points being: the employment of the double suspension mirror method of detecting small motions; a graphic method of determining the center of swing; keeping the beam under a constant strain; and using nearly the same scale divisions for deflections produced either by the attracting mass or by the riders. It was known that a movement of a centigram weight through a distance of 25 cm. on the beam would produce about the same deflection as that from the attracting masses, so that in the experiments one centigram weight was lifted and another let down at this distance to avoid moving them along the beam. The author so arranged his suspension mirror that the angle through which the balance beam turned was magnified 150 times, and the distance from the scale to the mirror was such that one division of the scale corresponded to an angular motion of 0.0001 radian. One scale division, therefore, implies an angular motion of about $\frac{1}{8}$ of a second of arc in the beam of the balance. By this method the motion of $\frac{1}{800000}$ of an inch in the masses is perceptible.

The gravitation constant and the mean density of the earth are functions of the masses and their distances apart. The measurement of these vertical and horizontal distances forms an important part of the work. The method of combining the observations to eliminate the effect of progressive changes deserves special notice. The scale was first read in the initial position, the riders were then moved, and the scale read again; following this, the scale was again read with the riders in the initial position. Then the masses were moved round and the scale read, and finally the reading was made with the masses in the initial position. This program was repeated as long as desirable. Three successive results being taken together, give a value free from progressive changes, and the ratio of the deflection by the mass to that by the rider is based on seven successive centers of swing. The final result obtained is 6.6984×10^{-8} for the gravitation constant, and 5.4934 for the mean density of the earth. Two sets were made, and these treated separately gave for the last quantity the value 5.52 and 5.46. It appears doubtful whether in the final results more than two decimal places are trustworthy. It is understood that the second set with the attracting masses in the opposite position will, to a great extent, eliminate the want of symmetry in the moving parts, but the discrepancies in the results after the interchange would seem to indicate that the final result is given to an accuracy not warranted by the observations. The quantity sought, Δ, is deduced from a formula depending on the distances between the masses and other quantities, such as the earth's compression, centrifugal force, etc. In this formula the difference between the ratios of the mass deflection to rider deflection for the two positions enters as a factor in the denominator. If we examine the values of this factor for the same set, we see differences greater than $\frac{1}{1000}$ part of the whole for the value of A, which really influence the value of $(A - a)$ to nearly the same extent, since a is small. Besides, a has uncertainties of its own, not here considered. If we differentiate the expression for Δ, it appears that the change in the factor $(A - a)$ gives a change in Δ seven times as great, and that the uncertainties in the measured quantities will make even the second decimal place uncertain. So that 5.49 would seem to be sufficient to characterize the result. It is doubtful whether, with all the various methods hitherto employed, we can assert that the mean density of the earth is known with an error less than one per cent of its value.

The bibliography at the beginning of the work is valuable. At the end appears a catalogue of about seventy pages of scientific and other books inserted by the publishers. We rather deplore this custom of binding extensive catalogues of works, many of them not in any way related to the subject matter of the volume, with scientific books. On the whole, Professor Poynting is to be congratulated on his work. His determination adds another trustworthy value to those already existing, and both the method employed and the care exercised in the observations entitle the result to confidence.

Erasmus D. Preston.

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THE

PHYSICAL REVIEW.

ON THE SOLUTION-TENSION OF METALS.

BY HARRY C. JONES.

FROM van't Hoff's theory of osmotic pressure, the molecules of dissolved substances exert a certain pressure in the solvent in which they are dissolved. Nernst[1] has pointed out that in order that a substance may dissolve in a solvent, it must have a certain "expansive force" to drive the molecules into the space where they will exert an osmotic pressure.

He has extended the idea of "solution-tension" also to the pure metals. When a metal is brought in contact with a solution of its salt, a very small part of the metal is conceived to be capable of taking plus electricity from the remainder of the metal and of passing into solution as ions, when the osmotic pressure of the metallic ions of the salt solution is not already greater than the solution-tension of the metal. To consider more closely concrete cases in the light of the suggestion of Nernst, let us consider a metal immersed in a solution of one of its salts. Let the solution-tension of the metal be T and the osmotic pressure of the metallic ions of the dissolved salt p. If the solution-tension be greater than the osmotic pressure, $T > p$, then a small part of the metal will pass into solution as ions. These ions will carry off a positive charge of electricity from the metal into the solution. The metal will then contain a corresponding negative charge, and the solution will be positive with respect to the metal. The negative electricity on the surface of the metal will attract electrostatically the positive ions in the solution, and on the surface of contact between the metal and solution there will be formed a double electrical layer. The existence of such a double layer has been shown to be probable by Helmholtz,[1] who thinks that such must always exist on the bounding surface of two conductors at different potentials.

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[1] Nernst, Zeits. Phys. Chem., 4, 129.

Under these conditions, where the metal is negative and the solution positive, the action of the double layer would be to drive the metallic ions out of solution and on to the metal, where they would lose their charge and pass over into the metallic condition. The attraction between this double layer, which tends to drive metallic ions out of solution, works against the solution-tension of the metal. Equilibrium will be established when these two contrary forces are equal, when in any given time as many metallic ions will be driven into the solution due to the solution-tension of the metal as will separate from the solution due to the electrostatic attraction of the double electrical layer. The difference in the potential of this double layer depends in part on the nature of the metal, but also in part on the concentration of the metallic ions in the solution, and therefore upon the osmotic pressure of the metallic ions in the solution. The action of the above two forces gives rise to an electromotive force between the metal and the solution in which it is immersed. Since $T > p$, the solution will be positive with respect to the metal, and the electromotive force will exist in the sense of producing a current from the metal to the liquid.

If $T < p$, the reverse will take place. Metallic ions will separate from the solution as metal on the immersed metal, which will become positive with respect to the solution. The positive charge on the metal will attract electrostatically the negative ions in the liquid, and an electrical double layer will be formed here, but in the reverse sense to that discussed above. Metallic ions will separate from the solution until the electrostatic repulsion of such ions by the positive metal is equal to the osmotic pressure tending to force them out of solution. The result of the action of these forces is to produce an electromotive force between the metal and solution. But since $T < p$, the metal is positive with respect to the solution, and the electromotive force will exist in the sense of generating a current from the solution to the metal, when the necessary connections are established.

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[1] Helmholtz, Wied. Ann., 7, 337.

This explanation of the source of the electromotive force, which exists between a metal and solutions of its salts, based on the relation between the osmotic pressure of such solutions, and the solution-tension of the metal, has been furnished by Nernst,[1] who has developed it essentially as given above. He has also called attention to the fact that the amount of metal which would pass into solution as ions under the condition $T > p$ is in general extremely small because of the great electrostatic capacity of the ions.

Ostwald[2] has developed a formula for calculating the solution-tension of metals, knowing the difference in potential between the metal and the solution of its salt in which the metal is immersed, and the osmotic pressure of the salt solution, thus:

Let a metal be immersed in a solution of one of its salts. By passing $N_e E_0$ (N_e = valence of the kation, $E_0 = 96540$ Coul.) of electricity through the metal, a gram-atomic weight of the metal is dissolved. Osmotic energy is transformed into electrical. Let P be the solution-tension of the metal, and p the osmotic pressure of the metallic ions in the solution. The isothermal change of the volume energy consists of two parts, the values of which are Pv and $\int_P^p vdp$. In terms of the gas law,

$$Pv = RT, \text{ and } \int_P^p vdp = RTln\frac{p}{P} = -RTln\frac{P}{p}.$$

The two together,

$$RT\left(1 - ln\frac{P}{p}\right).$$

But this same equation which applies to gases also holds for solutions where osmotic pressure is substituted for gas pressure.

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[1] Nernst, Zeits. Phys. Chem., 4, 150.

[2] Ostwald, Lehrb., Bd. II., 851.

But since the volume energy has decreased, the value is to be taken as negative, and the alteration of the volume energy then is

$$RT\left(ln\frac{P}{p} - 1\right).$$

The electrical work is $N_eE_0\pi$, which is just equal to the change in the volume energy. $\pi =$ the difference in potential between the metal and the solution.. Equating these two equal expressions, we have

$$\pi N_eE_0 = RT\left(ln\frac{P}{p} - 1\right),$$

from which

$$\pi = \frac{RT}{N_eE_0}\left(ln\frac{P}{p} - 1\right).$$

The electrolytic solution-tension seemed from what was hitherto known to be a constant for each metal, increasing in general with increase in temperature [1] as the vapor-tension and solubility increase with the temperature.

Ostwald [2] has calculated the solution-tension of a number of the metals from the formula which he developed. In these calculations the potential differences between metals and solutions of their salts as measured by Wright and Thompson,[3] and by F. Braun,[4] were employed. The difference in potential between a metal and a solution of one of its salts depends upon the solution-tension of the metal on the one hand, and the osmotic pressure of the metallic ions in the solution acting against the solution-tension on the other.

In the formula $\pi = \frac{RT}{N_eE_0}\left(n\frac{P}{p} - 1\right)$, in addition to the constant there are three values involved, the difference in potential between the metal and the solution of its salt in which it is immersed $= \pi$, the osmotic pressure of the solution $= p$, and the solution-tension of the metal $= P$. But the difference in potential between the metal and a definite concentration of its salt solution must be determined, and also the osmotic pressure of a solution of the same concentration, in order that the solution-tension may be calculated. Normal solutions containing a gram-atomic weight of the metal in a liter were employed. Such solutions of normally dissociated salts are about 80 per cent dissociated, and have an osmotic pressure of about 22 atmospheres. The solution-tension of the following metals was calculated from the formula given above. If in this we substitute the value of $\frac{RT}{E_0}=0.058$, passing from the natural to the Briggean logarithms, we obtain

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[1] Nernst, Zeits. Phys. Chem., 4, 172.

[2] Ostwald, Lehrb., Bd. II., 946.

[3] Wright and Thompson, Phil. Mag. [5], 19, 209.

[4] Braun, Wied. Ann., 16, 561, and 17, 593.

$$\log P=\frac{\pi N_0}{0.058}+\log p+0.434$$

The solution tension, in atmospheres, as calculated by Ostwald from this formula, using the measurements of potential differences between metals and solutions of their salts made by Wright and Thompson, and Braun, are given below: —

Magnesium 10^{44}
Zinc 10^{18}
Aluminium 10^{13}
Cadmium 10^{7}
Iron 10^{3}
Lead 10^{-2}
Copper 10^{-19}
Mercury 10^{-15}
Silver 10^{-15}

From the above figures, the metals show striking differences as to their solution-tension in water solutions of their salts. The calculated values are in general either very great or very small. Of the above metals, iron and lead have intermediate values. These values can be taken as expressing the approximate solution-tensions of the above metals.

The main point in mind when the work described in this paper was undertaken, was the determination of the electromotive force between solutions of salts in water and solutions of the same salts and of the same concentrations in other solvents, using the same metal as the kation of the salt as electrodes. Knowing the dissociation of a salt in water, it seemed probable that the dissociation in other solvents could be calculated from the electromotive force. But this calculation would be based on the assumption that the solution-tension of a metal is a constant and the same constant for the different solvents in which its salts were dissolved.

But little is known at present of the dissociation of salts in solvents other than water, and the above method based on the measurements of electromotive force seemed to promise much in this direction. The dissociation as calculated from electromotive force could then be compared with that calculated from conductivity and "migration velocities" of the ions in the different solvents, when such determinations shall have been made. Indeed, some work on the conductivity of alcoholic solutions of salts has already been carried out.[1]

Silver nitrate solutions alone have been carefully studied thus far in my work. A tenth-normal solution of this salt in water and a tenth-normal solution in ethyl alcohol were prepared. These were placed respectively in the glass vessels, *A* and *B*, Fig. 1. Both of these vessels were connected with the middle vessel, *C*, which was filled with the water solution of silver nitrate of the same strength as that in vessel *A*, by means of the side tubes, as shown in Fig. 1. The ends, *HH*, of these side tubes were closed loosely with a roll of filter paper, to diminish the diffusion from one vessel to the other, and the consequent mixing of the solutions. The liquid in the three vessels was then continuously connected, and at the same time could not pass rapidly from one to the other. Into the solutions in *A* and *B*, were dipped the silver electrodes *EE*. The silver wires, *FF*, were attached to the silver electrodes and surrounded by the small glass tubes *gg*, where they passed through the corks *DD*. The upper and lower ends of each glass tube were fused to the wire by means of fusing glass. The use of the short glass tube was to make closer contact in passing through the cork than would have been possible with the metallic wire alone.

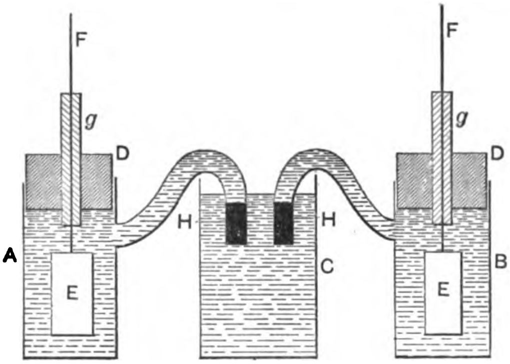

Fig. 1.

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[1] Völlmer, Dissertation, Halle.

The silver electrodes were cut from thin sheet silver and carefully cleaned. They were then covered with a layer of silver deposited electrolytically from a solution of potassium-silver cyanide. In order to secure a uniform deposit, the solution of the double cyanide must be very dilute, and the current not stronger than that of one Leclanché element. About a half-dozen such electrodes were plated together, all in connection with the same binding-screw. But even when thus plated together, the electrodes were not always exactly constant, the one against the other. They however became constant to within a few ten-thousandths of a volt if allowed to stand in combination with each other for some time in a dilute solution of silver nitrate in water. Such a solution of silver nitrate contains silver ions, and any small differences in potential between the electrodes immersed in this solution would gradually disappear.

At first measurements of electromotive force between one-tenth normal solutions in alcohol and in water were made, and then more dilute solutions were investigated. The same concentration of solution in both solvents was always employed. The following solvents were used for silver nitrate; water, ethyl alcohol, methyl alcohol, and acetone. The first series of measurements was made between water solutions and ethyl alcohol solutions. All measurements of potential differences were made by means of the most sensitive form of the Lippmann electrometer,[1] consisting of the upright capillary tube of very small diameter. Readings to 0.0002 volt could be made with a fair degree of accuracy. All measurements were referred to a Clark element which had been compared with a standard Clark. The usual volt element was prepared. This was connected against a Leclanché element through a convenient form of resistance box, and the whole with the electrometer. The point of equilibrium was then determined on the box. Then the Clark minus the volt, *i.e.* the difference, was connected against the Leclanché, and the equilibrium determined on the box. This gave data for calculating the real value of the supposed volt, knowing the temperature, and therefore the electromotive force of the Clark. Two determinations of the value of the supposed volt made some weeks apart showed it to be respectively 0.9955 and 0.9946 volt. Knowing the value of the Clark, and also of the supposed volt, their difference is known. In order that the very sensitive electrometer might be used, the Leclanché was closed through about 2000 ohms resistance, and then the difference between the Clark and volt was brought to equilibrium on the box against the Leclanché with this extra resistance. Electromotive forces were then measured on the box against the Leclanché with the above resistance. The Leclanché was never closed except during the short intervals in which measurements were actually being made, and was compared from day to day against the difference between the Clark and volt to detect any changes. The Leclanché was found to become weaker, but changed very slowly.

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[1] Ostwald, Lehrb., Bd. II., 924.

The following table contains the measurements of electromotive force between water solutions and ethyl alcohol solutions. The first column contains the concentrations of the water solutions in terms of normal solutions; the second, those of the alcohol which are the same; the third, the first series of measurements of electromotive force; the fourth, the second series of such measurements. The second series of measurements was made with newly prepared solutions.

Table I.

Concentrations in water solutions. Normal.	Concentrations in ethyl alcohol solutions. Normal.	Electromotive force. 1st series.	Electromotive force. 2d series.
0.1	0.1	0.0956	0.0951
0.05	0.05	0.1055	0.1045
0.01	0.01	0.1185	0.1199
0.005	0.005	0.1274	0.1284
0.001	0.001	0.1309	0.1300

The *alcoholic solutions* were always *positive* against the water solutions, and to the extent given by the above results. The difference increases with the dilution, as seen from the results. Since this work was begun, a publication has appeared from Mr. Adolfo Campetti (Dissertation, Turin), describing measurements of the electromotive force between alcohol and aqueous solutions of ammonium, lithium, calcium, copper, zinc, and cadmium chlorides, and zinc and cadmium iodides. He also found that the alcoholic solutions were positive against the water solutions, but does not seem to have seen any special significance in his results as far as the determination of the solution-tension of metals is concerned.

In order that the alcoholic solutions should be positive against the water solutions, silver ions must separate from the alcoholic solutions as metal on the electrode. Whether this takes place depends, as we have seen, on the relation between the osmotic pressure of the metallic ions in the solution and the solution-tension of the metal. The water solutions are far more dissociated than the alcoholic and the osmotic pressure is therefore greater in the water solutions, yet ions separate from the alcoholic solution on the electrode when connected against the water solution of the same degree of concentration. This separation of ions takes place from the alcoholic solution which has *less* osmotic pressure; therefore, the force opposing this separation — the *solution-tension* — must be *less* in alcoholic solutions.

While but little is known in general of the dissociation of compounds in alcohol, yet a few substances have been studied by Völlmer[1] as to their conductivity, and among these is silver nitrate. Knowing the dissociation of silver nitrate in water and in alcoholic solutions of the same strength, it seems to be possible to calculate the relative value of the solution-tension of silver in water and in alcoholic solutions from the electromotive force existing between the two.

Let π_1 be the electromotive force of the alcoholic solution in which the silver electrode is immersed; then,

$$\pi_1 = \frac{RT}{N_e E_0} \log \frac{p_1}{P_1},$$

where p_1 is the osmotic pressure of the alcoholic solution, and P_1 the solution tension of the silver in the alcoholic solution. Let π_2 be the electromotive force of the water solution; then

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[1] Völlmer, Dissertation, Halle.

$$\pi_2 = \frac{RT}{NE_0} \log \frac{p_2}{P_2},$$

where p_2 is the osmotic pressure of the water solution of silver nitrate, and P_2 the solution-tension of the silver in the water solution. The electromotive force determined was the difference between π_1 and π_2.

$$\pi_1 - \pi_2 = \frac{RT}{N_e E_0} \log \frac{p_1}{P_1} - \frac{RT}{N_e E_0} \log \frac{p_2}{P_2};$$

but since $\frac{RT}{E_0} = 0.058$ volt, this becomes

$$\pi_1 - \pi_2 = \frac{0.058}{N_e} \log \frac{p_1}{P_1} - \frac{0.058}{N_e} \log \frac{p_2}{P_2}.$$

But N_e = the valence of the kation for silver $= 1$, we have then

$$\pi_1 - \pi_2 = 0.058 \left(\log \frac{p_1}{P_1} - \log \frac{p_2}{P_2} \right).$$

In the last formula all the values are known except P_1 and P_2, taking the dissociation of silver nitrate in alcohol as found by Völlmer.[1] By making $P_2 = 1$, the value of P_1 in terms of P_2 can be calculated

$$\log P_1 = -\left(\frac{\pi_1 - \pi_2}{0.058} \right) + \log \frac{p_1}{p_2}.$$

The conductivity of the following concentrations which lie within the range of my work has been determined, 0.025 and 0.0025 normal. The molecular conductivity as found by Völlmer in alcohol, for these concentrations, is 136.3 and 246.3 respectively. The greatest molecular lowerings found for silver nitrate in alcohol were for 0.00002445 normal, 350.0, and for 0.000007733 normal, 353.0, and these values can be regarded as very nearly the maximum for such solutions. The limit seems, however, not to have been quite reached, and the true limit found by Völlmer by extrapolation was 356. The limit found by Kohlrausch for water solutions of silver nitrate was $M\infty = 1080$. The value of $M\infty$ for this and other salts in water, in comparison with the highest value found for alcohol, was about 3, with a mean value of 2.93. The value of $M\infty$ for water can be taken as representing complete dissociation; therefore the highest value found in alcohol would represent $\frac{1}{2.93}$ of complete dissociation if the "velocity of migration" of the ions was the same in alcohol as in water. Poisenilles has shown that the "internal friction" of alcohol is to water as 682 : 523. The above value $\frac{1}{2.93}$ must therefore be multiplied by $\frac{682}{523}$. $\frac{682}{2.93 \times 523} = \frac{1}{2.25}$ Therefore in the most dilute alcoholic solutions investigated by Völlmer only $\frac{1}{2.25}$ part of the molecules was dissociated. The true value of $M\infty$ for an alcoholic solution of silver nitrate corresponding to complete dissociation is therefore about $356 \times 2.25 = 800$.

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[1] Völlmer, Dissertation, Halle.

The dissociation of silver nitrate in alcohol is then for the concentration 0.025, $\frac{136}{800} = 17$ per cent, and for the concentration 0.0025, $\frac{246}{800} = 31$ per cent.

The dissociation of silver nitrate in water for these concentrations as found by interpolation in the results of Kohlrausch is for the concentration 0.025, 90 per cent; and for the concentration 0.0025, 97 per cent. Inserting these values for the dissociation in the formula,

$$\log P_1 = -\left(\frac{\pi - \pi_2}{0.058}\right) + \log \frac{p_1}{p_2},$$

for p_1 the dissociation in the alcoholic solution, and for p_2 the dissociation in the water solution, we have all the data necessary for calculating the solution-tension P_1. The value of P_1 for the above two concentrations will be calculated, using for $(\pi_1 - \pi_2)$ the value found by interpolation in my measurements of the electromotive force between alcoholic and water solutions.

The value of $\pi_1 - \pi_2$ for the concentration 0.025 is 0.1135 v.
The value of $\pi_1 - \pi_2$ for the concentration 0.0025 is 0.1295 v.
The solution-tension P_1 calculated for the conc. 0.025 is 0.048.
The solution-tension P_1 calculated for the conc. 0.0025 is 0.053.

As has been seen, this calculation is based on the dissociation of alcoholic solutions as given by the work of Völlmer, following also his line of reasoning. If these dissociations for alcoholic solutions should be shown to contain any error, it will be perfectly simple to insert the true dissociation of such alcoholic solutions in the above formula, and recalculate the solution-tension of the metal in the solution.

The value of P_1 as calculated for the concentration 0.025 normal, agrees well with that calculated for one-tenth this concentration, using the dissociation values of Völlmer and Kohlrausch for the alcoholic and water solutions respectively, and my own measurements of the electromotive force of the solutions against each other. From these results the solution-tension of silver in alcoholic solutions of silver nitrate is only about one-twentieth of that in aqueous solutions of the same salt.

If we regard 356 as representing complete dissociation of silver nitrate in ethyl alcohol, then the calculated values of P_1 for the two concentrations are respectively 0.021 and 0.024, while $p_2 = 1$.

The increase in the electromotive force of the alcoholic solutions of silver nitrate against the water solutions, with increase in dilution, is due doubtless to the more rapid increase in the dissociation of the alcoholic solutions of the salt for the concentrations used.

It was desired to test the effect of changing the anion of the silver salt on the solution-tension of the metal in alcoholic solutions of the salt. To accomplish this, attempts were made to measure the electromotive force of alcoholic solutions of silver benzoate and of silver acetate against water solutions of the same concentrations, using silver electrodes as before. But on account of the slight solubility of these salts in ethyl alcohol, no satisfactory results could be obtained. The question, then, as to the effect of the anion of the silver salt on the solution-tension of the metal in alcoholic solutions of the salt remains at present entirely unsettled.

The electromotive force between 0.01 n. silver nitrate in water, and 0.01 n. silver nitrate in ethyl alcohol, is about 0.1190 volt. The electromotive force between two solutions of 0.01 n. silver nitrate in water or in alcohol would be of course zero. It is then of interest to add varying amounts of the solution in one solvent to the solution in the other, and measure the potential difference against the solution in one or in the other solvent. Varying amounts of the water solution 0.01 n. were added to a constant amount of the 0.01 n. alcohol solution. Except for the slight change in volume, the mixtures remained then 0.01 n.; the difference in potential of these mixtures, against 0.01 n. solution of silver nitrate in water was then determined. The results are given in Table II. A = the amount of the alcoholic solution (10 c.c.), W = the different amounts of the water solution added to the 10 c.c. alcoholic solution. E = the electromotive force found.

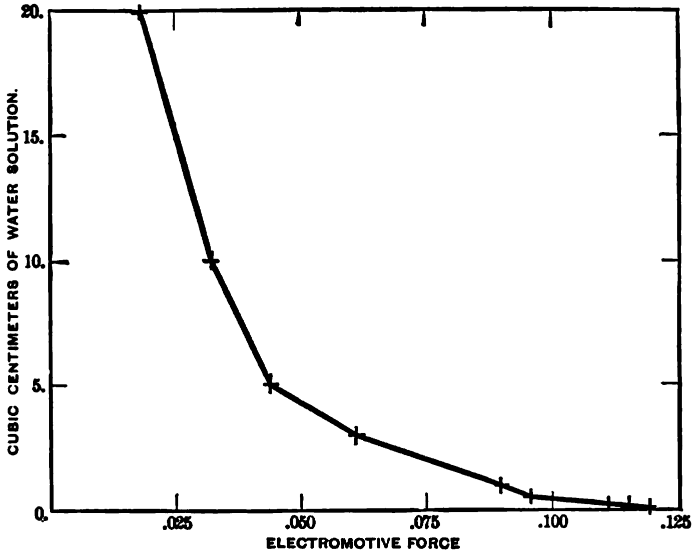

Fig. 2.

These results are plotted as a curve, Fig. 2. The first measurement, where the largest amount of water was used, was omitted for convenience in selecting the scale for plotting the remaining results.

TABLE II.

A	*W*	*E*
10 c.c.	100 c.c.	0.0030
10 "	20 "	0.0179
10 "	10 "	0.0319
10 "	5 "	0.0438
10 "	3 "	0.0605
10 "	1 "	0.0901
10 "	0.5 "	0.0961
10 "	0.2 "	0.1110
10 "	0.1 "	0.1150
10 "	0.0 "	0.1192

Solutions in Methyl Alcohol.

The methyl alcohol used, like the ethyl alcohol, was purified and freed from water, and the boiling-point determined. The electromotive force of solutions of silver nitrate in methyl alcohol against solutions of the same concentration in water, was determined in all respects as with ethyl alcohol. Below are the results tabulated as those with ethyl alcohol.

TABLE III.

Concentrations, normal in water solutions.	Concentrations, normal in methyl alcohol solutions.	Electromotive force. 1st series.	Electromotive force. 2d series.
0.1	0.1	0.0906 volt	0.0892 volt
0.05	0.05	0.1000	—
0.01	0.01	0.1131	0.1117
0.005	0.005	0.1180	—
0.001	0.001	0.1230	0.1219

The methyl alcohol solutions were always positive against the water solutions of the same concentration and to the amounts given in the above table. The methyl alcohol solutions are, however, somewhat less positive against the water solutions than the ethyl alcohol solutions, as seen by comparing the electromotive force values in the above table with those in Table I. This is what might be expected from the composition of the two alcohols with respect to water.

When it was found that the ethyl alcohol solutions gave greater potential differences against water solutions than the corresponding methyl alcohol solutions, it was supposed that the ethyl alcohol solutions would be positive against methyl alcohol solutions of the same concentration. The reverse was found to be true. For the concentrations used, the methyl alcoholic silver nitrate was always positive against the ethyl, and to the amounts given below.

TABLE IV.

Concentrations, normal in ethyl alcohol solutions.	Concentrations, normal in methyl alcohol solutions.	Electromotive force. Methyl alcohol positive.
0.1	0.1	0.0058 volt
0.05	0.05	0.0093
0.01	0.01	0.0098
0.005	0.005	0.0058
0.001	0.001	0.0097

This seems to indicate an electromotive force at the contact between the water and alcohol solutions, and that this differs in amount for the contact between water and ethyl alcohol, and water and methyl alcohol solutions. It appears to be possible to calculate the electromotive force on the contact between any two different concentrations of the same electrolyte in the same solvent, knowing the velocity of migration of the anion and kation in that solvent, and the dissociation of the two solutions. But here we have to do with two solvents and the calculation of the electromotive force at the contact is probably at best only roughly approximate. Ostwald[1] has deduced a formula for calculating the electromotive force on the contact of two different concentrations of the same electrolyte, from the work of Nernst,[2] thus: π = the electromotive force to be calculated, p_1 = the osmotic pressure of both ions in the more concentrated solution, p_2 = the osmotic pressure in the more dilute, u = the velocity of migration of the kation, v = that of the anion. By passing E_0 electricity through the contact of the two solutions, $\frac{u}{u+v}$ gram-equivalent of the kation will migrate in the positive sense, and $\frac{v}{u+v}$ of the anion in the negative. This amount of the kation $\frac{u}{u+v}$ passes from where the pressure is p_1 to where the pressure is p_2, and the corresponding osmotic energy is $\frac{u}{u+v}RTln\frac{p_1}{p_2}$. The anion has meanwhile passed from a lower pressure into a higher, supposing the stream to pass from the more concentrated to the more dilute solution and the energy spent is $\frac{v}{u+v}RTln\frac{p_1}{p_2}$. The gain in energy is therefore the difference between the above values = $\frac{u-v}{u+v}RTln\frac{p_1}{p_2}$. Placing the electrical energy πE_0 equal to the osmotic, we have

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[1] Ostwald, Lehrb., Bd. II., 843.

[2] Nernst, Zeits. Phys. Chem., 4, 129.

$$\pi=\frac{u-v}{u+v}\frac{RT}{E_0}ln\frac{p_1}{p_2},$$

$$=\frac{u-v}{u+v}0.0002\ T\log\frac{p_1}{p_2}.$$

If the velocity of migration of the ions were the same in alcohol as in water solutions, the electromotive force π on the contact for 0.025 normal silver nitrate would be apparently about 0.002 volt. Another term, $u_1 v_1$, for the velocity of migration of the kation and anion respectively, in the alcoholic solutions can be inserted in the above formula, when these shall have been more accurately determined. The value found for π would then probably be too small to account for the difference in potential between ethyl and methyl alcohol solutions of silver nitrate when measured against water solutions, and when measured directly against each other.

From the results, the solution-tension of silver in methyl alcohol solutions of silver nitrate, as in ethyl alcohol solutions, is less than in water solutions. Since but little is known of the dissociation of salts in methyl alcohol, the value of the solution-tension of metallic silver in methyl alcohol cannot at present be calculated.

Solutions of Silver Nitrate in Acetone.

Silver nitrate was dissolved in acetone which had been purified and whose boiling-point had been determined. A standard solution was prepared as with methyl and ethyl alcohol solutions, and from this the dilute solutions were made. A few measurements were made of acetone solutions against water solutions of the same concentrations.

TABLE V.

Concentrations, normal in water solutions.	Concentrations, normal in acetone solutions.	Electromotive force. 1st series.	Electromotive force. 2d series.
0.01	0.01	0.1521 volt	0.1512 volt
0.005	0.005	0.1692	——
0.001	0.001	0.1814	0.1829

The acetone solutions were always positive. Since the dissociation of silver nitrate in acetone is not known, it is not possible at present to calculate the solution-tension of silver in acetone solutions of silver nitrate. But the line of reasoning which was employed in the case of ethyl alcohol solutions would lead to the conclusion that the solution-tension is also *less* in acetone than in water solutions.

The electromotive force here, as with methyl and ethyl alcohol, increases with increase in the dilution, but somewhat faster with acetone than with the alcohols. This probably means that the dissociation of the salt in acetone increases slightly faster with respect to the water solutions for these concentrations than the dissociation of the salt in the alcohols.

I also tried to make a series of measurements of the electromotive force of alcoholic solutions of zinc chloride against water solutions. Mr. Campetti[1] described such measurements. I used different forms of zinc electrodes, — amalgamated zinc, polished zinc, and electrodes covered with zinc deposited electrolytically. The results which were obtained for one-tenth normal solutions in the two solvents, water and alcohol, varied from 0.15 to 0.25 volt. But the *alcoholic solutions* were *always positive* against the *water*. If the electrodes were constant when introduced respectively into the water and alcoholic solutions, after standing a short time in the solutions they were no longer constant against each other. A white precipitate was frequently observed in small amounts in the alcoholic solution after standing for some time in contact with the zinc electrode, indicating some action between the solution and the electrode. The electromotive force frequently changed very considerably on allowing the two solutions to stand for some time in contact with the electrodes. I hope to continue the work with other metals.

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[1] Campetti, Dissertation, Torino.

The point of chief interest brought out in connection with this investigation is that the *solution-tension of silver is not a constant for all solvents of its salts*, but depends upon the nature of the solvent in which its salts are dissolved. The same fact will probably be found true for other metals.

In conclusion I wish to express my sincere thanks to Professor Ostwald for valuable suggestions during the progress of this work, which was carried out in his laboratory.

LEIPZIG, PHYS-CHEMICAL LABORATORY,
March, 1894.

ON ELECTRIC STRENGTH.

By G. W. Pierce.

In a paper by Dr. Macfarlane and the writer, published in the *Physical Review*, November–December, 1893, were given the results of a series of experiments on the electric strength of paraffined paper, beeswaxed paper, and kerosene oil. The apparatus consisted simply in two discharging tables connected in parallel with the poles of a Toepler-Holtz machine. Between a pair of brass disks about four inches in diameter, on one of the tables, was placed the solid or liquid to be tested. The system was gradually charged, and the pair of similar disks on the other table were slowly separated, causing the air gap between them to grow larger and larger until the spark ceased passing through the air, and broke down the insulator tested.

The air thickness was obtained by the reading on a micrometer-millimeter scale carried by one of the tables. The thickness of the dielectric near the hole made by the discharge was measured by a spherometer.

In the experiments described below, as in the earlier experiments, I have compared the different dielectrics with air, and have then attempted to reduce them to absolute measure by taking from the papers of Dr. Macfarlane,[1] Mr. Steinmetz,[2] and M. Baille[3] the difference of potential corresponding to the equivalent air values obtained.

Table I. gives the results obtained at different temperatures for melted paraffin, of which the melting-point is 52° C. In most of the tests it appears that raising the temperature strengthens the dielectric. I was unable to determine this point definitely.

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[1] Trans. Royal Soc. Edinb., Vol. 28, p. 633, and Vol. 29, p. 561. Proc. R. S. E., Vol. 10, p. 555; Phil. Mag., December, 1880; Trans. Amer. Inst. El. Eng., March, 1893.

[2] Trans. Amer. Inst. El. Eng., February, 1893.

[3] Thomson's Recent Researches in Electricity and Magnetism, p. 70.

Figure 1 represents the results graphically. Curve I. gives the relations between the thickness of the liquid and the corresponding thickness of air. Curve II. (using Baille's air equivalent), curve III. (using Macfarlane's air equivalent), and curve IV. (using Steinmetz's equivalent) give the difference of potential as a function of the length of the spark produced through the paraffin.

I also tested in the melted condition a paraffin of higher melting-point (76°–80° C.) and obtained the result that the electric strength is constant and somewhat greater than the 52° paraffin. Compare Table II. and Fig. 2 with Table I. and Fig. 1.

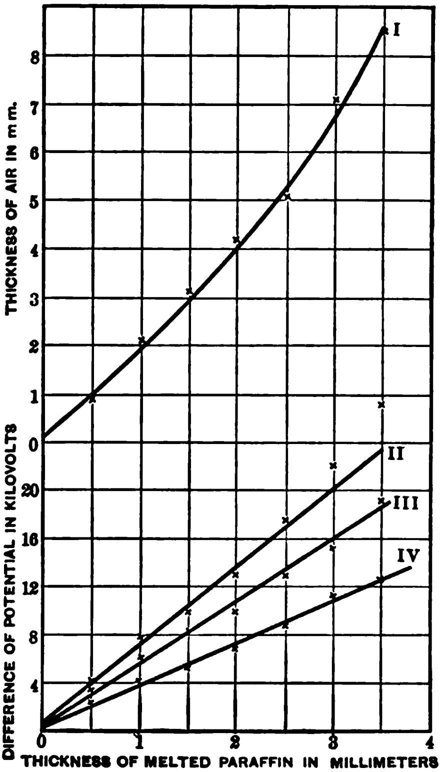

Fig. 1.

In order to test solid paraffin, I prepared a number of thin cakes by pouring the melted paraffin on the surface of hot mercury and allowing to cool, so that the cakes were of almost absolutely uniform thickness, and practically free from air bubbles. The paraffins used, with the exception of the 52° paraffin, were prepared by Drs. Bender and Hobein of Munich. In the case of the 52° paraffin (see Table I. and Fig. 3) the electrodes were the plates used in the experiments on paraffined and beeswaxed paper. I observed that with such a large surface of the dielectric exposed to the electrodes, the spark generally found a hole or defect to go through; and it is probable that the gradients of Table III. are somewhat too small. To obviate this difficulty in the subsequent experiments, balls of 2.5 cm. diameter were used for the paraffin electrodes, the air electrodes being balls (Table IV.) or plates (Tables V. and VI.). In Table IV., testing the strength of a solid paraffin of melting-point 76°–80° C., the values for the equivalent thickness of air and corresponding difference of potential are also evidently too low, as in other tests, at different points of the cake, the same pieces would not break down under the greatest electromotive force that could be obtained by the machine. In the measurements of the strength of solid paraffin of melting-points 42°–44° C., and 46°–48° C., Tables V. and VI., some of the results may have been vitiated by the balls pressing into the soft wax. I attempted to eliminate these errors by measuring the thickness of the cake after the disruption and at the point punctured.

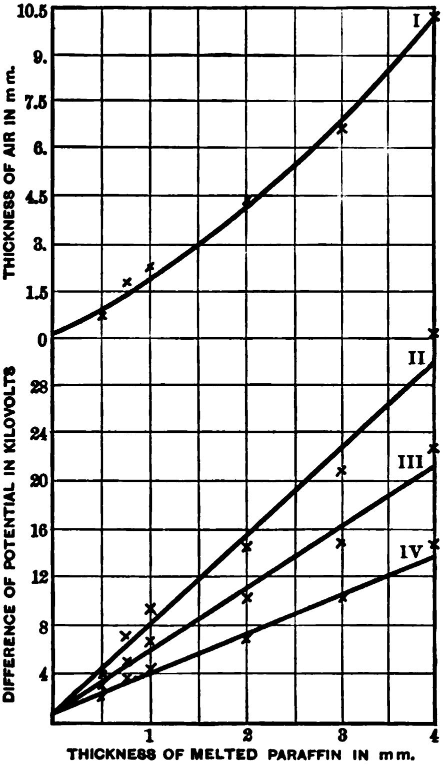

Fig. 2.

Though the results obtained with solids for the electrostatic gradient do not agree with one another as well as might be desired, still there is nothing to disprove the supposition that it is constant, and the more carefully the experiments were performed, the more nearly this quantity approached constancy.

Results.

FUNCTIONS FOR V (KILOVOLTS), r BEING MILLIMETERS.

Dielectric.	Own observations. Air reduced to P. D. by				Macfarlane. (*Trans. R. S. E.* Vol. 28.)	Steinmetz. (*Trans. A. I. E. E.* Vol. 10, No. 2.)
	Macfarlane's values.	Steinmetz's values.	Steinmetz's values modified as suggested at p. 104 below.	Baille's values.		
Paraffined paper	36.0 r	26.8 r	60.7 r	—	—	33.9 r
Beeswaxed paper	55.2 r	36.4 r	82.3 r	—	—	—
Kerosene oil	4.5 r	3.4 r	77.2 r	—	—	—
Melted paraffin. M. P. 52°	5.5 r	3.7 r	8.4 r	7.4 r	—	—
" " M. P. 76–80	5.9 r	4.0 r	9.1 r	8.1 r	—	8.1 r
Solid paraffin. M. P. 52° .	18.6 r	12.4 r	28.1 r	24.6 r	13.0 r	—
" " M. P. 42–44	30.0 r	18.6 r	42.2 r	42.0 r	—	—
" " 76–80 . .	—	23.3 r	52.9 r	53.0 r	—	—
" " 46–48 . .	30.0 r	15.2 r	34.5 r	43.5 r	—	—
Turpentine	—	—	—	—	9.4 r	6.4 r

Discharge along the Surface of a Solid Dielectric.

In my experiments with melted paraffin, I observed that the plates, dipping down into the vessel containing the paraffin, were attracted towards one another, making the real distance between the plates less than the reading on the scale. To overcome this difficulty, Mr. Steinmetz separated his plates by pieces of mica or glass of measured thickness. In this way he kept the disks at constant distance, but probably introduced an error; for a break in the continuity of a dielectric weakens it, as I have shown by comparing the length of a spark in air with the length of one produced by the same electromotive force along the surface between air and glass. I had observed in the first experiments with solid dielectrics that the spark would pass much farther along the surface of paraffin than through an uninterrupted air gap. At first I attributed this phenomenon to the presence of particles of conducting dust along the surface of the dielectric. On the 25th of April, 1894, I examined this subject more carefully, made a series of measurements of the surface lengths and equivalent air gaps, and obtained the results given in Table VII. and Fig. 5. Pieces of plate glass, broken so as to expose a smooth edge, were placed flat between one pair of the brass disks, with the smooth edge of the glass about the center of the disks; the machine was put in operation, and the other pair of disks were gradually separated until the spark preferred to pass along the surface of the glass. The distances between the pairs of plates were measured. I was greatly surprised to find that the ratio of the length of the spark in free air to that of the spark along the surface was very approximately constant, as expressed by the equation

$r = .442\,s$; where
r = sparking distance in air,
s = sparking distance along the surface between air and glass.

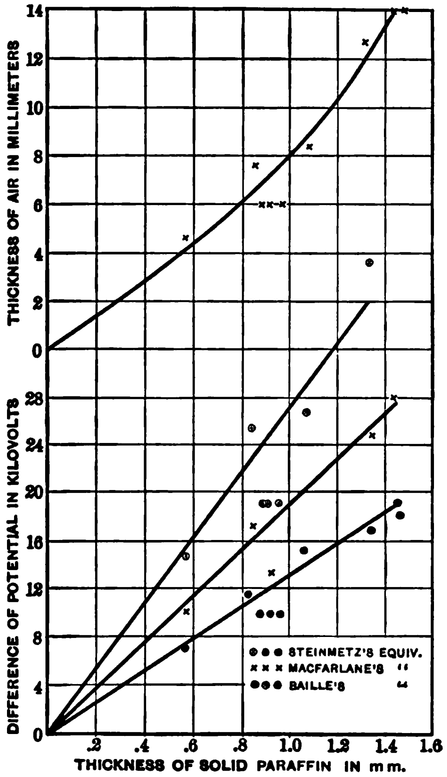

Fig. 3.

As all dust and moisture had been previously removed from the pieces of glass by rubbing them with chamois skin, this weakening of a gaseous dielectric by placing along with it a stronger dielectric could not have been due to the introduction of conducting particles. I have not had an opportunity to investigate the discharge along the boundary of other dielectrics, but believe that such an investigation would throw considerable light on the anomalous behavior of gaseous media.

Theoretical Examination of Results.

My experiments have not been conclusive. Not having at command the means of making absolute measurements of E. M. F., I have been compelled to make use of the air equivalents obtained by other experimenters. Their results do not agree. Dr. Macfarlane informs me that, while he is certain that his values are relatively correct, having carefully reduced them all to the same scale, he is not certain as to the correctness of the absolute magnitude. Mr. Steinmetz, in determining the electric strength of air, separated his plates by pieces of glass and mica, as he had done in his experiments on liquids, thus causing the spark to pass at much smaller P. D. than when the air between the plates was uninterrupted. He obtained the equation

$$\underset{\text{milli-cent}}{r} = 36(e^{-1.3\,V} - 1) + 54\,\underset{\text{kilovolts}}{V} + 1.2\,V^2.$$

According to my measurements above, Table VII. and Fig. 5, this value of r should be multiplied by .442, which would give, dropping the exponential term as it vanishes for all but very low values of r,

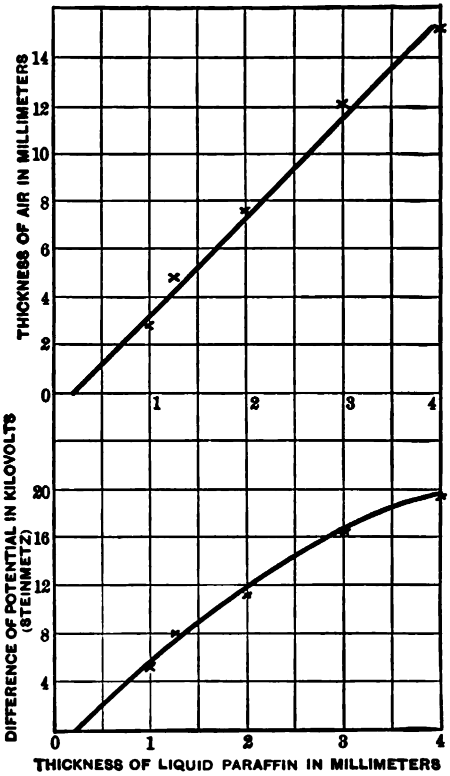

Fig. 4.

$$r = 24.268\,V + .5304\,V^2.$$

Steinmetz's measurements, thus modified, agree very well with those made by other experimenters, as may be seen by referring to Table IX. This table contains only observations of the length of sparks through air between parallel metal plates, produced by the given difference of potential.

The values of Mr. Steinmetz and Dr. Macfarlane are both probably correct relatively. The values given by Baille, in *Annales de Chimie et de Physique,* [5] 25, p. 486, 1882, appear to be the most reliable as to absolute measurement. Notwithstanding these discrepancies, I think we may consider it established that for solids and liquids the difference of potential necessary to force a spark through a dielectric is proportional to the length of the spark, or the electric strength is constant. This may be expressed by an equation of the form $V=kr$, in which V is the electromotive force, r the length of the spark, and k a constant depending on the substance tested.

For gaseous dielectrics, the electrostatic gradient diminishes as the distance between the plates is made greater, or the dielectric weakens as the length of the spark increases. In this case the potential difference can be expressed as a function of the thickness of the dielectric only by an equation of a higher degree.

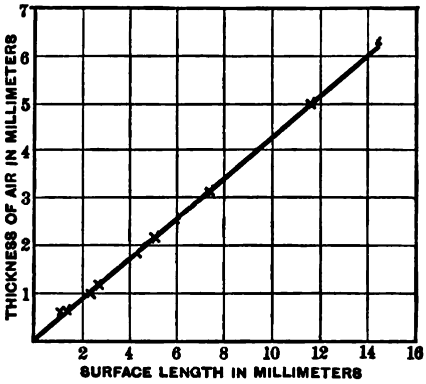

Fig. 5.

Upon examination of results, it appeared that this difference of behavior is due to convection in the case of the gas. This belief was strengthened by observing that when particles of dust or other solid impurities were suspended in a liquid, and the current passed as before, the liquid became very much agitated. The little particles were repelled from both plates, met in the middle with such energy as to heap up well-defined ridges at the surface, and then seemed to return to the plates as if they were alternately charged and discharged, as is the case with the air or dust particles in convection. When the liquid behaved in this way, I obtained a curve of results that, instead of being straight, was concave to the axis of abscissæ like the air curve. See Table VIII. and Fig. 4.

This assumption of convection seems to give the equations required.

First let us consider pure disruption. The energy of the condenser manifests itself as stress on the intervening insulating medium. At the moment of disruption, the total energy being dissipated, we have

$$W=\frac{AK}{8\pi r}V^2=Arf,$$

in which A=area of one of the plates, r=the distance between the plates, and f=the quantity of energy per unit of volume required to disrupt the dielectric. Whence

$$V^2=\frac{8\pi f}{K}r^2, \quad V=\sqrt{\frac{8\pi f}{K}}r.$$

In pure disruption f is constant and proportional to the resistance to the passage of electricity from each molecule to the next adjacent molecule.

When there is convection, a part of the energy is used up in charging the layers of molecules of the insulator in contact with the plates.

(The subsequent motion of the particles across the intervening space is a remanifestation of the energy expended in charging them, and does not require consideration.)

The molecules along the surface being dielectric, have a certain repugnance for electricity. The total surface resistance will be proportional to the area of the surface and repugnance per unit of area, as expressed by the equation $R=Ap$.

In discharge by both disruption and convection, as in air and other gases, this quantity must be added to Arf above, and equated to the total energy; whence,

$$\frac{AK}{8\pi r}V^2=Arf+Ap,$$

$$V^2=\frac{8\pi f}{K}r^2+\frac{8\pi}{K}rp,$$

$$V=\sqrt{\frac{8\pi}{K}}\sqrt{fr^2+pr}.$$

This formula agrees with that for air obtained experimentally by Dr. Macfarlane while at Edinburgh; .namely,

$$V = 66.94\sqrt{r^2 + .20503\, r},$$

and seems to be the rational expression of V in terms of r, if the discharge is purely disruption and convection. It is possible that, when the plates are exceedingly near each other, the condensation of the gas at the surface may introduce other terms. These approach zero as the distance increases, and may be neglected except for very small values of r. The formulæ deduced above apply only to discharge between plates of very great area compared with the distance between them.

Physical Laboratory, University of Texas.

Table I.

MELTED PARAFFIN. MELTING-POINT 52° C.

Thickness of paraffin in millimeters.	Thickness of air in millimeters.						Dif. of potential in kilovolts. (Average.) (Macfarlane.)	Dif. of potential in kilovolts. (Average.) (Steinmetz.)	Dif. of potential in kilovolts. (Average.) (Baille.)
	Temp. 83°–85° C.	Temp. 75°–80° C.	Temp. 70°–73° C.	Temp. 60°–65° C.	Temp. 55°–58° C.	Average.			
0.5	—	0.80	1.0	1.0	0.80	0.90	3.30	2.2	4.0
1.0	2.2	1.90	1.9	2.1	2.15	2.05	6.00	4.0	7.7
1.5	3.0	2.70	2.8	3.0	3.50	3.06	8.10	5.3	10.6
2.0	4.1	3.70	4.0	4.1	4.60	4.12	10.20	7.0	13.8
2.5	5.7	5.00	5.7	5.3	5.20	5.40	12.95	8.9	17.6
3.0	7.3	6.70	—	7.2	—	7.07	15.60	10.9	22.5
3.5	8.8	8.35	—	8.4	—	8.50	18.90	12.3	27.0

January 24, 1894.

Table II.

MELTED PARAFFIN. MELTING-POINT 76°–80° C. TEMPERATURE 80° C.

Thickness of paraffin in millimeters.	Thickness of air in millimeters.	P.D. in kilovolts. (Steinmetz.)	P.D. in kilovolts. (Macfarlane.)	P.D. in kilovolts. (Baille.)
0.5	0.90	2.2	3.3	4.0
0.75	1.90	3.6	4.8	7.4
1.00	2.25	4.1	6.6	8.4
2.00	4.25	7.0	10.2	14.2
3.00	6.50	10.2	15.0	20.6
4.00	10.25	14.7	22.5	32.1

Table III.

SOLID PARAFFIN. MELTING-POINT 52° C.

Thickness of paraffin in millimeters.	Thickness of air in millimeters.	P. D. in kilovolts. (Steinmetz.)	P. D. in kilovolts. (Macfarlane.)	P. D. in kilovolts. (Baille.)	Gradient kv. per mm. (Steinmetz.)	Gradient kv. per mm. (Macfarlane.)	Gradient kv. per mm. (Baille.)
0.92	6.0	9.7	13.5	19.2	10.5	14.6	20.9
1.32	12.6	17.0	25.2	39.7	12.9	19.1	30.0
0.96	6.0	9.7	—	19.2	10.1	—	20.0
1.44	14.0	18.5	—	—	12.8	—	—
0.58	4.6	7.5	10.8	15.2	12.9	18.6	26.2
1.42	14.0	18.5	28.2	—	13.0	19.9	—
0.91	6.0	9.7	—	19.2	10.7	—	20.1
0.82	7.8	11.5	17.1	24.9	14.0	20.8	30.3
1.06	8.3	15.2	—	26.3	14.3	—	24.8
Average gradient					12.4	18.6	24.6

Table IV.

SOLID PARAFFIN. MELTING-POINT 76°–80° C. BALL ELECTRODES.

Thickness of paraffin in millimeters.	Thickness of air in millimeters.	P.D. in kilovolts. (Steinmetz.)	P.D. in kilovolts. (Baille.)	Gradient kv. per mm. (Steinmetz.)	Gradient kv. per mm. (Baille.)
1.00	13.7	18.2	—	18.2	—
0.72	13.5	18.1	—	25.0	—
0.535	10.5	15.0	33.0	28.0	61.7
0.915	11.5	16.0	38.0	17.5	41.5
0.495	9.4	13.7	29.7	27.7	55.9

April 23, 1894.

Remark. — The above values for the thickness of air and difference of potential are evidently very low, as in other tests the same paraffin cakes would not break down under a much greater difference of potential.

TABLE V.

SOLID PARAFFIN. MELTING-POINT 42°-44° C. AIR ELECTRODES, PLATES; PARAFFIN ELECTRODES, BALLS.

Thickness of Paraffin in millimeters.	Thickness of air in millimeters.						Difference of potential in kv.			Gradient kv. per mm.		
	1st Trial.	2d Trial.	3d Trial.	4th Trial.	5th Trial.	Mean.	Macfarlane.	Steinmetz.	Baille.	Macfarlane.	Steinmetz.	Baille.
0.46	6.0	6.6	7.1	5.6	6.3	6.32	14.4	10.0	20.0	31.3	21.4	43.7
1.08	12.8	10.1	11.8	11.2	11.4	11.46	—	16.0	—	—	14.8	—
0.47	5.2	7.4 6.0	5.4	5.2	5.2	5.73	13.5	9.2	18.5	28.8	19.6	40.0
1.65	24.6	24.8	—	—	—	24.70	—	—	—	—	—	—
	Average gradient									30.0	18.6	42.0

TABLE VI.

SOLID PARAFFIN. MELTING-POINT 46°-48° C. AIR ELECTRODES, PLATES; PARAFFIN ELECTRODES, BALLS. PARAFFIN SOFT.

Thickness of paraffin in millimeters.	Thickness of air in millimeters.						P. D. in kilovolts.			Gradient kv. per mm.		
	1st Trial.	2d Trial.	3d Trial.	4th Trial.	5th Trial.	Average.	Steinmetz.	Macfarlane.	Baille.	Steinmetz.	Macfarlane.	Baille.
0.56	6.2	6.6	5.0	6.2	6.0	6.00	9.5	13.5	19.0	17.0	24.11	34.0
0.87	8.4	7.8	8.4	8.5	7.1	8.00	12.0	17.4	25.5	23.0	20.00	29.3
1.04	11.8	10.5	10.6	12.7	11.2	11.27	15.5	—	—	14.9	—	—
0.45	8.7	7.4	6.7	7.9	—	7.70	11.7	17.1	24.6	26.0	38.00	54.6
1.28	13.8	11.3	13.5	12.2	—	12.70	17.2	—	—	13.4	—	—
0.28	4.8	4.7	4.5	4.0	4.7	4.54	7.7	10.5	15.5	27.5	37.50	55.5
0.66	9.6	9.6	9.6	9.4	9.0	9.44	13.5	20.8	28.7	20.3	31.50	43.4
1.57	12.5	11.8	11.0	—	—	11.80	16.2	—	—	10.3	—	—
	Average gradient									15.2	30.00	43.5

April 27, 1894.

Table VII.

DISCHARGE ALONG THE SURFACE OF GLASS COMPARED WITH AIR.

s. Length of surface of glass in millimeters.	r. Thickness of air in millimeters.	Specific resistance. r/s.	Remarks. Average of
1.40	0.65	0.464	2 readings.
1.30	0.64	0.492	2 "
2.27	1.00	0.441	2 "
2.42	1.05	0.434	2 "
4.86	2.10	0.432	2 "
4.26	1.80	0.422	2 "
7.19	3.10	0.431	2 "
11.74	5.00	0.426	2 "
14.10	6.20	0.440	2 "

Mean specific resistance, 0.442.

$r = 0.442\,s$;

Where r = striking distance in air,

s = striking distance between glass and air.

April 25, 1894.

Table VIII.

CONVECTION EFFECT. MELTED PARAFFIN. MELTING-POINT 52° C. CONTAINING DUST PARTICLES PRODUCING PERCEPTIBLE CONVECTION. TEMPERATURE 70°–65° C.

Thickness of liquids in millimeters.	Thickness of air in millimeters.	Difference of potential in kilovolts. (Macfarlane.)	Difference of potential in kilovolts. (Steinmetz.)
1.00	3.0	7.01	5.4
1.25	4.8	11.40	8.0
2.00	7.6	16.65	11.5
3.00	12.1	—	16.5
4.00	15.0	—	19.6

TABLE IX.

DIELECTRIC AIR. LENGTH OF SPARK IN MILLI-CENTIMETERS.

Difference of potential in kilovolts.	Steinmetz — Disks 5 cm. diameter. (*Trans. Am. Inst. El. Eng.* February, 1893.)	Steinmetz's observations multiplied by 0.442.	Baille — plates. (*Thomson's Recent Researches in Electricity and Magnetism*, p. 70.)	Alex. Siemans — Disks of 10 and 3.7 cm. diameters. (*Electrotechnische Zeitschrift*, 1892, No. 19, p. 246.)	Bourne — Plates of 6.3 cm. diameter. (*Electrical Engineer*, N.Y., 1892, No. 205, p. 361.)	De la Rue and Müller. (*Mascart and Joubert*, Vol. 2, p. 195.)	Sir Wm. Thompson — Disks. (*Mascart and Joubert*, Vol. 2, p. 197.)	Macfarlane — Disks of 10 cm. diameter. (*Trans. Roy. Soc. Edinbr.*, 1878, Vol. 28, p. 633.)
1	30	13.3	12.5	24	—	21	13	—
2	77	31	35	48	—	43	42	—
3	132	58	60	73	—	66	62	—
4	192	85	90	105	—	91	100	112
5	255	113	120	139	150	118	127	135
6	323	143	150	171	—	147	—	160
7	396	174	177	206	—	180	—	172
8	468	207	210	240	—	215	—	213
9	540	239	245	277	—	250	—	260
10	622	275	280	317	300	286	—	305
12	800	354	350	400	—	365	—	405
14	988	437	420	480	—	—	—	505
15	—	—	450	—	570	—	—	—
16	1173	518	485	598	—	—	—	605
18	1320	583	555	755	—	—	—	705
20	1468	649	630	940	1000	—	—	800
25	2200	972	—	—	1300	—	—	—

A GENERAL THEORY OF THE GLOW-LAMP.[1] I.

By H. S. Weber.

THE phenomena of the glow-lamp are perhaps the simplest in the domain of radiation. The lamp consists of a conductor in a nearly complete vacuum, traversed by an electric current under the action of which it is heated to a stationary temperature such that the radiation from its surface consists in considerable part of visible rays. The simplicity of the conditions lead to the supposition that between the size and the temperature of the radiating surface, the wave-length and intensity of radiation, and the quantities by means of which the electrical energy developed in the filament are determined, there exist relations which are capable of expression in some simple form.

Up to the present time, only purely empirical attempts have been made to determine the relation between the amount of light emitted by the glow-lamp and its electrical constants. Jamieson and Preece believed the brightness of the glow-lamp to be proportional to the sixth power of the strength of the current. Voit concluded from the observations of Jamieson, and from numerous measurements made at the electrical exhibition of Munich upon many varieties of lamps, that the light was approximately proportional to the third power of the energy expended. Hess has shown that whenever the range of intensities becomes considerable, a member must be added to Voit's expression which is proportional to the energy. Finally, Slotte has drawn the conclusion from simultaneous measurements of the strength of the current and the intensity, that the light of the lamp is closely related to the fourth power of the current strength.

These experimental investigations could lead to no final result, since the most important element, viz. the temperature of the glowing filament, was left altogether out of account. Finding that the means of determining this temperature were lacking, only the most unreliable estimates concerning it, running from 2250° to 1200°, being attainable, I have spent several years in an attempt to establish an exact physical theory of the electrical glow-lamp which should cover all the observed facts, and have for this purpose made exact measurements upon more than thirty types of lamp, in order to determine the characteristic properties and to obtain the values of such constants of radiation for carbon as are necessary to a completion of the theory. I believe that I have now reached a point in these attempts where all the phenomena of the glow-lamp may be expressed by means of a few formulæ. Permit me to report briefly in the following pages the results of my investigations.

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[1] A paper read at the Frankfort Congress for Electrotechnics. Translated for the Physical Review, by E. L. N.

I.

The starting-point of my work was the establishment of a general expression by means of which the intensity of homogeneous radiation of a given wave-length emitted by a solid body at a given temperature might be expressed in terms of the wave-length and temperature, and of the size and nature of the radiating surface. No such general formula existed in the realm of radiation.

In 1888 I showed that the result of a great variety of observations could be expressed in this form, —

$$d^2s = \frac{dF \cdot dF_1 \cdot \cos w \cdot \cos w_1}{r^2} \cdot \frac{c}{\lambda^2} \cdot e^{aT - \frac{1}{b^2\lambda^2T^2}}, \qquad (1)$$

in which dF is an element of surface of the radiating body, dF_1 an element receiving this radiation, r the distance between the two elements of surface, w and w_1 the angles between the elements of these normals and the direction of the line joining them (r). This equation expresses the amount of energy of homogeneous radiation of the wave-length which is sent out from the surfaces of the element dF at the absolute temperature T to the surface of the element dF_1 in the unit of time. In this equation e has the usual signification; a is a constant common to all solids, viz. the value 0.0043; b^2 is a constant which should have the same, or nearly the same, value for various bodies; and, finally, c is a constant which varies from substance to substance. To distinguish them, I have named these three, respectively, constants of temperature (a), the illuminating power (b^2), and the constant of emissivity (c).

From Equation 1 it follows, in the first place, that the amount of energy radiated from the entire surface F in all directions in the unit of time is

$$s = c \cdot \pi \cdot F \cdot e^{aT - \frac{1}{b^2 \lambda^2 T^2}}. \qquad (2)$$

This quantity is to be called the intensity of homogeneous radiation.

It follows, further, that the sum of the intensity of all the homogeneous rays which are emitted at the temperature T (the total radiation of the body) is expressed by the equation

$$S = \int_{\lambda=0}^{\lambda=\infty} s \cdot d\lambda = c \cdot \pi \cdot F \cdot \frac{\sqrt{\pi}}{2} \cdot b \cdot T \cdot e^{aT} = C \cdot F \cdot T \cdot e^{aT}. \qquad (3)$$

The quantity $C = \frac{1}{2}\pi\sqrt{\pi c \cdot b}$ is to be called the constant of total radiation.

If the radiating body k is placed in an enclosed space, the walls of which have the common temperature T_0, the total loss of energy which K suffers in consequence of the radiation in the unit of time is given by the formula

$$\Delta S_{T,T_0} = C \cdot F(T \cdot e^{aT} - T \cdot e^{aT_0}), \qquad (4)$$

it being assumed that the surface F of the radiating body is very small as compared with the surfaces of the surrounding walls, an assumption which is admissible for all incandescent lamps. I have assured myself that Equation 2 expresses the result of all trustworthy measurements hitherto made upon the relation between wave-length and temperature of homogeneous rays. And it can be shown also that Equation 4 agrees completely with the carefully determined results of Schleiermacher upon the total radiation of platinum and the oxide of copper for temperatures between 273° and 1173°. Measurements of my own upon the total radiation of carbon between 300° and 700° were also capable of being expressed by Formula 4.

II.

Equation 4 affords a simple method for the determination of the temperature of the glow-lamp. After the temperature of the filament under the action of the current has become stationary, the amount of energy developed by the current in each unit of time plus the energy which the filament receives from the surrounding walls is equal to the energy which it gives out, so that for every such time-interval the equation

$$\Delta S_{T,\,T_0} = C \cdot F(T \cdot e^{aT} - T_0 \cdot e^{aT_0}) = i \cdot \Delta P$$

holds true, where i and ΔP are the current intensity which is regarded as constant, and the difference of potential between the terminals of the filament, respectively.

In the case here assumed, where $\Delta S_{T,\,T_0}$ is given in heat units, the equation takes the following form:

$$\Delta S_{T,\,T_0} = C \cdot F(T \cdot e^{aT} - T_0 \cdot e^{aT_0}) = \frac{i \cdot \Delta P}{J},$$

in which J is the mechanical equivalent of the calorie. It follows from this equation that by means of measurements of the quantities i, ΔP, T and F, the temperature of the filament can be determined, provided the constant C is known. This can be found by the estimation of the stationary temperatures which the filament attains under the action of known currents and potential differences. The value of these temperatures is given from the resistance of the filament, provided the dependence of resistance upon temperature has been determined by a series of experiments for the intervals of temperatures not given. The following table gives an example of the determination of the constant C, the total radiation of the filament of a specimen of the new Cruto lamp, the surface of which had been found by the length of the carbon and its cross-section at eleven points distributed along the filament at equal distances. This surface was found to be 0.647 sq. cm. The resulting value of $C = 0.0000171$ refers to gram-calories, centimeters, and seconds.

NEW CRUTO LAMP.

$F = 0.647$ sq. cm.

T	W	T	W
290°.0	332.08 Ω	440°.4	318.09 Ω
340°.3	327.68	490°.0	313.03
390°.3	323.01	540°.5	307.88

T_0 in the beginning = 290°.0; at the end = 290°.1.

i	ΔP	W	E	$\Delta S_{T,T_0}$	T	$\frac{T}{T_0}e^{a(T-T_0)}-1$	C
0.006526 A.	2.1472 V.	329.02 Ω	0.01401 W.	0.003344 cal.	325°.0	0.3027	169.2×10^{-7}
0.010051	3.2729	225.62	0.03289	0.007850	362°.5	0.7096	171.0
0.13176	4.2449	322.17	0.05593	0.013349	398°.9	1.1971	172.4
0.16542	5.2655	318.31	0.08710	0.020740	438°.2	1.8577	171.0
0.19986	6.2863	314.53	0.12564	0.029983	475°.3	2.6399	173.9
0.23358	7.2637	310.97	0.16966	0.040451	510°.2	3.5315	175.4
0.26690	8.2022	307.31	0.21892	0.052248	546°.1	4.6677	171.4

$M = 171.8 \times 10^{-7}$.

Following this plan, I have in the course of years determined the constant of total radiation for thirty-three varieties of glow-lamp. The summary of the results obtained shows that these thirty-three filaments belong to two groups. Three carbons, that of the Edison lamp, the new Cruto lamp, and of the lamp of Woodhouse and Rawson, have a radiation coefficient of 0.0000169 and 0.0000174; while thirty filaments give for the radiation-constant values between 0.0000127 and 0.0000132. The three carbons first-named are black; the others show gray surfaces. The mean value of C for these two different varieties of filament stand in the relation of 100 to 75.5. This is almost exactly the same relation which Leslie, at the beginning of this century, found for lampblack and graphite; viz. 100 to 75. It may, therefore, be asserted that incandescent lamp filaments with gray surfaces should be classified as graphitic carbon.

A knowledge of the constant C makes it possible to determine the temperature of the filament for every condition of incandescence when we know the size of its radiating surface. It is interesting to know what the temperature of the filament is when the incandescent lamp is in operation at normal brightness. The result is a surprising one; viz. that the normal temperature of all sorts of lamps is very nearly the same, and is included in an interval between $T=1565°$ and $T=1580°$ of the absolute scale. In the case of some lamps of very great brilliancy — that is to say, of lamps with thick filaments, which can be brought to higher temperatures without serious damage, and which can consequently be operated at higher economy — we find the normal temperature to be about 40° higher.

It is, further, of importance to know between what temperatures the filament changes when the candle-power of the lamp is varied through considerable range: from 2 candles to 30 candles for a 16 candle-power lamp, for example, or from 20 to 300 candles in the case of a 200 candle-power lamp. The result of measurements shows that the temperature variations which correspond to such changes in brightness amount to about 180°. In practice, accordingly, glow-lamp illumination covers a range extending from 1400° to 1600° for small lamps, and 1450° to 1650° for large lamps. If we confine ourselves in the study of incandescent lamps to this range of temperatures, we may write, in place of Equation 5, the simple form —

$$\Delta S_{T,T_0}=C\cdot F\cdot T\cdot e^{aT}=\frac{i\Delta P}{J}, \qquad (6)$$

since in these cases the quantity $T_0\cdot e^{aT}$ is very small as compared with the quantity $T\cdot e^{aT}$. This is shown by the following small table: —

T_0	$T_0 \cdot e^{aT_0}$	T	$T \cdot e^{aT}$
273 + 50°	1.295 × 10^8	1400°	576 × 10^8
+ 75°	1.554	1450°	740
+ 100°	1.854	1600°	1554
+ 125°	2.203	1650°	1989

In this simpler form, Equation 6 indicates in what degree the current must increase in order to bring about a given rise of temperature in the filament. To make this matter clearer, I have drawn up the following table: —

T	$T \cdot e^{aT}$	T	$T \cdot e^{aT}$	T	$T \cdot e^{aT}$
1400°	576 × 10^8	1500°	948 × 10^8	1600°	1554 × 10^8
1410	606	1510	996	1610	1634
1420	637	1520	1047	1620	1717
1430	670	1530	1100	1630	1803
1440	704	1540	1156	1640	1894
1450	740	1550	1214	1650	1989
1460	777	1560	1276	1700	2541
1470	816	1570	1340	1800	4136
1480	858	1580	1408	1900	6712
1490	902	1590	1479	2000	10860

From these data, and from the expression

$$\frac{d(T \cdot e^{aT})}{dT} \cdot \frac{1}{T \cdot e^{aT}} = \frac{1}{T} ta,$$

it may be seen that for the entire range of temperature from 1400° to 1600°, the following simple law holds true: viz. that every increase in the energy of the current amounting to ½ per cent raises the stationary temperature of the incandescent lamp 1°.

To show in greater detail the changes which the filament suffers by change of current and potential difference, and the changes which the resistance of the filament and the amount of light emitted suffer through change of temperature, I introduce the four following tables, which may also serve to indicate the manner in which the investigation of thirty-three types of incandescent lamps was carried out. In the eight vertical columns of the tables are given respectively ten values: The strength of the current i, the difference of potential ΔP, the resistance W, the energy E, the candle-power H, the quantity $\frac{H}{E_8}$, the efficiency $E_1 = \frac{E}{H}$, and the absolute temperature T. As unit of brightness, the British standard candle has been selected, and H denotes the mean horizontal intensity. Of the lamps to which these four tables refer, two had black carbons and two gray carbons. The quantity R in each table refers to the radiation factor for brightness—the factor, namely, by which the mean horizontal candle-power is to be multiplied in order to get the mean spherical candle-power. These quantities were determined by measuring the lamp in 132 positions as evenly distributed through space as possible.

WOODHOUSE AND RAWSON'S LAMP.

("60 V., 30 C.P.")

$F = 0.889$ sq. cm. $R = 0.780$. $C = 0.0000169$.

i	ΔP	W	E	H	$q = \frac{H}{E_3}$	$E_1 = \frac{E}{H}$	T
0.8768 A.	41.41 V.	47.23 Ω.	36.31 W.	1.82 N.K.	38.1×10^{-6}	19.95 W.	1400°
0.9855	45.18	45.85	44.52	3.60	40.8	12.37	1441
1.0967	49.01	44.69	53.74	6.55	42.2	8.20	1479
1.2108	52.79	43.60	63.90	11.07	42.4	5.77	1514
1.3264	56.72	42.76	75.23	17.85	41.9	4.21	1548
1.3853	58.60	42.30	81.18	22.38	41.8	3.63	1561
1.4460	60.59	41.90	87.61	27.61	41.1	3.17	1578
1.5058	62.48	41.49	94.08	34.13	40.9	2.76	1592
1.5675	64.46	41.12	101.05	41.16	39.9	2.55	1606
1.6290	66.32	40.71	108.63	48.35	38.4	2.23	1620

SUNBEAM LAMP.

("80 V., 5 A., 200 C. P.")

$F = 5.04$ sq. cm. $R = 0.711$. $C = 0.0000131$.

i	ΔP	W	E	H	$q = \frac{H}{E_1}$	$E_1 = \frac{E}{H}$	T
4.064 A.	53.90 V.	13.26 Ω.	219.0 W.	22.72 N.K.	2.16×10^{-6}	9.64 W.	1463°
4.218	55.74	13.21	235.1	28.47	2.19	8.26	1478
4.442	58.49	13.17	259.8	38.77	2.21	6.70	1498
4.747	62.26	13.12	295.5	56.82	2.20	5.20	1524
5.056	66.07	13.07	334.0	81.23	2.18	4.11	1549
5.377	70.00	13.02	376.4	115.1	2.16	3.27	1573
5.689	73.82	12.98	420.0	156.9	2.12	2.68	1595
5.980	77.49	12.96	463.4	205.8	2.07	2.25	1615
6.282	81.29	12.94	510.7	263.7	1.98	1.94	1636
6.584	84.99	12.91	559.6	334.7	1.89	1.69	1654

LAMP OF THE ALGEMEINEN ELEKTRIZITÄTS-GESELSHAFT, BERLIN.

("100 V., 16 C.P.")

$F = 0.760$ sq. cm. $R = 0.801$. $C = 0.0000129$.

i	ΔP	W	E	H	$q = \frac{H}{E_1}$	$E_1 = \frac{E}{H}$	T
0.4212 A.	77.19 V.	183.3 Ω.	32.51 W.	2.99 C.P.	87.2×10^{-6}	10.87 W.	1464°
0.4431	80.89	182.5	35.85	4.13	89.3	8.67	1483
0.4668	84.80	181.7	39.58	5.60	90.3	7.07	1503
0.4903	88.83	181.2	43.55	7.41	89.7	5.88	1522
0.5136	92.87	180.8	47.70	9.71	89.5	4.91	1541
0.5360	96.71	180.4	51.84	12.42	89.2	4.18	1557
0.5588	100.60	180.0	56.21	15.76	88.7	3.57	1574
0.5823	104.58	179.6	60.90	19.70	87.2	3.09	1591
0.6057	108.60	179.3	65.78	24.25	85.2	2.71	1607
0.6295	112.57	178.8	70.85	29.41	82.7	2.41	1621

NEW CRUTO LAMP.

("100 V., 0.59 A., 16 C.P.")

$F = 0.632$ sq. cm. $R = 0.734$. $C = 0.0000174$.

i	ΔP	W	E	H	$q = \frac{H}{E_3}$	$E_1 = \frac{E}{H}$	T
0.3948 A.	78.53 V.	198.9 Ω.	31.00 W.	2.21 C.P.	74.2×10^{-6}	14.03 W.	1434°
0.4233	83.40	196.9	35.30	3.32	75.9	10.63	1460
0.4506	87.56	194.3	39.45	4.69	76.4	8.41	1482
0.4924	93.79	190.5	46.18	7.58	77.0	6.08	1514
0.5180	97.60	188.4	50.56	9.93	76.8	5.09	1532
0.5444	101.12	185.7	55.05	12.75	76.4	4.32	1550
0.5706	104.41	182.9	59.58	16.10	76.1	3.70	1566
0.5990	108.27	180.4	64.85	20.67	75.8	3.14	1582
0.6305	112.11	178.7	70.68	26.60	75.3	2.66	1600
0.6579	115.99	176.3	76.32	33.06	74.2	2.31	1618

A RELIABLE METHOD OF RECORDING VARIABLE CURRENT CURVES.

By Albert C. Crehore.[1]

Introduction.

A PRACTICAL problem that has in more recent years presented itself to the electrician and physicist alike is "How shall we measure the exact current which flows in a conductor at any instant of time, and record all the irregular changes to which it is subject?" Probably every one who has thought of such matters at all has considered this problem in some of the phases which it presents. The importance of the question, since the introduction and extensive use of the alternating current, has emphasized the fact that we need a "reliable method" of measuring the instantaneous values of a variable current, which is not a "method by points," but "a method which continuously records the current."

Under "a method by points" is included any method in which the current is obtained from readings (usually of an electrostatic voltmeter) due to the charge of a condenser which may be connected in at any point of time. The essential characteristic of the method is that the current is *supposed* to repeat itself exactly during successive periods, or more generally when the conditions are exactly repeated. There can be no doubt that the current *does repeat* itself under exactly similar conditions, but can we be *sure* that those conditions *are exactly repeated?* By this method a number of points are found, the time occupied being at least several minutes, and the collection of points properly arranged is a representation of the current during as short a time as the one-hundredth of a second, perhaps. Yet this method has proved to be a very useful and practical one, and has given us information concerning the currents and potentials of generators and transformers which is of paramount importance. Yet all will agree that this "method by points" is too limited in its application, and does not show us any sudden temporary change taking place in a current which does not *repeat* itself. Such, for instance, as a sudden "make," or "break," or "change" in an alternating current would not be easily shown by this method. The second method, previously designated "*a method which continuously records the current*," is the one to which this paper more particularly refers. Under this head is included all methods which attempt to record the current by causing it, either directly or indirectly, to move a material something so that its displacement is some single valued function of the current. As an example of this method may be mentioned the well-known experiments of Froelich in which a telephone is used, upon the disk of which is mounted a mirror that permits a beam of light to be reflected from it. Any vibration of the disk gives an angular motion to the ray of light, and this motion is in turn recorded upon a moving photographic plate. Other examples might be mentioned in illustration of this method, for instance, a wire which is deflected in a magnetic field, or stream of mercury so influenced; but it will be noticed that in all of these cases an appreciable amount of *ponderable* matter is required to be moved backward and forward during each reversal of the current. When the current reverses hundreds of times per second, the unavoidable difficulty is introduced that the forced oscillations of this ponderable matter, no matter how small in amount, become so superimposed upon those of the current which it is desired to measure that they are inseparably mixed together; and the record does not show the true current, but the resultant vibrations of the instrument. That this is the case with the method of the telephone above referred to, has been established beyond a doubt it seems, by experiments conducted at Cornell University by Mr. Henry Floy. The current furnished to the telephone was carefully measured by the "method by points," and care was taken to see that the current as measured by points was the same as that used in the telephone. The vibrations of the telephone did not even approximately agree with the current as measured by well-established methods.

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[1] A paper presented at the General Meeting of the American Institute of Electrical Engineers in Philadelphia.

Bearing these points in mind, and remembering the high frequency of some of the oscillations which it is desired to record, may we not with some degree of certainty predict that any of these methods requiring the rapid motion of ponderable matter will be open to precisely the same objections which are noticed in the case of the telephone? Without answering this question, probably all will agree that the difficulty may *certainly* be avoided by using as a vibrator, instead of this so-called "*ponderable matter*," a vibrator that has *no* weight. It is to this question of finding a form of vibrator *without weight* that I invite your attention.

The Weightless Vibrator.

The idea of the weightless vibrator is perhaps already suggested in the beam of light. But how shall we cause a beam of light to have a change in direction simply by means of a current flowing in a circuit without the intervention of some moving material? A way of influencing a beam of light directly by an electric current (or more properly by its magnetic field) is that discovered long ago by Faraday. It is by means of the discovery of the rotation of the plane of polarization by an electric current that I propose a method of obtaining a weightless vibrator. The explanation will be made clearer by reference to the diagram of apparatus (Fig. 1). A beam of light is passed through a polarizer (Nicol prism), so that the vibrations of the beam take place in only one plane upon emergence. If it is then passed directly through an analyzer (Nicol prism) the latter may be set at such an angle as to prevent all light from passing through it, and thus produce darkness beyond the analyzer. Faraday's discovery was that if a beam of polarized light is passed through some *substance* in the direction of the lines of magnetization within that substance, there is a rotation of the plane of polarization in a direction which is the same as the direction of the current required to produce such a magnetic field. The direction of rotation is unaltered, therefore, whether the light beam advances in the same or the opposite direction to the magnetization, so that a beam reflected back and forth through the substance several times has its rotation increased by equal amounts each time. If the direction of the ray of light be at right angles to the lines of magnetization, there is no rotation produced. The amount of this rotation has been carefully investigated by Verdet, who announced laws by which it may be expressed. They are summed up in the following statement:

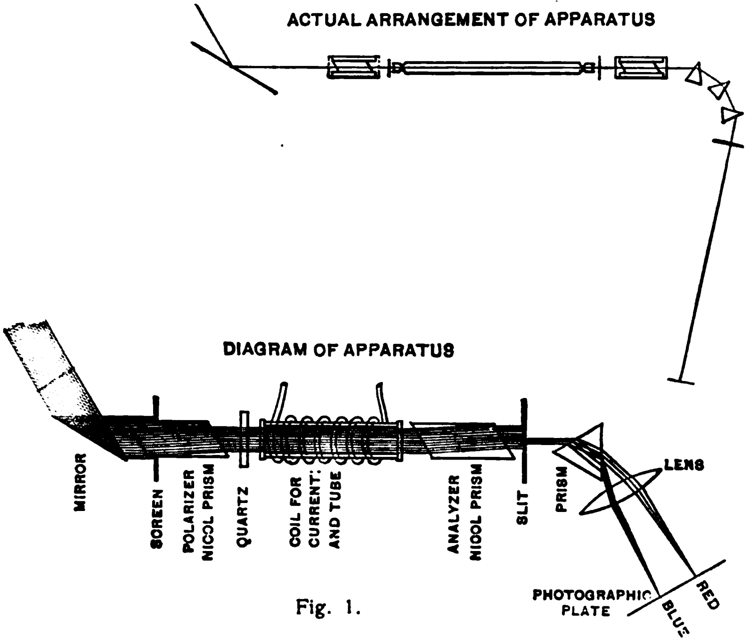

Fig. 1.

"The rotation of the plane of polarization for monochromatic light is in any given substance proportional to the difference in magnetic potential between the points of entrance and emergence of the ray"; that is, it is equal to a constant times this difference of potential, and is expressed by the formula

$$\theta = v V, \tag{1}$$

where θ=angle of rotation, V=difference in magnetic potential, and v for a given wave-length is constant in any one substance. This constant is known as Verdet's constant. If now the light is passed through the polarizer and then through a tube containing the substance used, around which is wound a coil of wire, and thence through the analyzer, an observer would find complete darkness upon looking through the analyzer when set in the crossed position. But if without moving the analyzer a current is sent through the coil on the tube, light appears to the observer. This is because the plane of polarization has been rotated by the current, and practically the prisms are no longer crossed. Now let the analyzer be rotated while the current is still flowing, and the observer will see a series of beautiful colors through the analyzer, a different one for each position of it; but as long as the current flows, he cannot produce darkness again by any amount of rotation of the analyzer.

This effect suggests what is known to be a fact, that the different wave-lengths composing white light are rotated by the current in different amounts, so that when the analyzer is turned to the angle corresponding to the yellow light, say, only the yellow light is prevented from passing through the analyzer. All the other rays, being rotated by different amounts, pass through the analyzer, and there being mixed together they give rise to the series of beautiful complex colors above mentioned. A different color is seen for each position of the analyzer because in each position a different color is subtracted from white light, and the observer sees what is left, or merely the complementary color.

The law which tells the amount of rotation given to the different colors is pretty accurately known; and theory in this case is in close accord with the observed facts. The equation which closely expresses the amount of the dispersion for the different wave-lengths may be written: —

$$v=\frac{cn^3}{\lambda^2}\left(1-\frac{\lambda}{n}\frac{dn}{d\lambda}\right), \tag{2}$$

where v is the so-called Verdet's constant, λ the wave-length, and n the index of refraction of the medium. c is a constant for any one medium, which is, however, for different media, inversely proportional to the permeability of the medium. This is a formula at which Maxwell arrived from his theory of molecular vortices, and we shall see how closely it is in accord with observation. We see by this formula that Verdet's constant depends not only upon the wave-length, but upon the index of refraction corresponding to that particular wave-length, and also upon the rate of change of the index with respect to the wave-length. If this rate of change of n with respect to λ is small, as would be the case with a substance where the dispersion is small, and the index of refraction regarded as approximately constant, then it is seen that the formula reduces to an extremely simple form, viz.: —

$$v=\frac{c_1}{\lambda^2} \tag{3}$$

Here Verdet's constant is inversely proportional to the square of the wave-length. Using this approximate form for the present, we see from Verdet's law, equation (1), that

$$\theta=vV=c_1\frac{V}{\lambda^2}. \tag{4}$$

But the difference of magnetic potential, v, is $\frac{4\pi Si}{10}$, where Si is ampere turns, and thus we have

$$\theta=4\pi c_1 Si/10\,\lambda^2=c_2\,i/\lambda^2, \tag{5}$$

where

$$c_2=4\pi c_1 S/10. \tag{6}$$

A reference to Fig. 2 will show this relation between angle of rotation, wave-length, and current. Several spirals are shown, corresponding to the several lines of the spectrum, known as *A*, *B*, *D*, *F*, and *G*. The radii of the circles which intersect these spirals are proportional to the current flowing in the circuit, while the angle, which the radius, drawn to any point of intersection, makes with *OP*, represents the rotation for that particular wave-length and current. The spiral *OA* is in the extreme red, and *OG* in the violet of the spectrum, and the diagram thus indicates that the red rays are not rotated so much as the blue. The direction of rotation in the diagram is as indicated by the arrow. Now, returning to the observer looking through the analyzer, if he could resolve the light there seen into the pure colors of the spectrum, what he should expect would be, with no current, a complete spectrum, since *all* rays are rotated by the current. But let him rotate the analyzer, and he finds that first one color then another disappears, and a *dark band* is seen to move across the spectrum as he rotates the analyzer. Again, let him rotate the analyzer to a certain angle and leave it there while he varies the current. He should expect that the band would move, but would vanish entirely with zero current, and thus prevent observation for small currents.

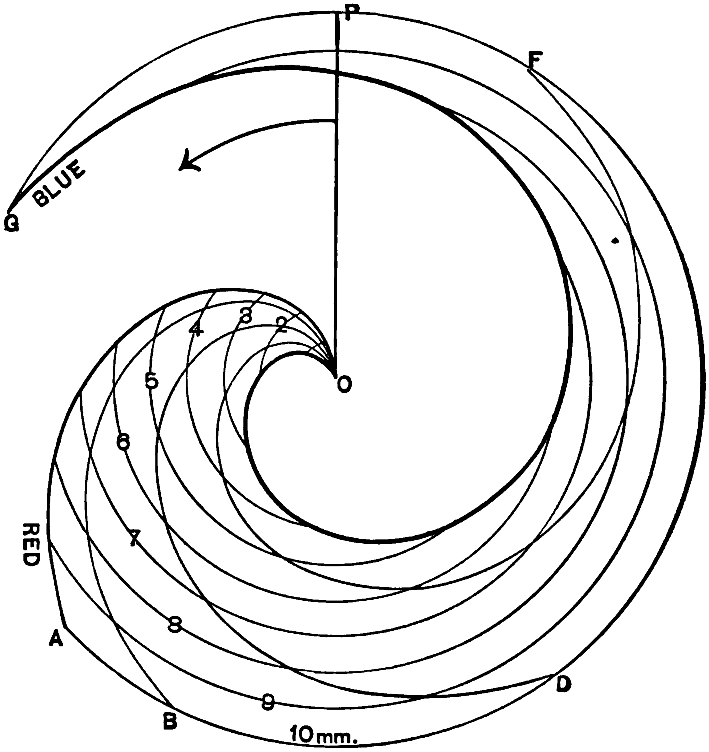

Fig. 2.

Fortunately we have substances which naturally rotate a beam of polarized light, for by means of this aid we may obviate the difficulty that the band vanishes with no current. For instance, a parallel plate cut from a crystal of quartz perpendicular to the optic axis has this property of rotating the plane of polarization. Quartz is selected for the material used because of its great transparency and high specific rotary power. The law of the rotation is similar to that already mentioned for the rotation by the current. The approximate law is that the rotation is inversely as the square of the wave-length, which may be expressed:—

$$\phi = c_8 e/\lambda^2, \tag{7}$$

where ϕ is the angle of rotation for the wave-length λ, c_8 is a constant, and e is the thickness of the plate. The thickness of the quartz plate is seen to correspond to the current in equation (5). Fig. 2 represents the actual rotation for different thicknesses of quartz, each circle corresponding to a plate one millimeter thick. The equivalent of a quartz plate one millimeter thick is represented approximately by 35,700 ampere turns wound upon a tube containing carbon bisulphide. This latter is the substance used, being selected on account of its high transparency and specific rotation.

If a quartz plate be placed between polarizer and analyzer, the effect is the same as if the current circulated around the tube of carbon bisulphide, and we may, by rotating the analyzer, move the dark band completely across the spectrum by means of the quartz plate without any current. But suppose we set the analyzer so that the dark band remains in the center of the spectrum, and then pass a current through the coil. We observe a motion of this dark band back and forth through the spectrum as the current is repeatedly reversed. For any given current its position is always the same, so that its motion may be calibrated by passing known currents through the coil. Have we not in this found a weightless vibrator that is sure to move in unison with the currents, if the term is allowed?

Before passing on to the more practical side of the question, it may be asked, will this band move back and forth so that its displacement is approximately proportional to the current? The answer to this question lies so near at hand that your attention is invited to it for a moment. The rotation of the plane by quartz is approximately represented by the formula:—

$$\phi = c_8 e/\lambda^2.$$

The rotation by the current is represented by

$$\theta = c_2 i\,/\,\lambda^2.$$

If both these rotations take place together, the resultant is merely the sum of the components, and

$$\chi=\phi+\theta=(c_8e+c_2i)\frac{1}{\lambda^2}. \tag{8}$$

The position of the dark band depends upon the position of the analyzer. Let the analyzer be set at some convenient angle, a, with the position of complete darkness, and let it remain there. Then the wave-length or color where the dark band occurs for the quartz plate (being called λ_0) is given by (7) above, and we have

$$a=c_8\frac{e}{\lambda_0^2}. \tag{9}$$

The wave-length corresponding to this constant angle, a, when both quartz and current are used, is given by (8), and we have

$$a=(c_8e+c_2i)\frac{1}{\lambda_2}, \tag{10}$$

in which λ is that wave-length corresponding to a certain current, i, and therefore i and λ are coördinate variables. Equating the values of a, we have

$$\frac{c_8e}{\lambda_0^2}=\frac{c_8e+c_2i}{\lambda^2}. \tag{11}$$

This may be written

$$\lambda^2=\frac{c_2\lambda_0^2}{c_8e}i+\lambda_0^2, \tag{12}$$

and in this form the relation between wave-length and current is seen to be represented by a parabola. In Fig. 3 are represented two sets of parabolas obtained from equation (12) by assuming that λ_0 and e take in succession different values. Where $e=2$ mm. the set of parabolas marked I. is obtained, and where $e=6$ mm. set II. is obtained. By giving λ_0 different values we merely vary the parameter of the parabola without changing the origin. It will be remembered that λ_0 represents that wave-length corresponding to the position of the dark band for no current. It is therefore the value of λ, when i is equal to zero, as appears from the equation independently. The axis of λ is the vertical line to the right of the figure, upon which the letters *A*, *B*, etc., are written. These letters show the positions of the various Fraunhofer lines, and one parabola is drawn for each line. Each parabola then corresponds to one setting of the analyzer that the dark band is found at these lines of the spectrum for zero current. The upper parabola is at the red end, and the lower at the blue end, of the spectrum. The axis of current is the base line of the diagram, and currents to the left of the vertical line are called negative, while those to the right are called positive. The axes of all the parabolas coincide with each other and with that of the current, and for a given quartz plate they all intersect this axis at the same point, so that taking different settings of the polarizer is equivalent to changing the parameter only of the parabola.

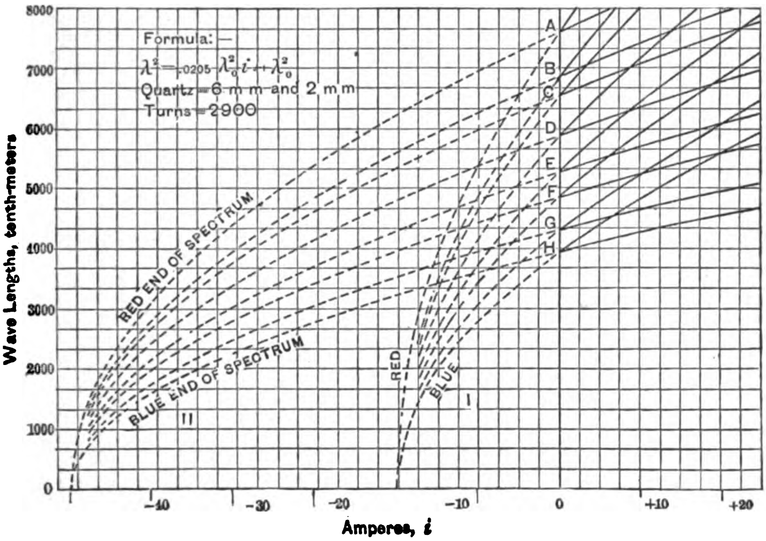

Fig. 3.

The interpretation of these results may be put as follows: If we have a spectrum in which the wave-lengths are proportional to the distances along the spectrum (which is the case with Professor Rowland's arrangement of a concave grating), then the displacement of the dark band to one side or the other, due to the current, will be exactly according to the shape of these parabolas near the zero point; that is, near the vertical line lettered *A*, *B*, etc. Since it appears that each parabola at such a great distance from the origin is nearly a straight line, the displacement in such a spectrum will be nearly proportional to the current.

A noticeable feature, easily revealed by the graphical construction, is that in the red end of the spectrum, where the inclination of the parabola to the i axis is the greatest, the motion of the band will be the greatest for a given current.

Of course it is understood that this construction has to do with the relation between the wave-length and the current, and not between the displacement and current, unless the wave-length and displacement are proportional. It does not apply, for instance, to the displacement in the spectrum of most prisms. In the prisms used the red rays were so crowded together that the motion as observed was nearly the same in the red as in the blue. The width of the band, however, is for this reason narrower in the red than in the blue, — a consideration of considerable practical importance.

Description of Some of the Apparatus.

The tube upon which the coil carrying the current was wound was a glass tube 1.4 cm. internal, and 1.8 cm. external, diameter, and 70.15 cm. long. The tube was filled with carbon bisulphide, which was confined in the tube by means of two plane parallel plates of glass, each 1.3 cm. thick, fitted tightly upon the ground ends of the tube. Upon this tube were wound six layers of No. 18 double cotton copper magnet wire, occupying a length on the tube of 61.5 cm. The wire was wound so that 100 turns occupied 12.7 cm. Thus the total number of turns, 2900, is very large considering the size of wire.

The light used was sunlight reflected from the mirror of the heliostat.

The Nicol prisms are two fine specimens which were obtained by Dartmouth College at a time when larger specimens could be obtained than may now easily be found. The width of the slit does not need to be very narrow. A width of a quarter to a half millimeter will do better than a narrower one, because more light is admitted to the photographic plate, and in passing through so many different substances even sunlight is rendered comparatively feeble by the time it strikes the photographic plate.

A further description of the apparatus is hardly deemed to be necessary, inasmuch as no claim is made to having obtained more than the most crude of first results, which may be the results obtained by apparatus arranged in a comparatively poor manner for the end sought. Yet the results obtained seem to be so promising for the future that the subject is presented to you at this early date in the experiment in the hope that it may soon receive an impetus from other experimenters who have better facilities than those at my disposal, and thus become a fruitful source of extending our knowledge of instantaneous current flow in conductors.

The objections which most naturally suggest themselves against this method of taking current curves are perhaps the following. The photographic plate must move so quickly that the time of exposure of any one part of the plate is extremely short. To meet this demand the most sensitive plates that can be made should be used. The width of the band with any given plate depends largely upon the time of exposure. Then, too, a plate is to be desired that will photograph toward the red end of the spectrum as well as in the blue. The band does not possess very sharp outlines, but gradually shades off from dark to light.

These objections do not have so much weight, however, in cases where the general direction of the variation of the current is what is wanted more than any exact measurement of its amount, and in the majority of cases this is really what is wanted. Yet it cannot be said that in these preliminary experiments the band used was nearly as sharp as may be obtained.

Another objection of a different nature that seems difficult to avoid is the fact that the coil, which is wound upon the tube, must necessarily possess a small amount of self-induction. It may be said, however, that even though we are prohibited from measuring certain currents on account of this self-induction, we are always sure that we are measuring the exact current which is flowing through the coil.

A More Exact Expression of the Relation between the Wave-Length and Verdet's Constant.

The approximate relation between the wave-length and Verdet's constant used above was that Verdet's constant varied inversely as the square of the wave-length. It is considered of sufficient interest to inquire just how nearly this is an approximate formula.

By reference to equation (2) it is evident that if we only knew the relation between the index of refraction and the wave-length, we might obtain the relation between the wave-length and Verdet's constant in terms of these two quantities alone and constants.

Such a relation is afforded by Briot's formula, which is a modification and improvement upon the well-known formula of Cauchy. This is

$$1/n^2 = k\lambda^2 + A + B/\lambda^2 + C/\lambda^4 + \cdots, \tag{14}$$

where n is the index of refraction corresponding to the wave-length λ, and k, A, B, etc., are constants for the given substance. Assuming that all terms beyond B/λ^2 are negligible, we may differentiate with respect to λ and obtain the equation

$$\frac{\lambda}{n}\frac{dn}{d\lambda} = n^2\left(\frac{B}{\lambda^2} - k\lambda^2\right). \tag{15}$$

Upon eliminating B/λ^2 between (14) and (15), we obtain

$$\frac{\lambda}{n}\frac{dn}{d\lambda} = 1 - 2\,kn^2\lambda^2 - An^2. \tag{16}$$

Substituting in (2) the expression thus obtained, we have

$$v = cn^5(2\,k + A/\lambda^2). \tag{17}$$

But by (14)

$$n^5 = (k\lambda^2 + A + B/\lambda^2)^{-\frac{5}{2}}. \tag{18}$$

Hence

$$v = c(2\,k + A/\lambda^2)(k\lambda^2 + A + B/\lambda^2)^{-\frac{5}{2}}. \tag{19}$$

This formula represents to a high degree of accuracy the observed values of Verdet's constant for carbon bisulphide. It probably would for any other substances, but carbon bisulphide is the only one to which it has been applied by me. The constants k, A, and B may be found by means of equation (14) and the observed values of the refractive index for known wave-lengths. The values used are those observed by Messrs. Gladstone and Dale. (See Glazebrock's Physical Optics, page 243.)

Line of spectrum.	Index of refraction for carbon bisulphide.	Line of spectrum.	Index of refraction for carbon bisulphide.
A 7621	1.6142	E 5270	1.6465
B 6870	1.6207	F 4861	1.6584
C 6563	1.6240	G 4308	1.6836
D 5893	1.6333	H 3969	1.7090

These values give curve I, in Fig. 4. The lines *A*, *F*, and *H* were selected and three simultaneous equations formed from (14), so that the resulting curve II. should pass through these three observed points.

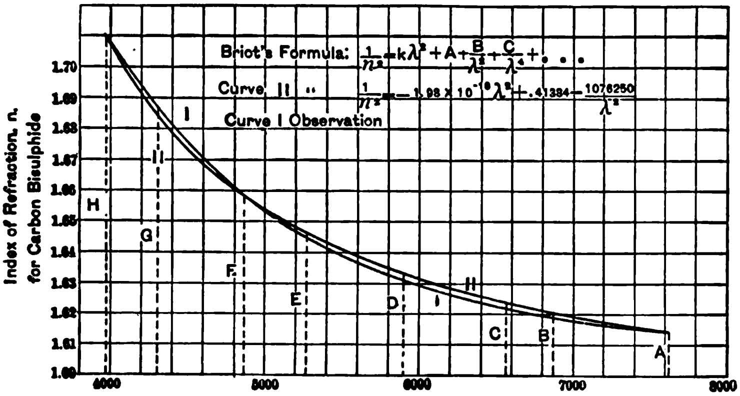

Fig. 4.

The values

$$k = -1.98 \times 10^{-10}, \tag{20}$$

$$A = .41384, \tag{21}$$

$$B = 1076250., \tag{22}$$

were obtained by the determinant solution of these three simultaneous equations. The resulting black curve may not appear to coincide very closely with the red, but it must be remembered that the origin of coördinates is a long distance below the paper, and the apparent differences are but a very small fraction of the whole. The unit is the tenth meter for wave-lengths.

Having these constants, they may now be substituted in (19), and *c* determined from the observed values of Verdet's constant. These observed values are: —

Line of spectrum. λ.	Verdet's constant for carbon bisulphide. v.
C 6563	0.0319′
D 5893	0.0415′
E 5270	0.0537′
F 4861	0.0667′
G 4308	0.0920′

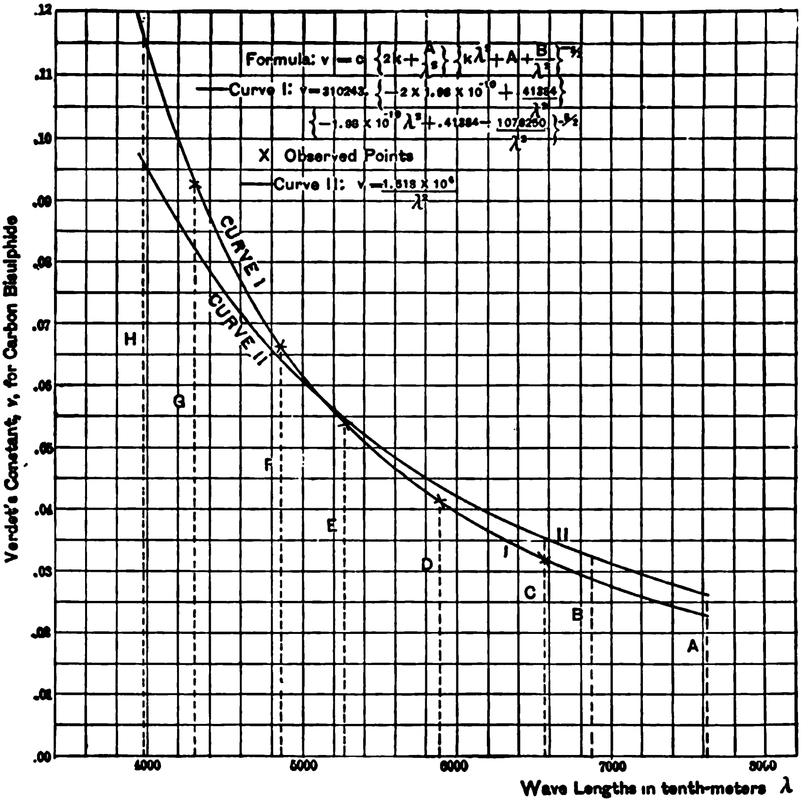

Fig. 5.

The constant thus determined, where a minute is the unit of angle, gives

$$c=310243. \tag{23}$$

Using these constants for equation (19), we obtain curve I. in Fig. 5. The points marked × are observed values, and the calculated curve practically passes through them all.

The curve II. in this diagram represents the approximate law that Verdet's constant is inversely as the square of the wavelength, and the degree of the approximation may be observed.

MINOR CONTRIBUTIONS.

A NEW FORM OF SPECTROPHOTOMETER.

BY EDWARD L. NICHOLS.

IN 1890 the writer described a form of spectrophotometer, the characteristic feature of which was the use of a horizontal, instead of a vertical, slit.[1] This instrument, which was intended for use upon the photometer bar, I called the horizontal slit photometer. It is shown in Figs. 1 and 2, which are taken from a model constructed since the writing of the paper just cited.

Recently further modifications have been made, and certain desirable features have been introduced. These it is the purpose of this note to describe.

In its original form the horizontal slit photometer is a direct vision instrument, in which the requirements of symmetry are carefully met. The light is introduced to the right and left halves of the slit by means of a pair of rectangular prisms for total reflection, one of the acute edges of each of which has been ground away so as to give two surfaces about 2 mm. in width (see Fig. 3). These narrow ground faces are placed together, as shown in the figure, in a plane which bisects the slit transversely; their image forms the boundary between the two spectra in the field of view. Thus in the method of introduction into the collimator tube, the two rays of light are treated in a precisely similar manner. Within the tube, and along the entire path to the eye, likewise, identity of treatment is main-

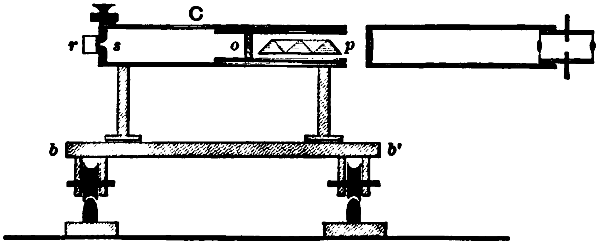

Fig. 1.

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[1] Trans. of the American Institute of Electrical Engineers, Vol. 7.

tained. This is a point in which many forms of spectrophotometer are at fault.

The prismatic spectrum, although superior to normal spectra as regards brightness, is objectionable for the purposes of spectrophotometry, because the less refrangible rays are crowded together. In the orange-yellow and yellow-green particularly, uniformity of tint cannot be secured unless the field of view be made extremely narrow, or high magnifying power be resorted to. In prisms of the densest flint, moreover, the violet is very dim, owing to the strong selective absorption in that region. The diffraction spectrum is free from these peculiarities.

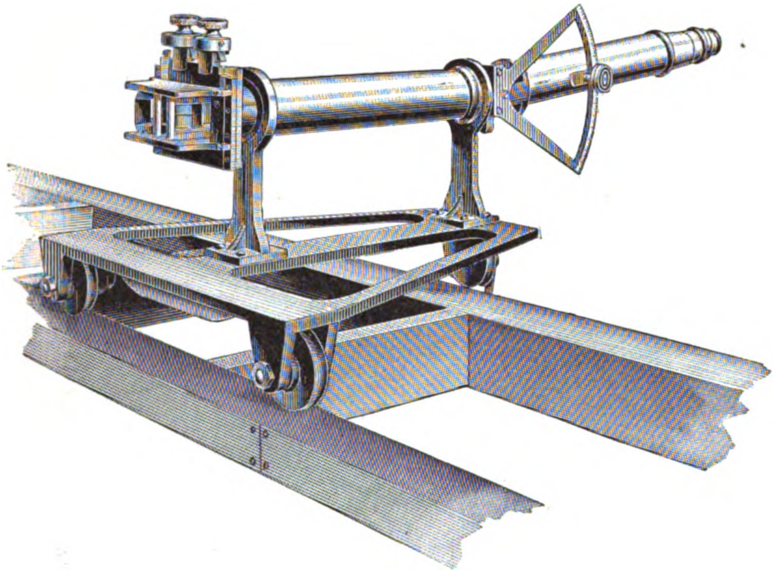

Fig. 2.

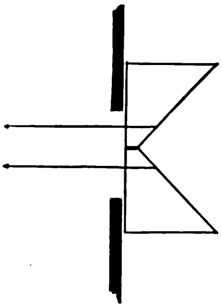

Fig. 3.

The modification of the horizontal slit photometer to be described here contains a grating instead of a prism, and it possesses the further advantage that it brings the slit, with whatever accessory apparatus may be selected for the adjustment of the spectra under comparison to equal brightness, within easy reach of the experimenter. The form of the instrument is shown in Figs. 4 and 5, the latter of which is a simple diagram of the essential parts. The collimator tube is mounted rigidly in a horizontal position upon a massive base. It carries a Vierordt by means of which the brightness of the two spectra may be adjusted.

Four years of constant use have verified my opinion as to the advantages of the horizontal slit in spectrophotometry: accordingly in the new instrument the slit is placed horizontally and the light is introduced by means of a pair of reflecting prisms such as have been indicated in Fig. 3. The observing telescope swings between a pair of vertical arcs which are rigidly attached to the base of the instrument, the axis of rotation being a horizontal line which cuts the optical axis of the collimator perpendicularly at a distance of about twenty centimeters from the objective of the latter. In an adjustable holder swinging in this same horizontal axis is mounted a plane Rowland grating which, receiving parallel rays from the collimator tube, produces two parallel sets of vertical diffraction spectra. The grating and the observing telescope being free to move independently about the same horizontal axis, any portion of any spectrum may be brought into the field of view. After adjustment the grating is held in position by means of a light arm which moves along the face of one of the sectors

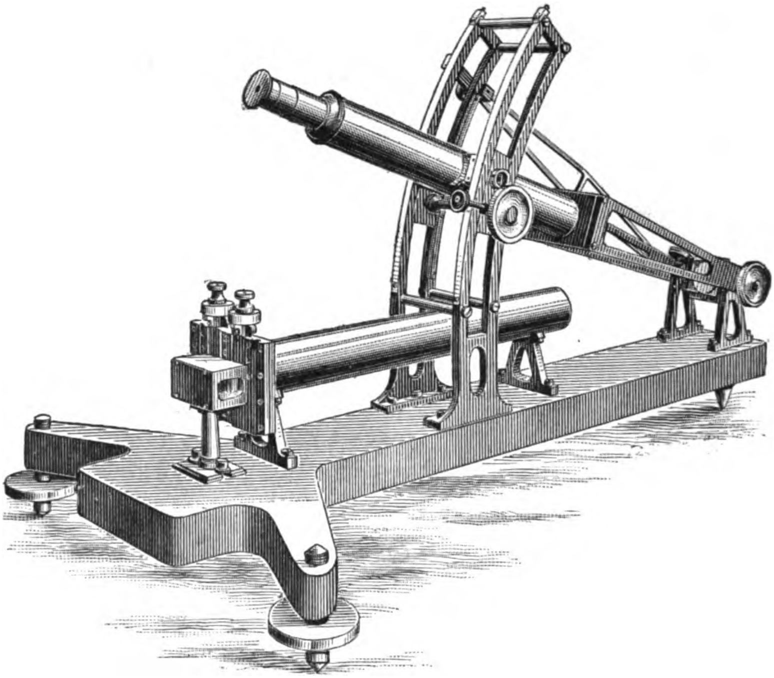

Fig. 4.

already described. The two sectors are provided with scales; the graduation of each is in equal parts.

This form of horizontal slit photometer, which, to distinguish it, may be called the *vertical arc* pattern, may be mounted upon a carriage and used in the photometer room like the earlier form, or two pairs of Nicol's prisms may be mounted to the right and left hand of the slit, by which means the spectra may be brought to equal brightness, or, best of all, where the character of the work will admit of such a method, the brightness of the two sources of light, the spectra of which are to be compared, may be adjusted by means of micrometer slits placed between them and the collimator slit. The use of such slits in combination with the ordinary Vierordt slit, which, taken by itself, can only be used to advantage between very narrow limits, gives a sufficient range, together with great simplicity of action, and the freedom from the errors to which polarizing spectrometers are subject. The method is applicable, however, only in cases in which the sources of light can be made to fill the field of these adjustable slits completely. Where this is possible this method is one of the most satisfactory of those which are available in spectrophotometry.

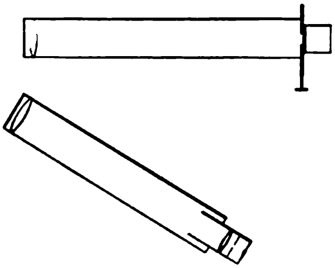

Fig. 5.

A PHONOGRAPHIC METHOD FOR RECORDING THE ALTERNATING CURRENT CURVE.

By C. J. Rollefson.

In this paper it is intended to give only a short description of a method devised by the writer for the study of alternate current phenomena. After a number of devices had been tried, the use of the phonograph suggested itself. The phonograph records vibrations of sound, which are similar to the vibrations produced by an alternating dynamo, and reproduces the quality of the sound to such a high degree of accuracy that it seems that it might prove an accurate instrument for recording the alternating current curve.

The method, in brief, consists of two operations: first, a record of the curve must be produced on the wax cylinder of the phonograph; second, the indentations produced on the cylinder in the first operation must be magnified by means of a suitable multiplying arrangement.

A section of the arrangement for producing a record of the curve on the wax cylinder is shown in Fig. 1. A small steel magnet, *a*, is fastened to the center of the glass diaphragm, *b*. Around a small bundle of fine iron wires, *c*, are wound a few turns of insulated copper wire, the small electro-magnet thus made being fastened in front of the small steel magnet attached to the diaphragm. Turning the cylinder, *C*, of the phonograph, and passing the alternating current through the coil of the electro-magnet, the small steel magnet, *a*, is alternately attracted and repelled, and the vibrating motion thus produced transmitted to the diaphragm, *b*, and the sapphire-arm, *l*, and thus indentations corresponding to the vibratory motion of the small steel magnet are produced on the wax cylinder.

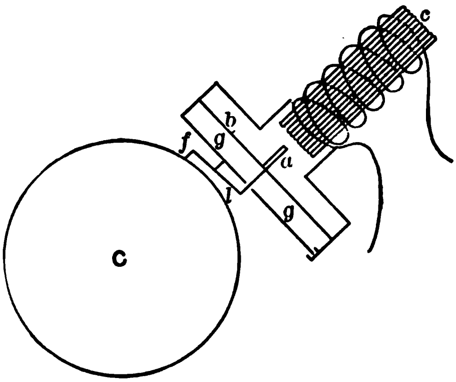

Fig. 1.

The first operation is now complete; it remains to magnify the indentations produced on the wax cylinder. A section of the apparatus which has proven successful for this purpose is shown in Fig. 2. Two small brass bars, *BB'*, are taken, the lower one being fastened rigidly to the diaphragm arm (not shown in the figure), and the upper being free to move. A rubber band, *n*, passing around the two bars holds the upper bar in position. Between the bars are placed two small rollers of very fine iron wire. To the end of one of the rollers is attached a small mirror, *m*, so that readings may be taken with a telescope and scale. The glass diaphragm is removed, and a fine wire, *d*, fastened to the sapphire-arm, *l*, and to one end of the movable bar, *B*. At the other end of the movable bar is fastened a rubber band, or spring, *r*, which serves to keep the reproducing-

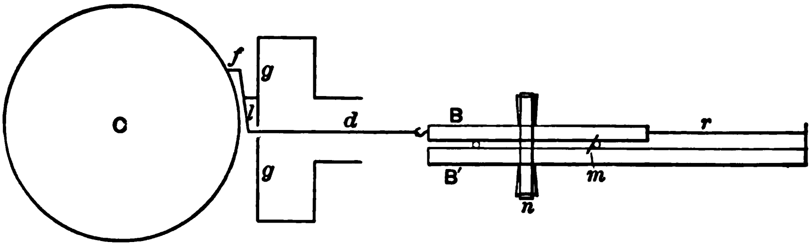

Fig. 2.

stylus, *f*, pressed against the wax cylinder. The reproducer-weight, *g*, to which the sapphire-arm is attached, is now fastened; otherwise this weight would move, and the movable bar remain stationary. The wax cylinder, *C*, is then turned successively through a very small angle by means of an arrangement not shown in the figure, and readings taken.

Only a few preliminary experiments have been performed, but the results obtained seem to be very promising. That the curve should be absolutely perfect is, of course, not to be expected, when inertia must be taken into account. The writer believes, however, that inertia does not play an important part in this method. The movement of the glass diaphragm when the curve is recorded on the wax cylinder may be made so small that the indentations produced on the cylinder are scarcely visible to the naked eye. In the second operation inertia does not enter, as there is no motion of any of the parts when the readings are taken.

This method seems to be especially adapted for the study of harmonics in the alternating current. By the aid of König's apparatus the number of harmonics present may be determined, and, perhaps, their relative intensities. Then, by plotting the curve, their relative positions may be determined. These operations may be repeated several times, and thus each operation serve as a check on the others.

The writer expects to continue investigations along this line the coming year.

ITHACA, N.Y., June 30, 1894.

ON THE RESISTANCE OFFERED BY A MEDIUM CONTAINED BETWEEN SPHERICAL SURFACES AND THEIR MERCATORIAL PROJECTIONS.

BY EDWIN J. HOUSTON AND A. E. KENNELLY.

IT may have escaped notice that a curiously simple relation exists between the resistance of a medium measured between concentric spherical surfaces and the resistance of the same medium between co-axial cylindrical surfaces.

CASE I.

Thus let a and A (Fig. 1) be the radii, in centimeters, of two concentric spherical shells, enclosing between them a homogeneous isotropic medium of resistivity ρ (thermal, electrostatic, electric, magnetic, or radiant);

then if R denotes the resistance of the medium between the spherical surfaces, we know that

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{A}\right).$$

If, however, Fig. 1 be taken to represent the cross-section of two coaxial cylinders instead of two concentric spheres, then the resistance R', between the cylinders per unit length of both is

$$R' = \frac{2\rho}{4\pi}\left(\log\frac{1}{a} - \log\frac{1}{A}\right);$$

or, as is usually written,

$$= \frac{\rho}{2\pi}\left(\log\frac{A}{a}\right),$$

from which it will be seen that the resistance per unit length of cylinders is obtained by doubling the "spherical resistance," after the terms within the brackets have been replaced by their respective natural logarithms.

Case II.

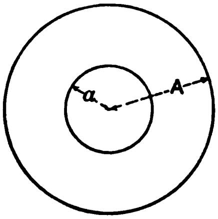

Fig. 1.

Let a and A (Fig. 2) be the sections of two spheres of radii a and A cm., situated at relatively large inter-central distances d cms. Then the resistance between them will be

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} + \frac{1}{A} - 2\frac{1}{d}\right).$$

If, however, a and A be regarded as the cross-sections of two indefinitely long parallel cylinders of radius a and A, separated by an interaxial distance d, the corresponding resistance between them per centimeter length of both is

$$R' = \frac{2\rho}{4\pi}\left(\log\frac{1}{a} + \log\frac{1}{A} - 2\log\frac{1}{d}\right);$$

or, as usually written,

$$R' = \frac{\rho}{2\pi}\log\left(\frac{d^2}{Aa}\right),$$

from which it will be seen that R' is again derived from R by doubling the value of R after the quantities in the brackets have been replaced by their respective logarithms.

When, in Case I., the spheres are not concentric, or when, in Case II., they are not small relatively to their intervening distance, the resistance between spherical electrodes becomes more difficult to compute, and the above simple relationship with the resistance between cylindrical electrodes is no longer retained. But the resistance between cylindrical electrodes can always be computed by simple formulæ for any eccentricity or intervening distance, and if any comparatively simple relation could be established between them and the corresponding spherical conditions by generalized formulæ, the application would be useful in practice. We have not succeeded in discovering this relationship for the general case, but there probably is, nevertheless, a comparatively simple one, since some of the numerical values calculated by Kelvin for resistance between spherical electrodes in close proximity, and published more than twenty years ago in the *Philosophical Magazine*, display suggestively simple relations at the critical values of the surd quantities in the corresponding cylindrical formulæ.

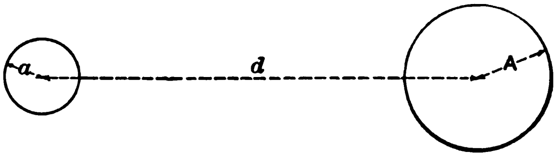

Fig. 2.

NEW BOOKS.

A History of Mathematics. By FLORIAN CAJORI, Professor of Physics in Colorado College. New York, Macmillan, 1894.

If history can be trusted, Alexander the Great was told by his tutor, Menæchmus, that there was no royal road to geometry (mathematics). But Menæchmus lived two thousand years ago.

The modern student who is acquainted with the ponderous syllogisms and awful notation or lack of notation of the ancients, is quite willing to admit that the modern road is royal and more than royal, and probably Menæchmus would agree with him.

Only one acquainted with the horrors of the "middle passage" from the ancient mathematical servitude, can be really appreciative of the modern release from the shackles of ancient conservatism and neophobia, and of the intellectual greatness of the men who could work with such tools as did our predecessors.

It is only when the student runs across some such expression as

$$x^{\bar{\upsilon}}\ \bar{\alpha}\ \bar{\varsigma}\ \bar{\varsigma}\ \bar{\eta}\ \pitchfork\ \delta^{\bar{\upsilon}}\ \bar{\epsilon}\ \mu^{\hat{o}}\ \bar{\alpha}\ \iota\ \varsigma\ \bar{\alpha},$$

which Diophantus used in place of our $(x^3 + 8x) - (5x^2 + 1) = x$, or

℞ v. cu. ℞ 108 p̄. 10 m̄ ℞ v. cu. ℞ 108 m̄ 10,

which Cardan, as late as 1545, used for $\sqrt[3]{\sqrt{108} + 10} - \sqrt[3]{\sqrt{108} - 10}$, or

25 ⓪ 3 ① 7 ② 9 ③,

used by Stevinus, as late as 1585, for 25.379, that he begins to understand the awful slough through which his predecessors have floundered and to appreciate "others have labored, and ye are entered into their labor."

Nesselmann recognized three stages in this floundering from ancient to modern nomenclature which he designates as the *Rhetorical*, the *Syncopated*, and the *Symbolical*. The *Rhetorical* method determined the unknown quantity by a long course of logical reasoning expressed in words without the use of any symbols whatever. The abbreviation of words of frequent occurrence gave rise to the *Syncopated*, which was gradually superseded by the modern *Symbolical* method.

Curiosity prompts the question, when and where did our present mathematical tools take shape; and gratitude, by whom were they invented. It is interesting, in fact necessary, for the mathematician to know the shoals and quicksands upon which his predecessors have foundered. Such knowledge would have saved the labor wasted by "the great army of circle squarers" who have "for two thousand years been assaulting a fortification which is as indestructible as the firmament of heaven."

It is a revelation to the reader to learn how

Equality was expressed by ι, Diophantus (ob. 330); by =, Ricorde (1540); by ‖, Xylander (1575), though generally written at length till the 1600's; by ∝, ∞, a contraction of *æqualis* (1600–1680); by : :, Oughtred (1631) and Wallis (1686).

Inequality by >, <, Harriott (1631); by $\overline{\sqsupset}\,\underline{\sqsupset}$, Oughtred (till ± 1700).

Addition by juxtaposition, Greeks and Hindoos; by *P*, *p*, Italians; by *e*, Pacioli (1494); by ϕ, Tartaglia (ob. 1557); by +, for excess, in 1489, and again in 1544, and generally in 1660.

Subtraction by ⋔, Diophantus (ob. 330); by ·, Hindoos; by *M*, *m*, Italians; by *de*, Pacioli (1494); by −, (± 1630).

Multiplication by ×, Oughtred (1631), Harriott (1631); by ·, Descartes (ob. 1650); by $\frown$, Leibnitz (1686).

Division by $a - b$, a/b, $\frac{a}{b}$, Arabs; by ·, Oughtred (1631); by ÷, Pell (1630), previous to which it had been ∺.

Powers by $\smile$, $\overset{1}{\smile}$, $\overset{2}{\smile}$, etc., Bombelli (1579); by ①, ②, ③, etc., Stevinus (1586); *Aq*, *Ac* for A^2, A^3, etc., Vieta (1591); *aa* for a^2, *aaa* for a^3, Harriott (1631); $a2$, $a3$ for a^2, a^3, Herigone (1634); a^2, a^3, etc., Descartes (1637); x^{-1}, $x^{2/3}$, etc., Wallis (1659); a^n, Newton (ob. 1727).

Roots by $\surd$, Rudolff (1525); $w\!/$ for 4th root, Rudolff and Schurbel (1551); $ww\!/$ for $\sqrt[3]{}$ by the same; $\sqrt{3}$, $\sqrt{33}$ for $\surd$, $\sqrt[4]{}$, by Stifel (ob. 1567); $\surd 2$, $\surd 3$ for $\surd$, $\sqrt[3]{}$ by other writers; $\sqcup 10$, $\llcorner 10$, $\lrcorner 10$ for $\surd 10$, $\sqrt[3]{10}$, $\sqrt[20]{10}$ by Napier.

He gets a new idea of the progress and mutations of mathematics as he views the transitions from

	1 *Zp* · 5 R*m* 4	
	1 Q + 5 N − 4,	Xylander (1575).
	1$\overset{2}{\smile}$*p* 5$\overset{1}{\smile}$*m* 4,	Bombelli (1572).
	1 ② + 5 ① − 4 ⓪,	Stevinus (1586).
to	$x^2 + 5x - 4$;	
and from	25 ⓪ 3 ① 7 ② 9 ②	Stevinus (1585),
	$25,\ 3'\ 7''\ 9'''$	Stevinus (1585),
	25 $\underline{379}$	Briggs (ob. 1631),
	25$\lfloor\underline{379}$	Later writers,
to	25·379	Napier (ob. 1617).

His respect for the inventors and early users of calculus will not be lessened by comparing the

$$\overline{\text{omn. } \overline{\text{omn. } l}\, \frac{l}{a}} = \frac{\overline{\text{omn. } l^2}}{2\,a}$$

of Cavalieri with

$$\int\!\!\int l\,\frac{l}{a} = \frac{\int l^2}{2\,a}.$$

All this and more will be found delightfully set forth in Cajori's *History of Mathematics.* In many points the book cannot be praised too highly, especially the portion which treats of the progress of mathematics in recent times.

Taken as a whole, we may say that it is the best text-book on this subject which has yet appeared; however, one cannot but regret that the author did not treat the subject on somewhat different lines, making it more truly a history of mathematics rather than a history of mathematicians. More diagrams illustrative of the problems referred to in the text would have increased the value of the book to students. The majority of mathematical readers would be more interested to know that the first curve rectified was a cubic parabola in 1660, the second a cycloid in 1673, that the polar triangle was introduced in the early 1600's, as also the word *focus*, than to know primarily by whom.

The author's explanation of the derivation of the word *algebra*, from *aldshebr walmukabala* (restoration and reduction, transposition and combination), is clear and interesting. The derivation of *sine*, from the Latin *sinus*, a translation of the Arabic word *dschaib*, which was mistaken for its homonym *dschaiba*, would have been improved if *jïva* the Sanscrit, original of *dschaiba*, had been explained to be a *bowstring.*

The description of the invention of logarithms would be clearer to students (for whom the book is intended) if attention had been called to the principle upon which Napier based his reasoning; viz.: In a geometrical progression, the terms of which are *numbered consecutively* (any term of the progression being found by multiplying together two previous terms and dividing by the first term of the series), *if* we add the *numbers* corresponding to the multiplier terms, and subtract that corresponding to the divisor term, the result will be the number corresponding to the quotient term.

Napier saw that if he could establish two corresponding series, the process of multiplication could be replaced by that of addition, especially if the first terms were respectively 1 and 0, as in the series

$$\begin{array}{cccccc} 1 & 2 & 4 & 8 & 16 & 32 \\ 0 & 1 & 2 & 3 & 4 & 5, \end{array}$$

in which case the operation of division and its correlate, subtraction, would be dispensed with.

To construct the two series, Napier conceived two points to move from the initial position with the same initial velocity, one at a constant velocity, the other at a velocity proportional at any instant to the space yet to be passed over.

The space described by one would increase in arithmetical progression, the other decrease in geometrical progression.

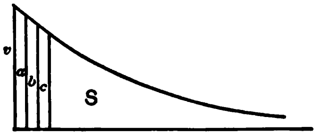

Fig. 1.

In Fig. 1, let the ordinates represent the velocities, v the initial velocity, and a, b, $c \cdots$ the increments of space at successive instants of time, and S the whole space to be described from the initial point.

From the method of generation

$$\frac{a}{b}=\frac{S}{S-a}, \text{ or } \frac{S}{a}=\frac{S-a}{b},$$

which by division becomes

$$\frac{S}{S-a}=\frac{S-a}{S-a-b}.$$

Similarly,

$$\frac{S-a}{S-a-b}=\frac{S-a-b}{S-a-b-c},$$

and so on. That is, the increments are proportional to the remaining space. Q. E. D.

Napier assumed the space S as 10^7, or what is the same thing, since the ordinates are proportional to the space s, $v = 10^7$, and proceeded to calculate the successive ordinates (geometricals) and the corresponding arithmeticals.

The ordinates might as well have been increasing, and the initial one equal to unity, in which case we should have as the equation of the curve $y = e^x$, the curve of the natural system of logarithms.

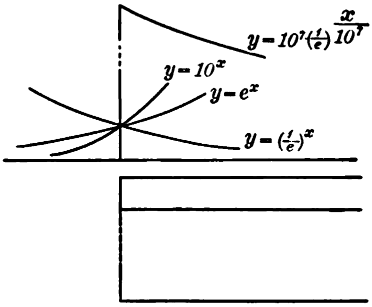

Fig. 2.

The different systems are shown in the second diagram, where $y = e^x$, $y = 10^x$, $y = \left(\frac{1}{e}\right)^x$, $y = 10^7\left(\frac{1}{e}\right)^{\frac{x}{10^7}}$ represent respectively the natural, common, inverse, and Napierian systems.

The lower band represents the equicrescent areas corresponding to the areas in geometrical progression in the upper part of the diagram. This band is double, one for areas generated with unit initial velocity, and the other for areas generated with 10^7 initial velocity. It is evident from the figure that any multiple of a system of logarithms is a new system of logarithms.

To get the Napierian curve, consider that the areas of the Napierian curve are 10^7 times the corresponding areas of the inverse curve $y = \left(\frac{1}{e}\right)^x$. Then

$$Y = \text{an area of the Napierian curve}$$

$$= 10^7\left(\frac{1}{e}\right)^x = 10^7\left(\frac{1}{e}\right)^{\frac{N.L.}{10^7}},$$

the arithmetical area x being divided by 10^7, since the Napierian arithmeticals are 10^7 as large as the arithmeticals of the inverse system.

The relation between the different systems is shown by

$$y = e^\lambda = \left(\frac{1}{e}\right)^\lambda = 10^l = 10^7\left(\frac{1}{e}\right)^{\frac{N.L.}{10^7}},$$

where λ = natural logarithm of y, l = common logarithm of y, and *N.L.* = Napierian logarithm of y.

From these equations we easily get

$$\lambda = \text{nat. log}\, y = \text{nat. log}\, 10^7 - \frac{N.L.}{10^7},$$

whence

$$N.L. = \text{Nap. log}\, y = 10^7(\text{nat. log}\, 10^7 - \text{nat. log}\, y)$$

$$= 10^7\, \text{nat. log}\, \frac{10^7}{y}.$$

In explanation of the word *logarithm:* In a series of magnitudes and their logarithms, the logarithm corresponding to any magnitude indicates the number of geometrical ratios necessary to produce the magnitude from unity; *e.g.:*—

1	2	4	8	16	32	. . . .	magnitudes,
0	1	2	3	4	5	. . . .	logarithms,
	$\frac{1}{2}=$	$\frac{2}{4}=$	$\frac{4}{8}=$	$\frac{8}{16}=$	$\frac{16}{32}$	. . . .	ratios,
0	1	2	3	4	5	. . . .	number of the ratio;

or

1	10	100	1000	10000	. . . .	magnitudes,
0	1	2	3	4	. . . .	logarithms,
	$\frac{1}{10}=$	$\frac{10}{10^2}=$	$\frac{10^2}{10^3}=$	$\frac{10^3}{10^4}$	. . . .	ratio,
0	1	2	3	4	. . . .	number of the ratio.

If we take 10^5 ratios between 1 and 10, then $\log 10 = 10^5$, and if we should form the series of ratios, we should find that 2 would be the consequent of the 30,103d ratio, and 3 of the 47,712th ratio, etc. That is, 30,103 would be the *number of the ratio* (λογος αριθμος, logarithm) corresponding to 2, and 47,712 that corresponding to 4, etc.

If we had taken 1 as the logarithm (λογος αριθμος) of 10, the logarithms of 2 and 3 and of all the other magnitudes would have been correspondingly smaller; viz. 0.30103, 0.47712, etc., as in the common system.

The author adheres to the term *centrifugal force*, which should be discarded or else accompanied by an explanatory plea of "confession and avoidance." He likewise uses *center of gravity* where he means *center of inertia*, regardless of the fact that comparatively few bodies have a center of gravity.

Frequent references to publications are given where the reader can find the material to satisfy the curiosity aroused by the text. The citations in the chapters on modern mathematics are particularly valuable, though not as numerous as they might have been.

If I may venture a friendly criticism, the author frequently fails to keep his subject down to the level of his readers.

For example, his reference to *Coss* and *Cossists* would be immediately intelligible had he added parenthetically that *Cosa* was the Italian for *thing*, the algebraic designation for the unknown quantity at that time.

His references to the combinatorial school, quadratic reciprocity, quadratic residues, etc., would have been more readable had there been a short explanation of their meaning accompanying them. A few additional lines would accomplish this. A certain knowledge of terms in current use, such as determinant, potential, Abelian function, Hessians, Pfaffians, etc., must be presupposed; but historical terms, if we may so designate them, should be explained in a history of mathematics, if anywhere.

This is the nearest approach to a text-book on this subject for classroom use that we have, and so far as it goes is excellent. But a rearrangement on the lines indicated would, I think, make it much more available. The history of mathematics has been too much neglected in our schools and colleges, and it is to be hoped that the appearance of this book may result in the introduction into the curriculum of this much neglected and sorely needed branch of mathematics. As the author says on his title page, "no subject loses more than mathematics by an attempt to dissociate it from its history."

The present work should be read by every student who loves high thinking and desires to know something of that science which makes the greatest demands on the intellectual forces of its votaries, and which has taxed to the utmost the mightiest minds that the world has produced.

Some will be scandalized to learn that Euclid's famous 12th axiom may perhaps be no axiom, and that possibly space itself may be finite; that the sum of the angles of a triangle may not be a straight angle, after all.

Many, too, will learn for the first time that mathematics is not a dead science, completed and mummified by Euclid and Newton, but that under the creative hands of Gauss, Riemann, Lobatchewsky, Grassmann, and of Sylvester, Cayley, Steiner, Klein, and a host of others, it has a vitality which promises as much for the future as has been done in the past.

It will be found to be a most readable and enjoyable book by every one who is interested in the intellectual progress of the race, and particularly by those who desire to know something of what is going on at the present day in the world of rigid scientific thought.

Like every new book it has some errors and misprints, which will doubtless be duly corrected.

ARTHUR LATHAM BAKER.

UNIVERSITY OF ROCHESTER,
April, 1894.

Electromagnetic Theory. Vol. I. By OLIVER HEAVISIDE, F.R.S. pp. xxi + 466. New York, Van Nostrand Company.

This work is a reprint of a series of papers which originally appeared in the London *Electrician* during the years 1891–93. It does not postulate so extensive a knowledge of mathematics as do the author's *Electrical Papers*, but it addresses all earnest students who have been trained to think with some degree of exactness. All such will find in it a valuable storehouse of ideas. The book is divided into four chapters, of which the first is introductory; the second presents an outline of the electromagnetic connections; the third expounds the elements of vectorial algebra and analysis; while the fourth gives the theory of plane electromagnetic waves.

In the introductory chapter some current delusions on the subject of mathematical investigation are exposed; for instance, that mathematics is a waste of time, or that it is a mere machine which can grind out only what is put in. The latter view has been advocated by pretentious philosophers, such as Mill and Huxley; in this they merely demonstrate that they have failed to understand the nature of exact analysis, the finest part of human knowledge. Mr. Heaviside takes the view that mathematics is reasoning about quantities. He says: "Even if qualities are in question, it is their quantities that are subjected to the mathematics. If there be something which cannot be reduced to a quantity, or, more generally, to a definite function, no matter how complex and involved, of any number of other quantities which can be measured (either actually or in imagination), then that something cannot be accurately reasoned about, because it is in part unknown." This view will appear too narrow to the pure analyst, probably even to the mathematical physicist, for has he not to consider identity and relation as well as equality?

The outline of the fundamentals of electromagnetic theory given in the second chapter is characterized by the use of rational units, vector analysis, and the duplex form whereby the electric and magnetic sides of electromagnetism are symmetrically exhibited and connected. "The forces and fluxes are the objects of immediate attention, instead of the potential functions, which are such powerful aids in obscuring and complicating the subject, and hiding from view useful, and sometimes important, relations."

In the electromagnetic system of units the unit magnetic pole is defined by means of the equation $f = mm'/r^2$. According to Mr. Heaviside f is not equal to the above function, but becomes equal when the function is divided by $4\pi c$, where c denotes the permittivity of the dielectric. If c be taken as 1, the equation becomes $f = mm'/4\pi r^2$. He shows that if the unit of quantity be defined by this last equation, then the electric and magnetic equations become not mere formulæ, but rational truths with transparent meaning. In this connection I may mention that, when writing *Physical Arithmetic*, I found that almost all the physical coefficients could be expressed in plain English, with the help of *per* and *by* and an occasional bracket, but that I encountered a difficulty in thus rationalizing the law of the inverse square. My solution (p. 298) was that the strength of a source is properly expressed by so many units of current per unit of solid angle, the unit of solid angle being defined after the analogy of the radian. This agrees precisely with our author's view. The existing system of units involves the assumption that the proper unit of solid angle is the ratio of the whole spherical surface to the square of the radius; that is, the steregon. But as the radian, not the whole circumference or the quadrant divided by the radius, is the proper unit of plane angle, so the steradian — that is, unit of spherical surface per square of the radius — is the correct unit of solid angle. The word *steradian* was introduced by Dr. Halsted thirteen years ago in his *Metrical Geometry*. There can be no question about the superiority of the rational units, but it does not follow that the legalized units ought to be changed immediately; for may not further light bring to view further desirable changes?

The third chapter expounds the elements of vector analysis. The exposition, while clear, and suitable for the class of readers addressed, contains principles which appear of doubtful validity to those who have made a special study of the matter. For instance, the scalar product of two vectors is not distinguished by any prefix, though the vector product is, apparently on the same principle that the eldest daughter is sufficiently distinguished by Miss, provided all the younger sisters have their Christian names appended. But the scalar product and the vector product are only partial products; the simpler name belongs properly to the complete product, which is their sum. Here we have an indication of one of the principal defects of the analysis; it is fragmentary, not a method *totus teres atque rotundus*. Again, the author founds his addition of vectors on the composition of successive translations. This plan, though usually adopted by writers on vector analysis, is inadequate, for it does not apply *simpliciter* to such vectors as forces applied at different points. Rotations are compounded in that manner, provided they are infinitesimal; to compound finite rotations requires ideas more complex than the simple vector.

The fourth chapter, devoted to the theory of plane electromagnetic waves, is mainly descriptive, and may be read with profit by many who are unable to follow the mathematical investigation from which the results are derived. The theory is applied to straight wires, and an account is given of the effects of self-induction and leakage, and the application shown to the problem of long-distance telephony.

ALEXANDER MACFARLANE.

Theory of Heat. By THOMAS PRESTON. 8vo, xvi + 719 pp. London, Macmillan, 1894.

Professor Thomas Preston's *Theory of Light* appeared in 1890, and has proved itself a serviceable book. It is now followed by his *Theory of Heat.* Such titles are significant of the stage upon which the more advanced work in physics has entered. They mark the attempt to weld together the results of modern research and those which have long stood as classical; to present a connected digest of established fact, and of theoretical doctrine which is on the safe side of the frontier of speculation. The public demand which these books imply, is one sign of a lively interest in the completer study of physics.

Some books of this type presuppose readers who are already familiar with elementary phases of the subject; a well-known instance is Lord Rayleigh's *Theory of Sound.* But the volume now before us concedes 100 pages of "Preliminary Sketch" to the needs of beginners, and demands only maturity of mind, with some knowledge of calculus, of those who are to follow its arguments.

The plan may have practical grounds, such as a desire to make the book available for a larger circle of readers, but it lies open to several objections, which are only confirmed by the attempt to execute it in the present instance.

It would be easy to point out a wavering conception of the intelligence of the reader, as shown by the selection of preparatory ideas, and to give examples of repetition, or separation of related matters, or "preliminary considerations" which are not followed up; but these would be minor criticisms; the cardinal defect is that the argument of the book is broken up, without any guarantee that the parted links form a complete chain. If the reader be really beginning the subject, he is either confused by unwarranted assumptions as to his fundamental ideas, or, more dangerous still, led to overlook the blank spaces in his own knowledge.

The truth is, that with the current opportunities for laboratory instruction, nobody should make the first approach to physics by reading merely; while if work has been done in a laboratory, many a precious page in a book like this, now devoted to A B C explanation, might be reserved to fill more serious gaps.

The work must, therefore, be judged from the standpoint of a student with some preparation. Such an one will find a part of the material offered already familiar, it may be, but he will recognize a freshness of treatment in it; and he will find a stimulus and an aid for the prosecution of his study in the references to literature which are liberally given throughout. There are, perhaps, a thousand of these in all; they include journals in English, French, and German, and more recent dates are *not* conspicuous by their absence. The German workers are least fairly represented; references to papers in that language form about one-sixth only of the total number, while the division is nearly equal between English and French. No one can expect such references to be uniform in value or novelty; but it is surprising that the author finds it advisable to refer us (p. 8) to Tait's *Sketch of Thermodynamics* for the nature of the candid scientific mind.

Although our author recognizes the necessity of selection in a case like this, and has, for reasons which he states, omitted several topics, his volume has, nevertheless, swelled to over 700 octavo pages. It therefore supports the protest of its preface against "pocket editions of the sciences." Not counting the preliminary sketch already spoken of, the book is equally divided between a group of five shorter chapters on Thermometry, Dilatation, Calorimetry, Radiation, and Conduction, and two longer chapters on Change of State and Thermodynamics.

The emphasis of the book is thus indicated as falling upon the energy relations of heat, and the series formed by the three states of matter. And this is quite natural in the days when molecular physics, upon a dynamical basis, seems to be the goal before us.

In all seven chapters the summaries of experimental work are such a prominent feature that this impresses one at the first glance. Historical order is preserved; a useful sifting and analysis of results is given in many important cases; and the process of scientific development through elimination of error is shown by the discussion of examples.

Notable instances of such work well done are: Chapter V., Section VI., "On the Continuity of State," where the discussion is made to include important later work; and the same chapter, Section VII., "On Equations relating to the Fluid State," in which the present condition of knowledge is worked out from the crude beginning made by Boyle. Minor examples are: "The Caloric Theory and its Overthrow"; "The Development of the Air Thermometer"; "The Radiometer."

The prevailing spirit of fairness is maintained when J. R. Mayer is set in his proper place, and when the attempt is made to purge Carnot of heterodoxy on the Caloric question. But is it intended to stigmatize Balfour Stewart as a "popular expounder" of science (p. 282)? And Maxwell's sorting-demon was hardly a mere "ingenious, but illusory, violator" (of the second law of Thermodynamics): this conception surely contributed to clearness of thought (p. 620).

Professor Preston is rightly a stickler for logical thinking. He is early at work upon the conception of temperature (p. 12), and his attempts to introduce logical order into our notions about thermometers are found throughout the book. But while he starts the fox, he can scarcely be said to run it down. Few will follow him in believing that "what we directly *observe* is temperature" (the author's italics). Do we not, in most cases, observe constancy, or change, of volume in the thermometric substance? Or, in some others, what proves to be a result of rate of transfer of energy?

In fact, it does not seem as if logic, and definition, and the more rigorous dynamical ideas of the subject were the strong points of the book. We are asked to believe (p. 58) that "the property that really is conserved throughout chemical processes" is weight (and not mass). That is, the weight at Panama of some mixed iron filings and flowers of sulphur equals the weight at San Francisco of the compound formed by uniting chemically the constituents of this mixture. Of course Professor Preston does not think this; but our ordinary notions need correction, he says; and this results from the way in which he would correct them. — A definition of molecule: "The smallest part of the body which can be separated from it and still possess all the characteristics which distinguish the substance" (p. 61). Fancy separating one molecule from a body and testing it!

The phrase "vibration of molecules" is used throughout the book. Careful search has failed to show, however, whether this is to be understood as meaning vibration of the molecule as a whole, or of the atoms which constitute the molecule.

Unless we know this, it is difficult to judge quite fairly the view of conduction presented (p. 506). But one is inclined to characterize it as misleadingly simplified. The tuning-fork analogy will help in some cases; but when we meet it here, the temptation is strong to quote the author's own words about glacier-motion: "To attempt to explain all the phenomena by attributing them to any single action is certainly not reasonable" (p. 283).

As regards the style, the author has constrained an evident tendency to exuberance which shows in the Introduction, Chapter I.; and has given us in the main sober English, as good as we usually find in scientific books. Occasionally, though, the pen slips. In the following examples the English almost obscures the sense. "The value of any hypothesis depends upon its convenience in systematizing observed facts, and to the extent to which it embraces all known phenomena must its utility be estimated" (p. 61). "We know that energy in immense quantities is perpetually passing through space with the velocity of light in the form of radiant heat and light, and that it exists also in the so-called potential state. The former exists in the ether, and probably also the latter" (p. 82).

We may say then that the book seems likely to be of service, like the *Theory of Light*, but more on account of the experimental results that it gives compendiously and accessibly, than because it proceeds with system and connection to build a structure of theory.

If it were rewritten so as to assume the proper basis of dynamical and experimental ideas, it would gain in compactness, and be more likely to secure recognition for permanent value.

FREDERICK SLATE.

UNIVERSITY OF CALIFORNIA,
April 17, 1894.

A Treatise on the Theory of Functions. By JAMES HARKNESS, M.A., Associate Professor of Mathematics in Bryn Mawr College, Pa., late Scholar of Trinity College, Cambridge, and FRANK MORLEY, M.A., Professor of Mathematics in Haverford College, Pa., late Scholar of King's College, Cambridge. 8vo. pp. ix+507. New York, Macmillan & Co., 1893.

The appearance of this treatise almost simultaneously with the larger work of Forsythe marks an era in the history of mathematical publications in England and the United States.

The earlier chapters are designedly elementary, and in their treatment of the subject leave little to be desired. The investigation of the real functions of a real variable in Chapter II. is helpful in paving the way for the complex variable, which occupies so large a share of the authors' attention. In Chapter III. the discussion of the infinitive series is particularly happy. The subject is considered at some length, and, for example in the case of the circle of convergence, is well stated.

The chapter on Abelian Integrals materially increases the value of the book. While, with regard to Chapter VIII., which treats of Double Theta-Functions, one can scarcely do better than to quote the words of the authors themselves contained in the preface: "Chapter VIII. is devoted to a somewhat condensed treatment of double theta-functions, which goes farther than is necessary for our immediate purpose, for the reason that the subject is not very accessible in the English language."

In the treatise, as a whole, the authors have erred on the side of conciseness. Theorems are stated and demonstrated, but the method of proof is so brief that unless one is quite in touch with the subject he will spend much valuable time in supplying the intermediate steps which are involved. Of course this very conciseness would enhance the value of the work to the reader who merely wished to verify a result. But the majority of us remember a conclusion better than the exact method by which it was attained, and it is frequently convenient to find the outline of a proof well filled in.

The arrangement of the type might be improved if the statement of the theorems differed from the demonstrations in appearance. The eye would assist the mind in detecting what was to be proved. In nearly every case the diagrams are exceedingly good, and their helpfulness leads one to wish that they were even more frequent.

The method of giving "References" at the close of each chapter to standard works on the subject which has just been considered deserves special commendation. The book is sure to prove a valuable addition to the library of any physicist.

L. M. PEIRCE.

The Incandescent Lamp and its Manufacture. By S. T. RAM.

Among the most important factors in the success of an incandescent electric light plant is the incandescent lamp; it is also the one of which many electric light manufacturers have the least knowledge. Possibly this state of affairs is largely because of the mystery with which lamp manufacturers have surrounded the factories and the methods of manufacture, but is also due to the fact that the incandescent lamp is not a piece of machinery having wearing parts to be lubricated, adjusted, and repaired, and the accurate operation of which can be readily understood.

Incandescent lamps have, even for those of the same make, complex relations between initial efficiency, efficiency during life, either total, or whatever may be considered as commercial, life, cost of lamps, cost of power, etc. The consideration of the advisable efficiency of lamps to use in any given case must either be a matter of judgment or must be based on data obtainable only from experiments of a lengthy and costly nature; such data have been obtained, but are not generally known, and are not in the working library of many electrical engineers.

Mr. Ram's work deals largely with the manufacture of the lamps. The methods described give the principles of operation and the result desired, but the apparatus is not generally such as will appeal to an American manufacturer, because it is not to any great degree automatic. It is extremely difficult to manufacture incandescent lamps by means of automatic machinery. Much, however, has been done in this respect, and very little is described by Mr. Ram.

In the direction of information on the economic use of the lamp, Mr. Ram gives some valuable data, but makes no reference to many important articles on this subject published in America by such investigators as Howell, Nicols, Merritt, Thomas, and others. To mention as "the most recently published tests" those of Professor Ayrton, 1892, is apparently an evidence of a lack of knowledge of what is being done in this country.

Mr. Ram states that all lamps should be operated at a standard efficiency, and the C. P. at such efficiency be marked. If such a course were followed, a factory would produce much unsalable stock, as the commercial conditions are that orders are placed for a certain C. P. and voltage, and must be filled, as nearly as may be, as ordered.

The users of a lamp can find much in Mr. Ram's book which will be of great value. He will appreciate the folly of specifying lamps of a uniform cold resistance, as has been done by alleged experts, and also the importance of maintaining P. D. at the lamps.

Any information which will assist to give a truer knowledge of a factor which amounts to 10 to 20 per cent of the total operating expenses of an electric light station must prove of value, and Mr. Ram has prepared a book from which such information is obtainable, and users as well as manufacturers can read and study it to great advantage.

E. P. ROBERTS.

Electrical Engineering Formulæ. By W. GEIPEL and M. HAMILTON KILGOUR. pp. xx + 796. London, The Electrician Publishing Company, 1894.

This pocket book marks by its bulk the increase in the mass of material which the modern engineer is compelled to have at his fingers' ends. The volume has the size of page most convenient for a pocket manual, but it approaches the limit, if, indeed, it has not passed it, at which smallness of page ceases to be a matter of economy.

Geipel and Kilgour's manual is much more than a collection of engineering formulæ. It is, indeed, in small compass a brief compendium of physical units, of data concerning the properties of materials, of mathematics, including trigonometry, hyperbolic functions, the calculus and differential equations, of hydraulics, of heat and its applications to boilers and the making of steam, of heat engines, and of applied electricity.

That portion of the book which deals with electrical engineering itself begins only upon the 543d page, and occupies scarcely one-third of the volume, so that the saying, attributed to Kelvin, which has done so much to bring into contempt in the minds of a certain class of electrical engineers the very subject with which primarily they have to do, — viz. that electrical engineering is nine-tenths mechanical engineering, — although not strictly verified in the distribution of materials in this work, finds some justification therein. The selection of material, aside from the introduction of so many data which lie within the province of the steam engineer, rather than of the electrician, appears upon the whole to be a judicious one. The tabulated matter is well arranged and accurate.

A glance at the tables indicates that progress is being made among electrical engineers in Great Britain towards the adoption and use of the metric system. It is encouraging to find in a pocket book designed for engineers, temperatures stated in degrees Centigrade as well as in the scale of Fahrenheit, and to find the centimeter taking its place alongside the inch. The necessity of duplicating data in order that those who have not yet accustomed themselves to the C. G. S. system may be able to use this manual, is one which we hope will disappear before many editions have been sent out. Another sign of the times is the disappearance altogether of the department of sub-marine cable testing. It is not many years since this was the chief subject with which British electrical engineers had to deal. In the present volume the cable is treated purely as a device for the transmission of energy in electric lighting and power circuits. Cables for signaling, either by telegraphy or by means of the telephone, are not given a place. The work is one which will be found useful in the hands of practical men on both sides of the Atlantic.

E. L. Nichols.

HERMAN von HELMHOLTZ (1876).

Volume II. *November–December, 1894.* *Number 3.*

THE

PHYSICAL REVIEW.

STUDIES OF THE LIME LIGHT.

By Edward L. Nichols and Mary L. Crehore.

I.

THE experiments to be described in this paper were made for the purpose of tracing the changes which the radiation from the hottest part of the incandescent lime cylinder used in the Drummond light undergoes from the moment of ignition until the lime has reached its final state of incandescence. It has long been known that the brilliancy of incandescent lime diminishes rapidly from the moment that the cylinder is brought under the action of the oxyhydrogen flame, but the law of its degeneration has not, so far as we are aware, been definitely studied. W. H. Pickering[1] in 1880 made some measurements of the lime light in its final condition. In 1889, one of the writers, in collaboration with Professor W. S. Franklin,[2] studied the spectrum of this source in the course of a series of investigations upon the quality of the light emitted by various artificial illuminants. In the latter research, freshly ignited cylinders, and also those which had been for a considerable time subjected to the temperature of the flame, were measured. It was found that the freshly ignited lime was whiter, that is to say, relatively stronger, in the shorter wave-lengths than the arc light or than any other artificial source excepting the magnesium light, to which it showed a very close resemblance. In its final state of incandescence, however, the lime light was found to fall decidedly below the electric arc in whiteness, being relatively about twice as bright in the blue as gas light. Measurements described in the paper just referred to showed that the spectrum of the lime in its final state of incandescence gave readings which corresponded closely to those obtained by Pickering in his investigation.

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[1] Pickering, Proceedings of the American Academy of Arts and Sciences, Vol. 15.

[2] Nichols and Franklin, American Journal of Science, Vol. 38, p. 100.

In the present paper it is proposed to trace more in detail these time changes, for which purpose the work was divided into two distinct portions. The first method pursued deals with the radiation from the lime taken as a whole, and with such portions of its radiation as are capable of being transmitted by water and glass. Throughout the determinations, diaphragms were mounted in front of the lime cylinder, these being so placed as to isolate a small region of the radiating surface where the lime was at its highest temperature. These rays were allowed to fall upon the face of the thermopile, which was in closed circuit with a galvanometer of high sensitiveness. Fig. 1 shows the arrangement of the apparatus. Interposed between the thermopile *T*, and the diaphragms *D*, *d*, already described, was a double, opaque screen, *S*, which could be withdrawn by the observer at the telescope, thus allowing the rays from the incandescent cylinder to fall upon the face of the thermopile during a single swing of the galvanometer. The throw thus produced served as an indication of the intensity of total radiation from that portion of the surface of the cylinder which was visible from the face of the thermopile.

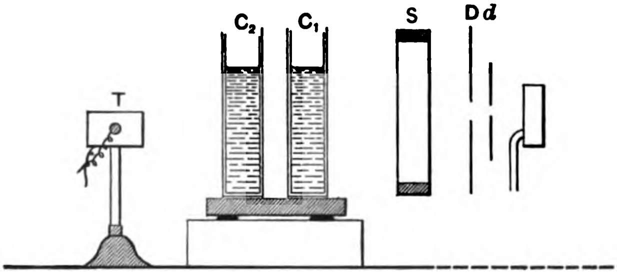

Fig. 1.

Upon a track between the thermopile and the lime light were mounted two cells, C_1 and C_2, containing water. These could be interposed in the path of the rays, thus cutting off the greater part of the non-luminous radiation. The deflection produced when these cells were interposed, when compared with that produced by the total radiation, afforded data for the computation of the radiant efficiency of the lime cylinder.

Total radiation and light-giving radiation were treated in these experiments as functions of the time, and their ratio, corrected for the amount of light which the cells absorbed, and for the amount of non-luminous energy which they transmitted, gave the radiant efficiency as a function of the time also. Fig. 2, which is from the average of three sets of measurements, shows the decadence of total radiation and of the radiation transmitted through the cells from time to time, as indicated by the falling off in galvanometer deflection under the two conditions already stated. It will be noted that the decadence in both cases is very marked. The deflection, for example, due to the uninterrupted rays, falls off in thirty minutes to one-third its initial value. The deflections for the rays transmitted by the water cells suffer similar loss.

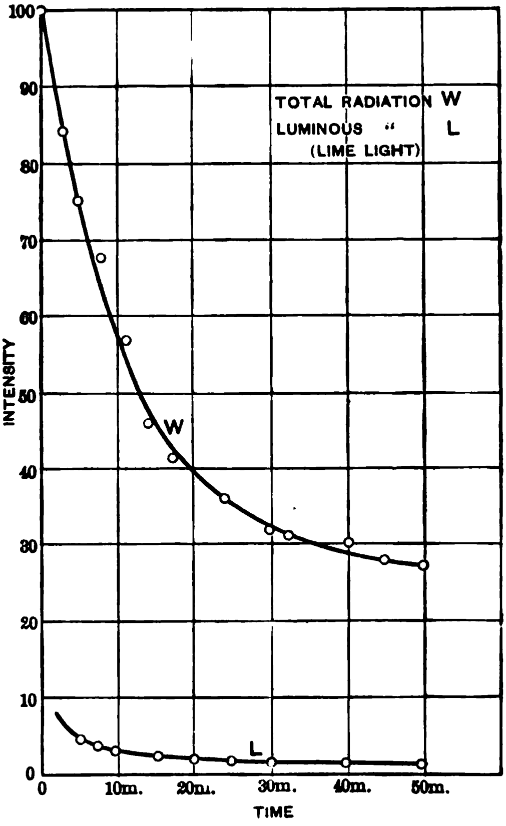

Fig. 2.

In order to compute from these two curves the radiant efficiency of the lime light, it is necessary to know how large a proportion of the light-giving radiation is transmitted by the water cells and also in how far these are impervious to the wave-lengths lying beyond the red. In nearly all previous studies of radiant heat, in which the method of Melloni has been used, it has been assumed that the proper material as regards opacity to such wave-lengths was a solution of alum in water. Recent observations by Mr. E. F. Nichols[1] upon the transmitting power of various substances in the infra-red have shown, however, that the addition of alum adds nothing to the adiathermancy of water, and that cells containing distilled water with no alum in solution will serve just as well for the purpose of obstructing the radiation of the infra-red as the alum solutions which have been used by so many observers. The precise relationship between the water and water containing ammonium and potassium and alums in solution is shown in the paper of Mr. E. F. Nichols already referred to. It will be seen from his diagram that these solutions all become opaque at the same point; viz. at wave-length 1.5. Also that they all reach about the same transparency at the boundary of the red, for which region they transmit about 80 per cent of the incident radiation.

It seems entirely justifiable in view of these results to use distilled water instead of a solution of alum for the purpose of this experiment. Measurements of the amount of light-giving energy transmitted by the two cells were made, and it was found that 55 per cent of the light incident upon the face of the first cell passed through both. An estimate of the proportion of the incident radiation of wave-lengths too long to affect the retina gave 0.004 as the transmitting power for such radiation.

By means of these two corrections it is a simple matter to compute the radiant efficiency of the lime light, which is given as a function of the time in Fig. 3. This curve shows that the radiant efficiency, which is the ratio of light-giving to total radiation, falls from about 14 per cent to a little more than 8 per cent during the first half-hour after the ignition of the lime.

It is interesting to note that these radiant efficiencies correspond very well with those which one would assign to the lime light upon the basis of our previous knowledge. It has already been pointed out that measurements of the spectrum of the freshly ignited lime showed a degree of incandescence corresponding closely with that of the magnesium light, while the spectrum of the lime in its final condition shows a state of incandescence somewhat slightly inferior to that of the electric arc. Mr. F. J. Rogers,[1] who has studied the magnesium lamp from the point of view of its radiant efficiency, finds, however, the value of the efficiency to be 0.137, a quantity which agrees very closely with the efficiency of the freshly ignited lime. The efficiency of the electric arc, on the other hand, according to numerous measurements, varies between 0.08 and 0.11, so that a comparison of the spectrum of lime in its final condition with the spectrum of the arc would lead us to a value for its final radiant efficiency, which is in excellent agreement with the values given in Fig. 3. Some measurements were made also upon the radiation of cylinders of magnesia and of zircon as a function of the time, and it was found that these sources, like lime, suffered marked decadence.

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[1] Physical Review, Vol. I., p. 1.

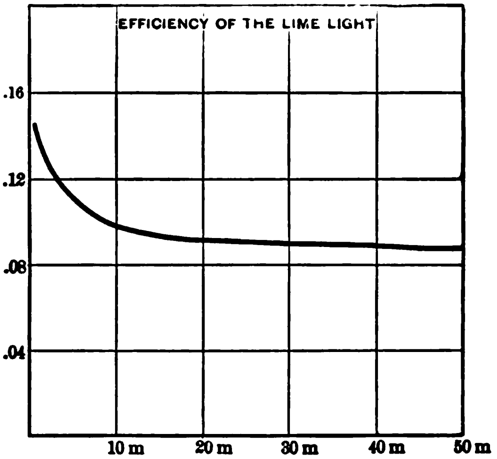

Fig. 3.

II.

The second portion of our investigation was made with the spectrophotometer, the object being to determine the law of decadence, wave-length by wave-length, throughout the visible spectrum. The arrangement of the apparatus is shown in Fig. 4.

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[1] Rogers, American Journal of Science, Vol. 43, p. 301.

It consisted of a one-prism spectroscope provided with a Vierordt slit. This device for bringing the brightness of comparison spectra to a common value is the earliest and simplest of spectrophotometric devices. It is useful only where the range of intensity is small, but for the purpose of the present investigation it gives reliable results.

The lime cylinder was mounted a short distance before the slit of this spectroscope, two diaphragms, as in the first portion of the work, being interposed. The upper half of the slit was covered by means of a rectangular prism for total reflection, one of the acute edges of which formed the boundary between the two spectra. This edge was made to coincide as accurately as possible with the middle of the slit in order to secure a well-defined band of separation between the two portions of the field which were to be compared.

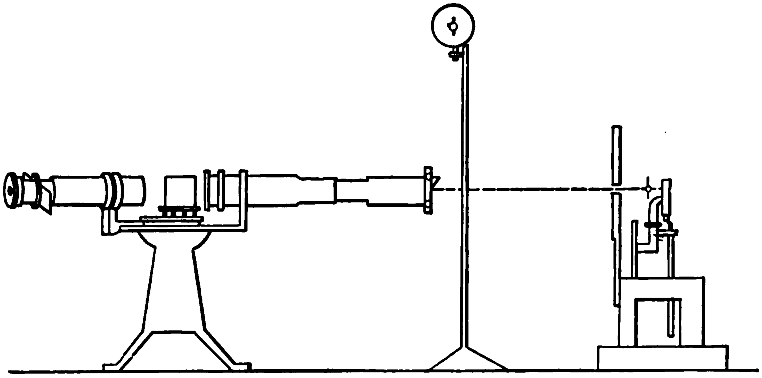


Fig. 4.

By means of this prism, rays from a glow-lamp placed about 30 cm. above the collimator tube were introduced into the upper half of the slit. This lamp was adjusted by previous trial to a degree of incandescence corresponding, as regards the distribution of energy in its spectrum, to gas light. For this purpose it was placed on a photometer bar, and compared with a bat's wing gas burner, the voltage of the lamp being adjusted until a photometer of a type very sensitive to color differences, showed no difference in the quality of the two lights. The voltage necessary to maintain the lamp in this condition was noted, and throughout the subsequent experiments the lamp was maintained at the temperature corresponding thereto. In this way a steady source of comparison was obtained, the quality of the light from which was as nearly as possible identical with the light from a gas flame. While this is no very satisfactory basis for the definition of the quality of a light source, it appears to be the one least open to objection at the present day.

The eye-piece of the spectroscope was provided with a diaphragm by means of which similar regions in the two spectra to be compared could be isolated. The measurements consisted, as in all spectrophotometric work, in bringing the corresponding wave-lengths of the two spectra to equal brightness. This was accomplished in the present case by the adjustment to the Vierordt slit. In consideration of the fact that the source of light with which we had to deal changed rapidly with the time, the following method of taking readings was adopted:—

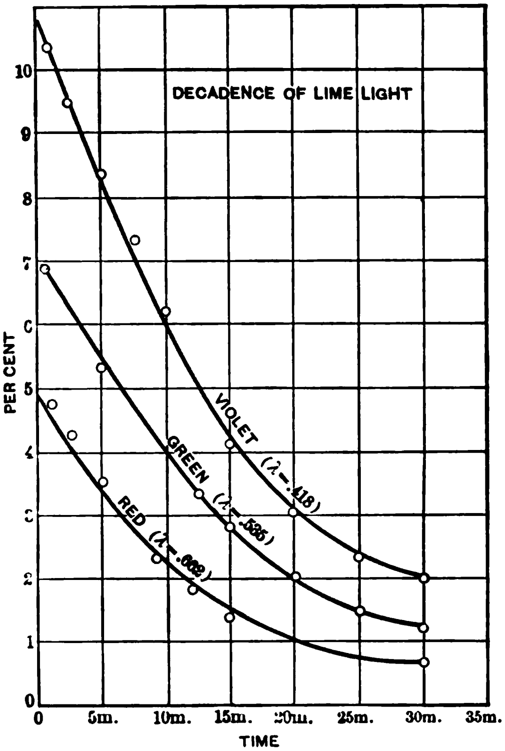
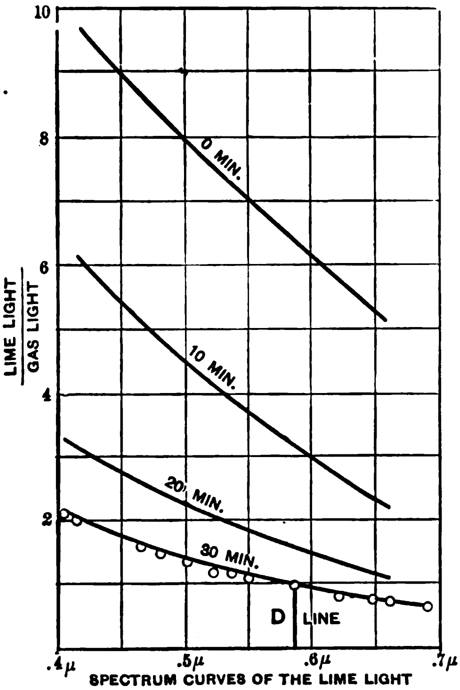


Fig. 5.

A fresh lime cylinder having been placed before the oxyhydrogen jet, the spectroscope was set to a chosen wave-length, the jet was ignited at a time carefully noted, and readings were taken at frequent intervals for this wave-length alone, until, after the lapse of about thirty minutes, it was found that the intensity of the region under investigation had ceased to undergo further appreciable change of brightness. A new lime cylinder was then taken, the spectroscope was set to another wave-length, and the readings were repeated. In this way three regions, one in the red (0.662), one in the green (0.535), and one in the violet (0.418), were measured as functions of the time. The results are given graphically in Fig. 5. Finally, an incandescent lime cylinder which had been subjected to the action of a flame for half an hour, by which time it had come into a condition of nearly stable incandescence, was explored for relative brightness throughout its spectrum. For purposes of convenience, the region of the *D* line in this spectrum was taken as unity, and a curve was platted, the ordinates of which were the ratio $\frac{\text{lime light}}{\text{gas light}}$, this ratio being by definition equal to unity in the yellow.



SPECTRUM CURVES OF THE LIME LIGHT

Fig. 6.

This 30-minute curve of relative brightness of the lime-light spectrum is given in Fig. 6. From this and the curves showing the decadence in the red, green, and violet as a function of the time (Fig. 5), it was possible to plat curves showing the character of the spectrum at any desired time after the ignition of the lime. Such curves are platted in Fig. 6 for the instant of ignition (0 min.), for 10 minutes and for 20 minutes.

In addition to these measurements some attempts were made to study the recuperative power of lime cylinders upon cooling. It was found that lime, after having been held in incandescence, even for several hours, would, upon being allowed to cool, recover to a marked degree, acting upon re-ignition almost as though it were a fresh lime. This power of recuperation did not, however, appear to be uniform in the various specimens tested. It was much more complete in some than in others The time at the disposal of the experimenters did not admit of an exhaustive study of this interesting property.

The experiments described in this paper make it clear that lime possesses the remarkable property, long since known to exist in the case of other oxides, of luminescence by heat. In other words, its radiation when freshly ignited is one corresponding to a temperature very much higher than that to which it is actually subjected. That this is the case with magnesia, when the incandescent oxide is produced by the burning of magnesium ribbon, has been abundantly shown by Rogers in the paper already cited. One of the present writers in collaboration with Professor B. W. Snow[1] has shown that zinc oxide, even at a temperature of 1000° C., possesses, when first brought to incandescence, a radiating power vastly greater than that of platinum at the same temperature and superior to the latter, especially in the shorter wave-lengths. Also that this power is lost in the course of a few minutes. This property, which appears to be common to the many metallic oxides, affords a very interesting problem to the student of radiant energy. The present study is a contribution extending only to a single member of that group.

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[1] **Nichols and Snow, Philosophical Magazine [5], Vol. 32, p. 401.**

A STUDY OF THE RESIDUAL CHARGES OF CONDENSERS AND THEIR DEPENDENCE UPON TEMPERATURE.

BY FREDERICK BEDELL AND CARL KINSLEY.

WHEN a condenser is subjected to a difference of potential for some time, part of the charge received is absorbed by the dielectric, and a part remains upon the surface of the condenser plates.[1] This latter part becomes discharged when the terminals of the condenser are connected together through a conducting circuit, and the condenser appears to be in a perfectly neutral condition. The absorbed charge, however, still remains in the condenser and will gradually come to the surface in the form of a residual charge of the same sign as before. If this be discharged, and the condenser be allowed to stand insulated, a second residual charge will collect in the same way as the first. A series of residual charges with rapidly diminishing values may thus be formed. Therefore, the condition of a condenser is dependent upon its past charges, some of which may have been held by the dielectric for weeks or even months before appearing upon the surface; a former charge of one sign may entirely neutralize a subsequent charge of opposite sign and smaller value. It has been justly said that the past history of a condenser must be known before it can be trusted.

The object of the present investigation was to determine this soaking-in effect of dielectrics. A study was made, first of the successive residual discharges of a neutral condenser and then of the effects produced by previously charging the condenser in the opposite direction. Figs. 1–10 show some results from this part of the investigation. The absorption by the dielectric was next studied by allowing a charged condenser to stand insulated and discharge through its own dielectric. The effect of a previous negative charge upon this absorption and upon the apparent resistance of the condenser was thus studied, as shown in Figs. 11 and 12. An investigation was then made of the influence of initial potential upon the discharge curves, and the insulation resistance, the results being shown in Fig. 13. An examination of temperature effects was then undertaken. The influence of temperature changes upon discharge curves through various commercial oils, and upon the resistance of the oils (see Figs. 14, 15, and 16), was ascertained preliminary to similar experiments upon condensers. Finally the influence of temperature upon the absorption by the dielectric of a condenser was obtained by a study of the successive residual discharges (Figs. 17 and 18), and by discharge curves (Fig. 19) at different temperatures. We will discuss separately the several parts of investigation.

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[1] The part absorbed depends directly upon the duration of the time of charge; the part remaining on the surface is practically independent of the length of charge, for the condensers used in the present investigation. This was shown in a preliminary investigation by Messrs. W. M. Craft and H. B. Henderson, who showed the first discharge, as measured by the throw of a ballistic galvanometer, to be independent of the length of charge.

The Effect of a Previous Negative Charge upon Successive Residual Discharges.

Previous to the final charge, the conductor was subjected, for this series of experiments, to a charge of opposite sign. The final charge was in every case in the same direction (arbitrarily called positive) and for the same length of time, viz. 30 seconds. Experiments were made with previous negative charges; *i.e.* charges opposite in direction to the final charge, of different duration. The value of the positive or negative potential to which the condenser was charged was in all cases about 114 volts. After the final positive charge of 30 seconds, the condenser was allowed to stand insulated for five seconds, and was then short-circuited for 15 seconds. After this discharge by short-circuiting, the condenser was allowed to stand insulated for definite intervals of time, during which the absorbed charges worked out to the surface. It was then discharged through a galvanometer, the throw of which furnished a comparative measure of the discharges.

Successive residual discharges were thus obtained by allowing the condenser to stand after each discharge for an interval of time, during which the residual charges worked to the surface. The first, second, third, and fourth residual discharges were obtained with equal intervals of time between the successive discharges. This was repeated, allowing different intervals of

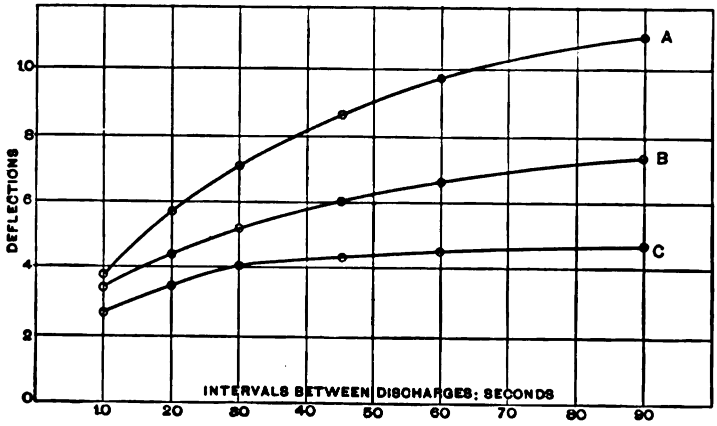

Fig. 1.

First Residual Discharge. Curve A for no previous charge; curve B for a previous negative charge for 30 seconds; curve C for a previous negative charge for 60 seconds.

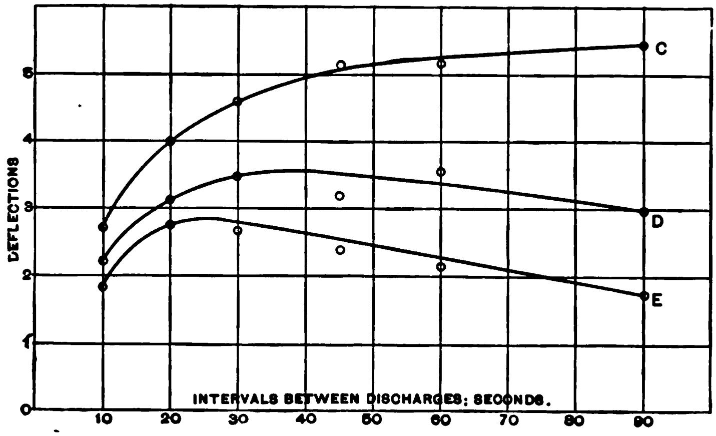

Fig. 2.

First Residual Discharge. Previous negative charge as follows: Curve C 60 seconds; curve D 90 seconds; curve E 120 seconds.

time between the discharges, and the results are shown in Figs. 1–8. In these figures, curves *A*, *B*, *C*, *D*, and *E* represent the residual discharges corresponding to different lengths of previous negative charge. In the case of curve *A*, there was no previous charge of any kind, the condenser being as nearly neutral as possible when the final positive charge of 30 seconds was given. The measurements for curve *B* were made upon the same condenser, which was given a previous negative charge for 30 seconds, was then allowed to stand insulated for five seconds, and then short-circuited for 15 seconds previous to the final positive charge. For curve *C*, the condenser received a preliminary negative charge for 60 seconds, stood insulated for five seconds, and was then short-circuited for 15 seconds previous to the final positive charge. For curves *D* and *E*, the previous negative charges were 90 and 120 seconds respectively, all other operations being as before. The absorption due to a positive charge deflected the needle to the right, and such deflections are plotted above the *X*-axis; deflections to the left, due to a negative charge,

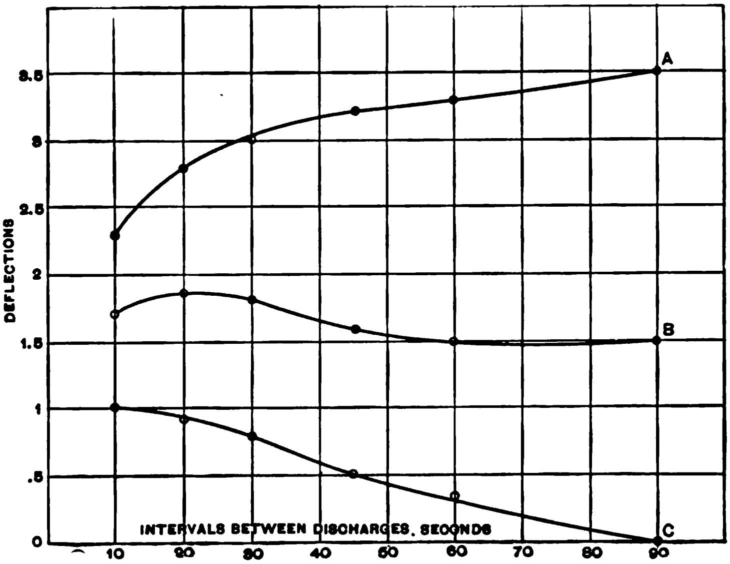

Fig. 3.

Second Residual Discharge.

are plotted below the axis. In taking curves *D* and *E*, the sensitiveness of the galvanometer was changed, and curve *C* was, therefore, determined twice, in order to show the relation between curves *A* and *B* and curves *D* and *E*.

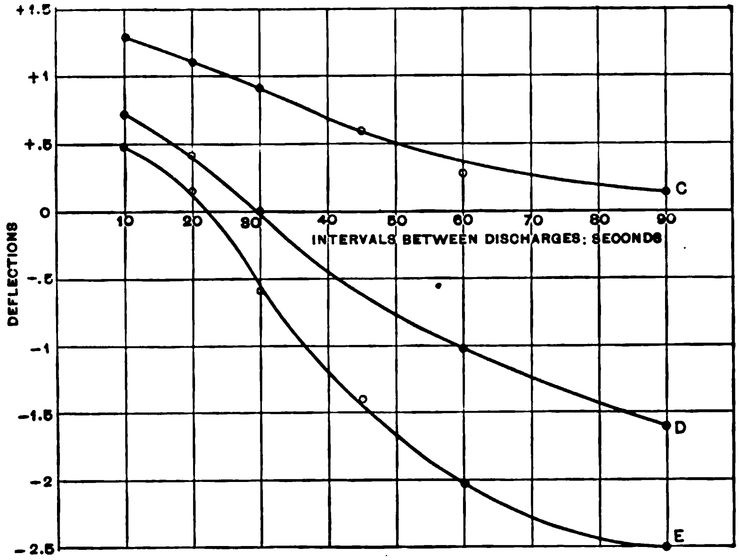

Fig. 4.
Second Residual Discharge.

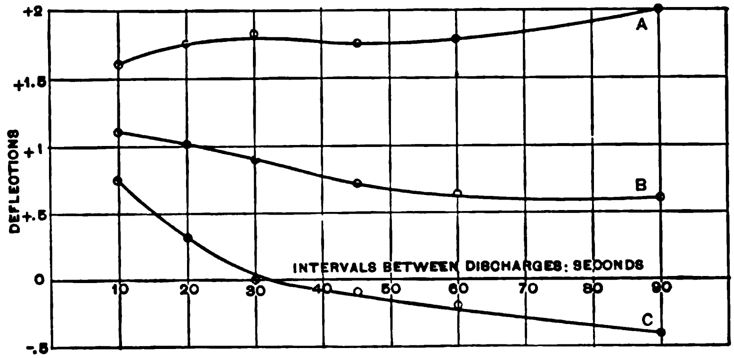

Fig. 5.
Third Residual Discharge.

The effect of the previous negative charge in partially or wholly neutralizing the final positive charge is clearly seen by an inspection of the curves. This influence becomes more and more marked with each successive discharge, and still more apparent as the intervals between the discharges are made greater. This would naturally be so, since the previous residual charge has time to work out and neutralize the absorption due to the final charge.

In the first residual discharge curves, Figs. 1 and 2, the previous charge has evidently the most influence where the intervals between discharges are the longest, and although the discharges are all positive, it is not at all unlikely that curves *D* and *E*, at any rate, would be brought to zero, or even made negative by increasing the intervals between discharges.

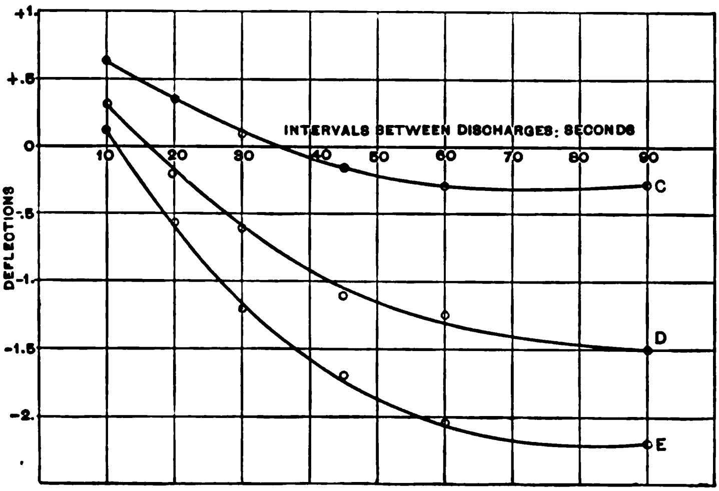

Fig. 6.
Third Residual Discharge.

In the curves for the second residual discharges, Figs. 3 and 4, curve *C* is brought to zero, while curves *D* and *E* actually cross the *X*-axis, indicating a negative discharge; that is, one opposite in direction to the final charge. The evident turning again of the lower part of the curves indicates that the previous negative charge is becoming weakened.

All these effects are more marked in the curves for the third residual discharges, Figs. 5 and 6, where in the case of curves *D* and *E*, the 10-second interval is the only one giving a residual discharge in the direction of the final charge. These curves are very nearly of the form which they would have if the final positive charge had not been given to the condenser at all, and the residual discharges were due only to the previous negative charge, as is more plainly seen by inverting the figure. As is usual, the fourth residual discharge curves are rather flat, but curves *D* and *E*, as they indicate the presence of merely a negative charge, resemble the usual second discharge curves, and are considerably concaved towards the *X*-axis.

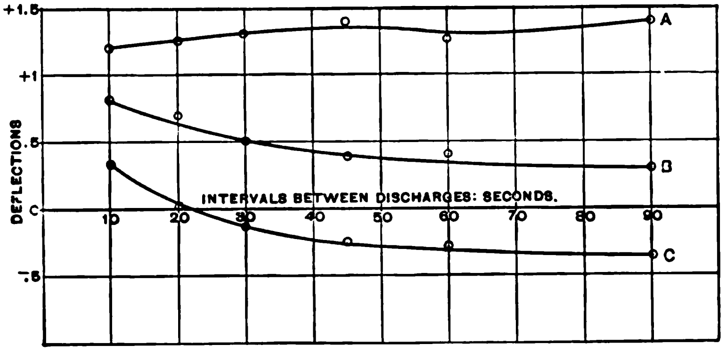

Fig. 7.

Fourth Residual Discharge.

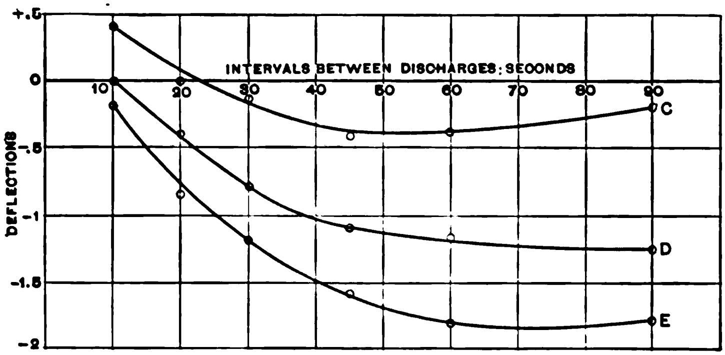

Fig. 8.

Fourth Residual Discharge.

The fifth and sixth discharge curves were obtained, but were not sufficiently marked to warrant reproduction; they merely emphasized the foregoing observations.

The curves shown in Figs. 9 and 10 are obtained from the preceding curves, and show the effect of a change in the length of the previous negative charge given to the condenser before the final charging. The residual discharges, as measured by the throw of the galvanometer, are represented as ordinates, and the lengths of the previous negative charges as abscissæ. Two sets of curves are drawn, obtained with intervals between the successive discharges of 45 and 90 seconds, respectively. In the case of the curves corresponding to intervals of 90 seconds, the residual discharges decrease with the increase of the previous negative

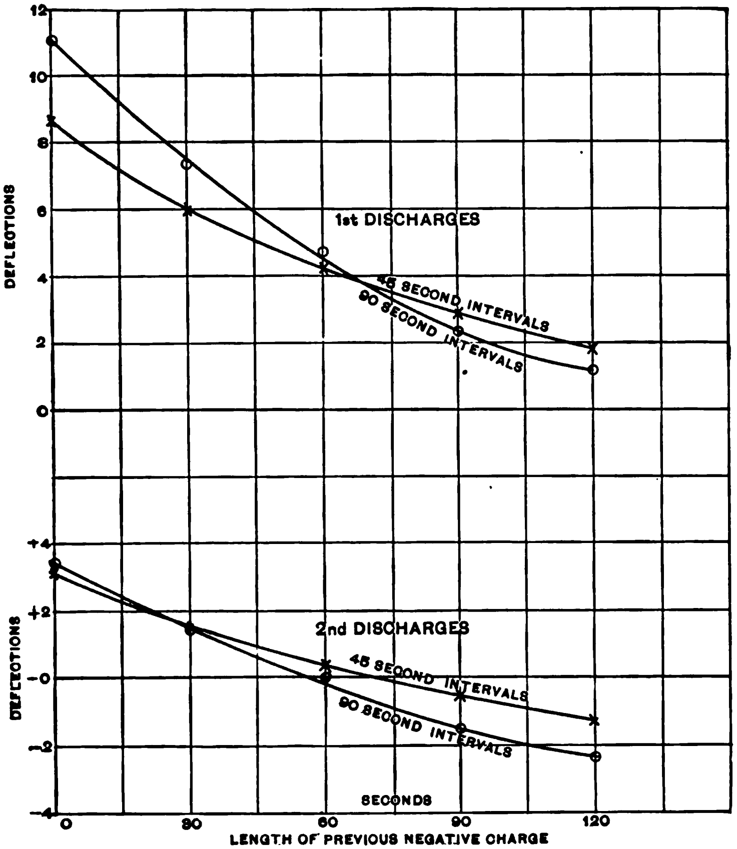

Fig. 9.

Effect of Previous Negative Charge upon Residual Discharges.

charge, more rapidly than for the corresponding curves taken with 45-second intervals between the successive discharges. This is in accordance with the interpretation of the previous results. These curves show plainly that the absorption of the previous negative charge becomes greater as the time of this charge is increased; and the smoothness of the curves would indicate that the dependence of absorption upon time follows some definite law. The curves would probably be asymptotic to some line determined by the condition of the condenser and the nature of the dielectric. This line would then represent the limit to the amount of absorption.

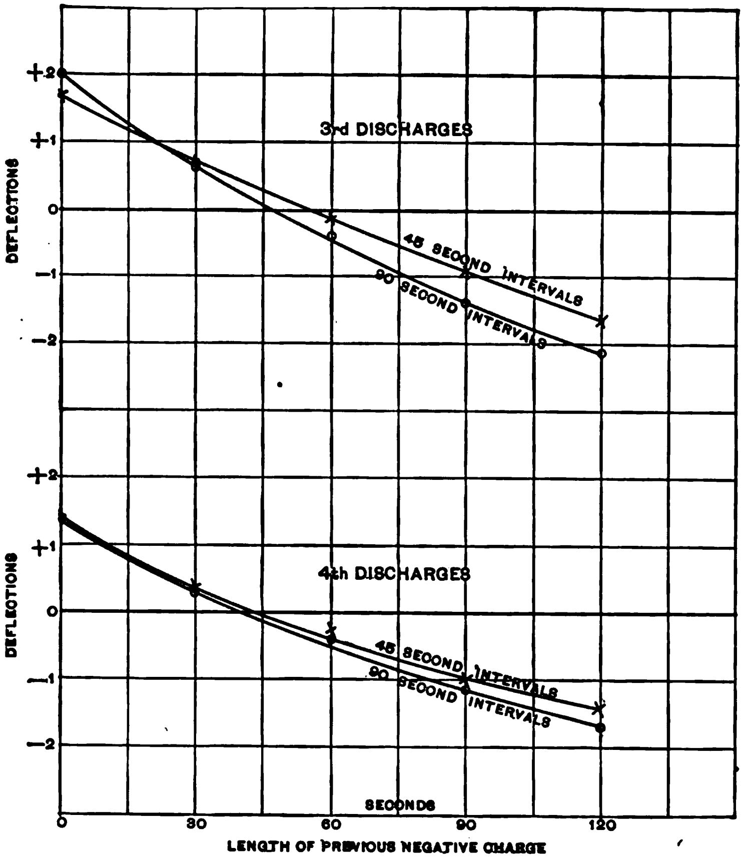

Fig. 10.

Effect of Previous Negative Charge upon Residual Discharges.

The reflex nature of the curves, particularly of *D* and *E*, Figs. 4 and 6, suggested that they might be made to again cross the axis of *X* if three charges were given them, the first being in the direction of the final charge. This was done, and upon short-circuiting the condenser and then taking residual discharges at 10-second intervals, the galvanometer first showed positive, then negative, and finally again positive residual discharges.

With longer intervals between discharges it was found that residual charges of opposite sign more thoroughly neutralized each other. Hence, a condenser, if charged in both senses for different periods, would show before long a residual discharge merely in the direction of the greatest absorption. If the period were made long enough this would likewise leak away, and the curve would approach the axis of *X*; although, probably, the condenser would never become absolutely neutral.

The Effect of Absorption upon the Discharge Curves.

The curve for the discharge of a condenser through its own dielectric deviates widely from an exponential curve on account of the absorption of charge in the dielectric; for, as a condenser discharges, this residual charge works out to the surface and

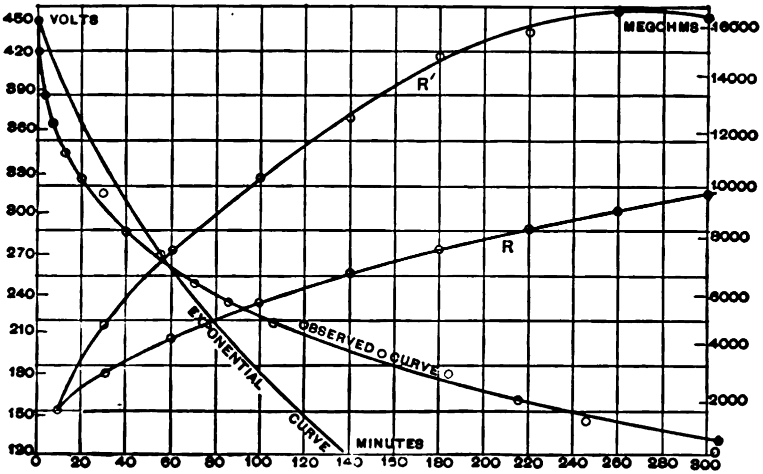

Fig. 11.

Condenser Discharge Curve and Computed Resistance. (Condenser No. 3.)

becomes effective. The form of the discharge curve depends likewise upon previous charges, whether of the same or of opposite sign. The apparent resistance of a condenser is similarly dependent upon this absorption.

The discharge curves of the condenser were obtained by measuring the difference of potential at its terminals, by means of a multicellular voltmeter, as the condenser was allowed to discharge through its own dielectric. The leakage through the voltmeter was found to be negligible. Assuming that the discharge of the condenser follows the exponential law, we have

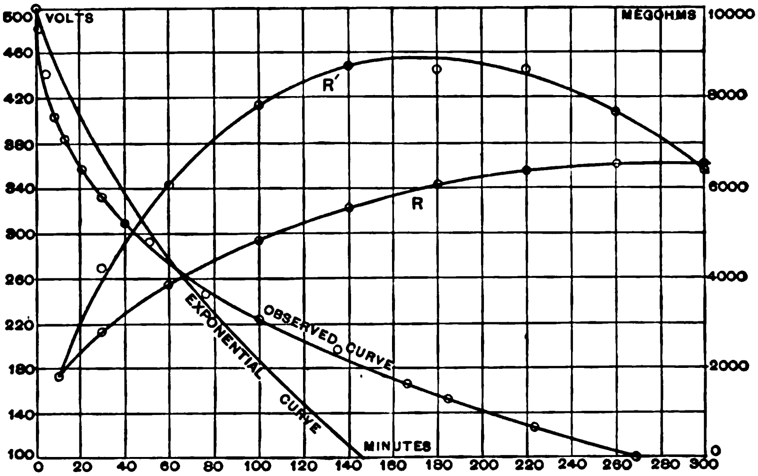

Fig. 12.

Condenser Discharge Curve and Computed Resistance. (Condenser No. 3.)

$$V_t = V_0 \epsilon^{-\frac{t}{RC}},$$

where V_0 represents the initial potential; V_t the potential at some time t counted from the beginning of discharge; C the capacity of the condenser; R the insulation resistance, and ϵ the base of the Napierian logarithms. This gives

$$R = \frac{t}{C \log_\epsilon \frac{V_0}{V_t}}.$$

This value of the resistance is the value which would be true if the discharge curve were exponential from the beginning of discharge to the time t.

The resistance may be otherwise computed by taking brief intervals of time and assuming the curve to be exponential for this time. Thus, if V_1 and V_2 are the potentials at the time t_1 and t_2, we have

$$R' = \frac{t_2 - t_1}{C \log_e \frac{V_1}{V_1}}$$

The value of the resistance thus obtained, which we will call R', is a closer approximation than the resistance R obtained above, and becomes more nearly a true value for the apparent resistance of a condenser as the interval between t_1 and t_2 is shortened.

In Fig. 11 is given the observed discharge curve from a condenser, which, although used for some time, had always been charged in the same direction. An exponential curve is also plotted with the assumed constant resistance of 4250 megohms. The derived curves showing the variation of the resistances R and R', obtained as explained above, are also given.

This condenser was left short-circuited for 88 hours, and then charged in the opposite direction, and the curve of discharge obtained as above. The results are shown in Fig. 12. The previous charge affects the shape of the discharge curves, but its influence is more clearly seen in the curves for the variation of resistance. The increase in apparent resistance, shown in Fig. 11, is due, as has already been explained, to the absorption in the dielectric. In Fig. 12 the resistance increases at first, due to the absorption of the final charge, and then decreases as the previous absorption of opposite sign begins to be effective. From this it is evident that the soaking-in effect, which is the cause of the usual rise of resistance, must have acted in the opposite direction, and so must have hastened instead of hindering the discharge. Where the curve for resistance R', calculated for short intervals of time from the discharge curve, is horizontal, the two absorbed charges neutralize each other, the resistance is constant, and the discharge curve is truly exponential.

The exponential curves, of course, cross the actual discharge curves when the resistance of the condenser is the same as the constant resistance assumed for the exponential curve. The resistance assumed in the case of an initially neutral condenser should be so small that the exponential curve should lie entirely below the observed discharge curve, inasmuch as the apparent resistance is increased by the coming out of the charges previously absorbed.

The Influence of Initial Potential upon the Average Resistance.

To show more definitely that the departure of the discharge curve from an exponential form, and the variation of resistance were due to absorption, and were consequently functions of the time rather than potential, discharge curves were taken through a wide range of initial potential, and the average resistances calculated by taking the mean values of the resistance obtained after 60, 160, and 260 minutes, according to the two methods above explained. Two condensers were used to check the accu-

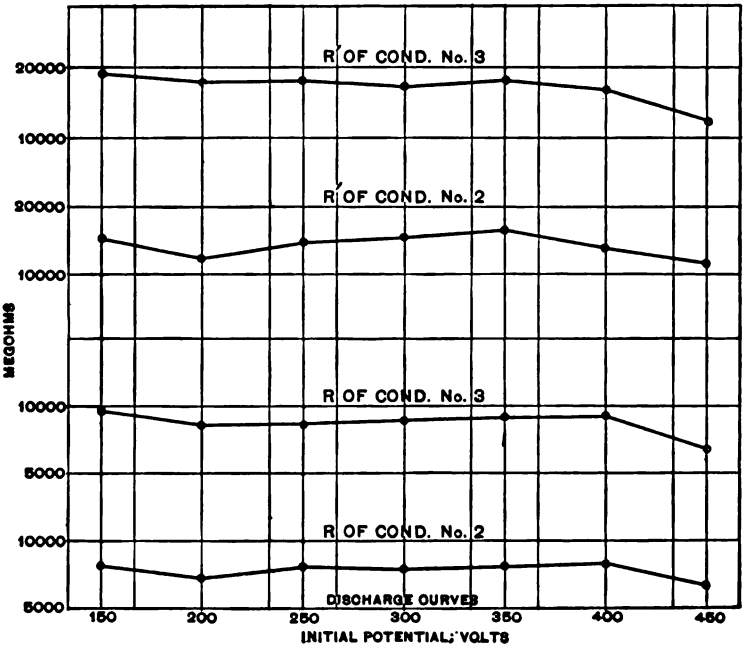

Fig. 13.

Average Resistance corresponding to Different Initial Potentials.

racy of the work. The results agree throughout, and show a practically constant resistance throughout the range of initial potential employed; viz. from 150 to 450 volts, as shown in Fig. 13. The condensers were designed to stand a pressure of 500 volts, so we may conclude that while the insulation remains unstrained the changes which occur are due to what may be termed local causes — such as the conditions of the atmosphere and of previous charge — rather than to the initial potential of the condenser.

The Effect of Temperature on the Resistance of Oils.

The foregoing results show the effects of absorption in a solid dielectric. The results which follow show that the absorption in ordinary dielectrics, as shown by the residual discharges and the curves of discharge of a condenser through the dielectric, are closely dependent upon temperature changes, and that at high temperatures the effects of absorption and residual discharge disappear entirely. An investigation of the effect of temperature changes upon the resistance of various commercial oils was made preliminary to a similar investigation upon solid dielectrics. The experiments for the determination of these temperature effects

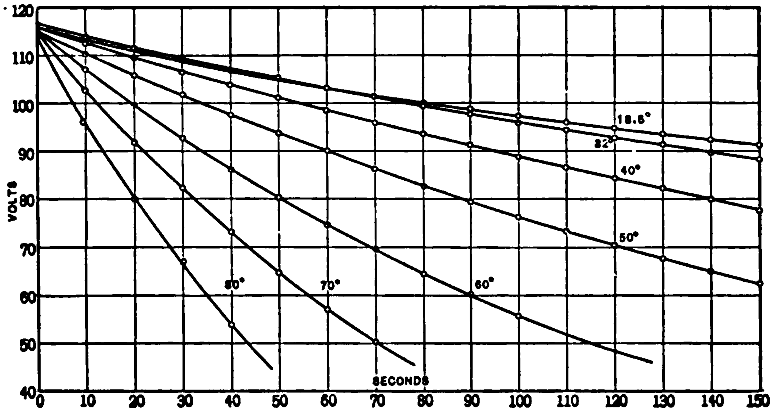

Fig. 14.

Discharge through Vacuum Oil at Different Temperatures.

were conducted, under the direction of one of us, by Mr. E. G. Gilson and Mr. E. G. Mason, and are described at length in their report[1] upon the work.

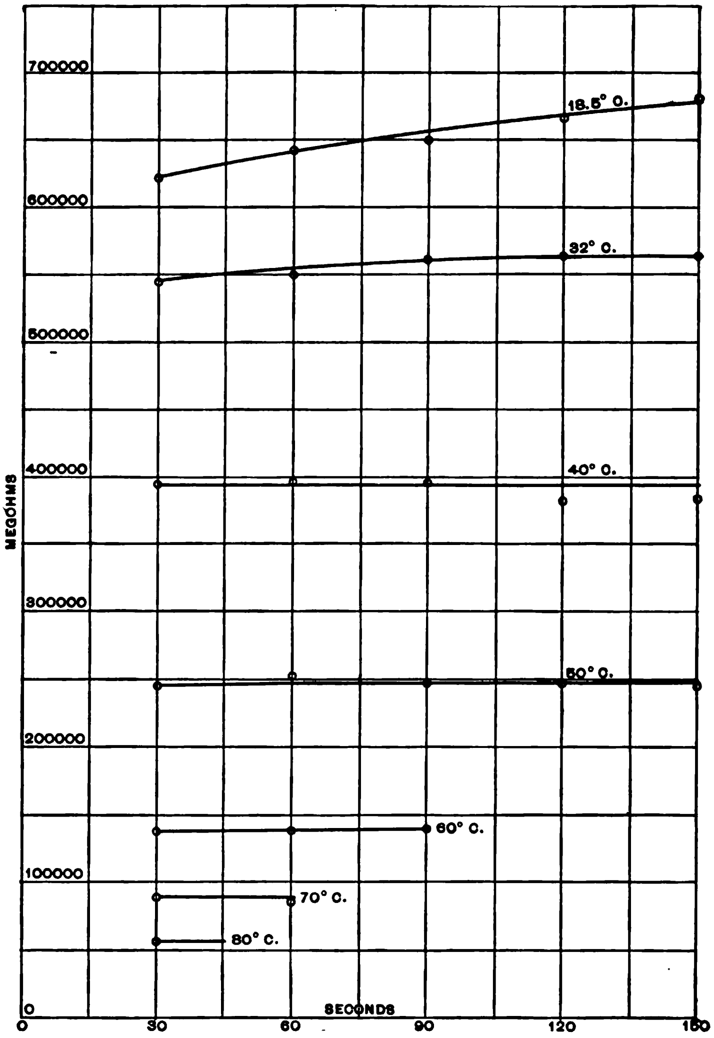

Fig. 15.

Resistance of Vacuum Oil at Different Temperatures.

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[1] See MS. thesis in the Cornell University Library, "Effects of Temperature upon Dielectrics," by E. G. Gilson and E. G. Mason.

In perfect oils there should be no absorption of charge, and the curves for the discharge of a condenser through such oils should follow the exponential law. Two surface plates were immersed in the oil experimented upon; these plates were 25.25 cm. long by

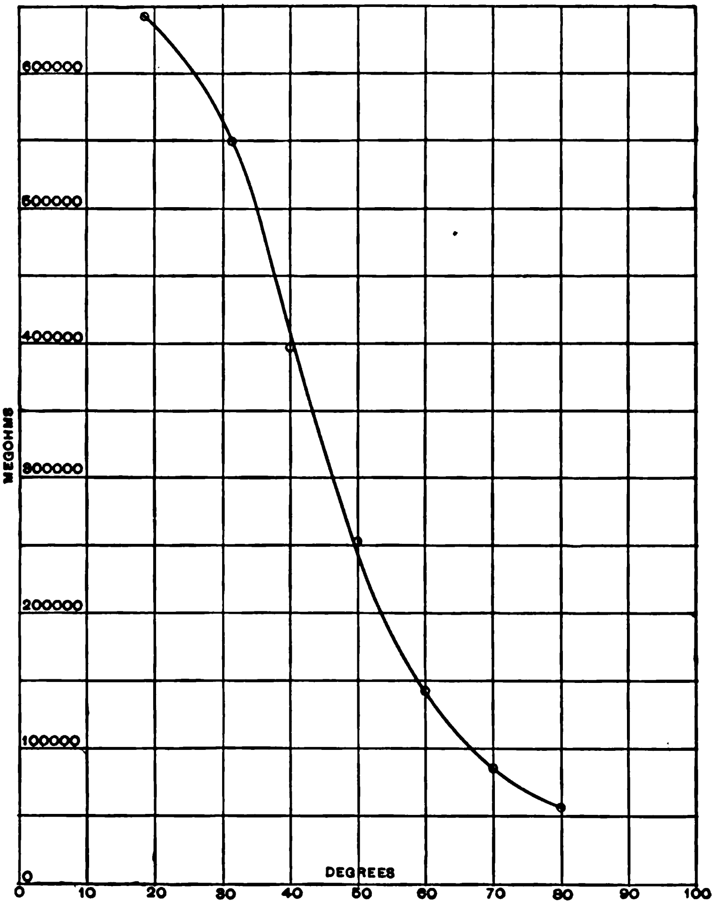

Fig. 16.

Resistance of Vacuum Oil at Different Temperatures after 60-second Electrification.

17.5 cm. wide, and were separated by three hard-rubber pillars 1.47 cm. high. The resistance of the oil was determined by discharging a condenser of known capacity through the oil by connecting the terminals of the condenser to these two surface plates.

The potential at the terminals of the condenser was measured by multicellular voltmeter, and the resistance computed according to the exponential law. The resistance of the rubber pillars, the multicellular voltmeter and of the condenser insulation were so small compared with that of the oil, that no correction for them was found necessary.

The results for different oils were similar, and curves are, therefore, given for vacuum oil alone.

Fig. 14 shows the curves for the discharge of a condenser through vacuum oil at different temperatures. The temperature of the condenser and its capacity were constant. The resistance of the oil computed from the discharge curves according to the first method already described, is plotted in Fig. 15. For the lower temperatures there is a slight increase in the resistance during the discharge. This has already been commented upon in the case of solid dielectrics, and indicates that there has been some absorption, possibly due to impurities in the oil. That the resistance is more nearly constant, and the discharge curves exponential at the high temperatures, indicates that absorptive effects disappear at these temperatures.

Fig. 16 shows the resistance of vacuum oil at different temperatures after an electrification[1] of 60 seconds. This decrease in resistance is most rapid between 30 degrees and 60 degrees where the resistance diminishes over 80 per cent. The same phenomena were observed in the experiments with boiled linseed oil, petroleum oil, and cylinder oil. The results are given in the accompanying tables.

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[1] This word as here used may be misleading. It is used to denote the time the condenser stands electrified after being separated from the source of potential.

RESISTANCE IN MEGOHMS OF 1 CU. CM. OF VACUUM OIL.

Time in seconds.	18.5°	32°	40°	50°	60°	70°	80°
30	622,000	547,550	395,000	247,500	139,200	91,250	58,600
60	643,500	549,500	396,500	252,500	140,100	88,000	
90	649,900	560,000	396,600	249,500	142,000		
120	666,000	564,500	384,000	249,000			
130	682,000	564,000	383,500	245,000			

RESISTANCE IN MEGOHMS OF 1 CU. CM. OF CYLINDER OIL.

Time in seconds.	20°	40°	50°	60°	70°	80°	90°
30	141,100	136,000	78,200	60,300	40,850	36,830	37,420
60	141,000	140,950	78,750	63,200	42,700	38,750	39,300
90	162,950	142,500	80,350	64,200	53,600	39,550	40,450
120	165,800	146,900	82,100	63,950	44,220	40,300	40,500
150	185,200	148,200	83,600	64,900	44,550	40,850	41,650

RESISTANCE IN MEGOHMS OF 1 CU. CM. OF BOILED LINSEED OIL.

Time in seconds.	20°	30°	40°
10	—	1263	770
30	2310	1181	1084
60	2340		

RESISTANCE IN MEGOHMS OF 1 CU. CM. OF PETROLEUM OIL.

Time in seconds.	13°	20°	30°	40°	50°
30	44,700	38,900	28,150	20,200	14,200
60	45,650	39,990	28,210	19,740	
90	46,550	40,850	28,720		
120	47,150	40,200			
150	47,550				

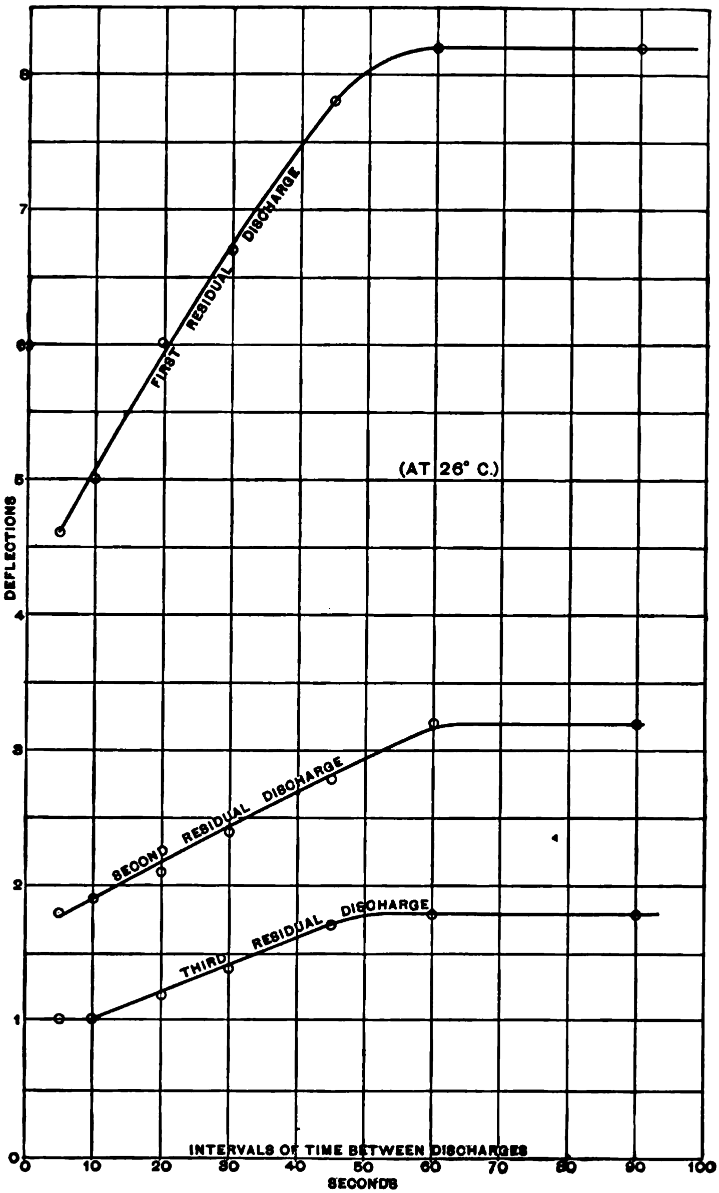

Fig. 17.

Residual Discharges of Condenser at 26° C.

Temperature Effects upon Solid Dielectrics.

For this investigation the same condensers were used as in the experiments described in the earlier parts of this paper. An investigation of the dielectric hysteresis of these condensers and the effects of temperature upon capacity have already been described at length in a paper[1] in the PHYSICAL REVIEW, where a detailed description of the condensers is also given.

A determination of the capacity of the condenser was made in connection with these earlier experiments, and also during the present investigation a year later. The values are here given for comparison.

Condenser.	No. 2.	No. 3.	No. 4.
Capacity (Microfarads) in 1893	1.571	1.52	1.539
" " " 1894	1.592	1.535	1.554

The measurements were made in each case by the method of mixtures, but with different apparatus, so that some of the variation may be due to the change of standards. The condensers, however, had stood severe treatment, being in continual use for a year under all sorts of conditions, and a slight change of capacity might be looked for.

Temperature Effects upon Residual Discharges.

The first, second, and third residual discharges are obtained at different temperatures by the method used in the experiments already described. The sensitiveness of the galvanometer was not kept constant, so that the deflections cannot be compared so as to show the relative values of the absolute quantity of electricity constituting the residual discharges under different conditions. The shape of the curves plotted in Figs. 17 and 18, however, indicates the relative amount of absorption at different temperatures, which is the point in question.

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[1] Bedell, Ballantyne, and Williamson, "Alternate-Current Condensers and Dielectric Hysteresis," PHYSICAL REVIEW, Vol. i., No. 2.

The residual discharges at 26° C. show a large amount of absorption, as it took long intervals between discharges to equalize the residual discharges.

Residual discharges at 32° C. give curves much flatter; the interval between discharges does not largely affect their amount. Residual discharges at 40° C. give remarkably flat curves, the intervals between discharges having, practically, no effect upon

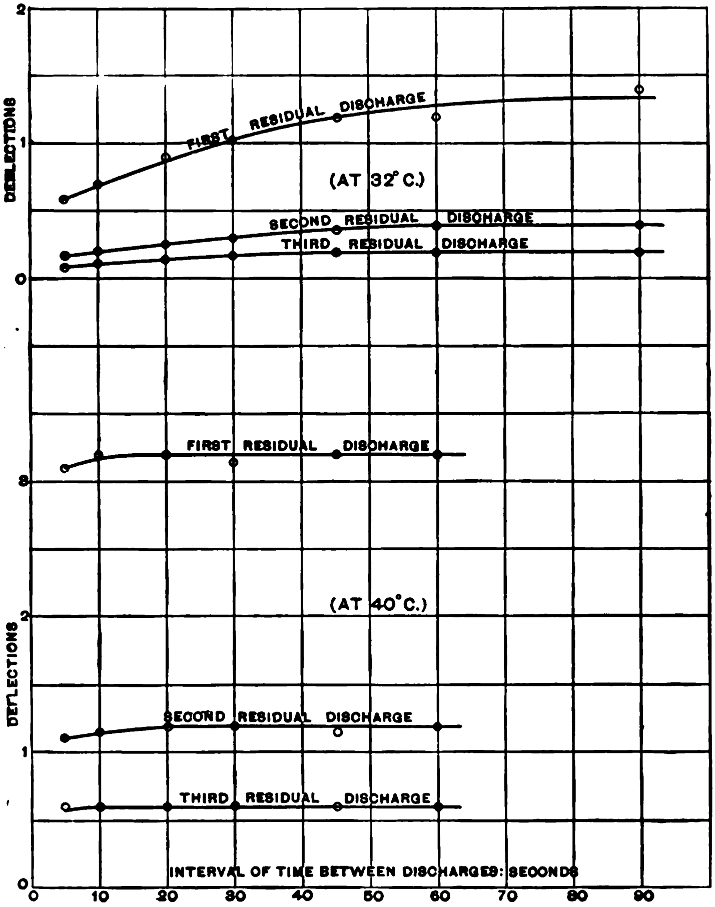

Fig. 18.

Residual Discharges of Condenser at 32° C. and 40° C.

the discharges. This indicates that there was but little absorption. Whatever residual charge there was, so readily passed out that a large increase in the intervals between discharges did not affect the quantity of the residual discharge.

These experiments show conclusively that absorption, as shown by residual effects, disappears at high temperatures.

Temperature Effects upon Discharge Curves and Insulation Resistance.

The insulation resistance of a condenser was obtained by allowing the condenser to discharge through its own insulation, and then solving the usual exponential equation for the resistance. The capacity of the condenser at different temperatures was known as the result of the experiments already referred to.

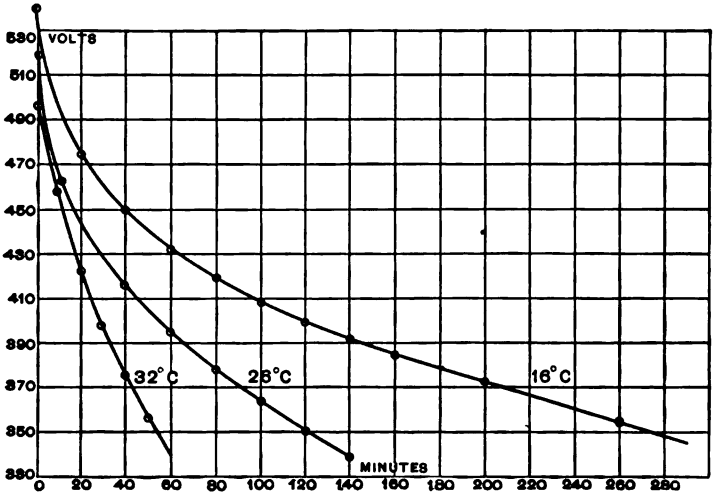

Fig. 19.

Condenser Discharge Curves at Different Temperatures.

The discharge curves at different temperatures are given in Fig. 19; the differences are due, however, to change in both capacity and resistance.

The variation in resistance during the discharge, computed according to the first method before described, is shown in Fig. 20, and is to be compared with Fig. 15, which shows the corresponding results for vacuum oil. Fig. 20 shows that the resistance remains more nearly constant at high temperatures. Inasmuch as this variation of resistance during discharge is due to absorption, it is evident that there is less and less absorption as the temperature rises, a conclusion already reached from a study of residual discharges.

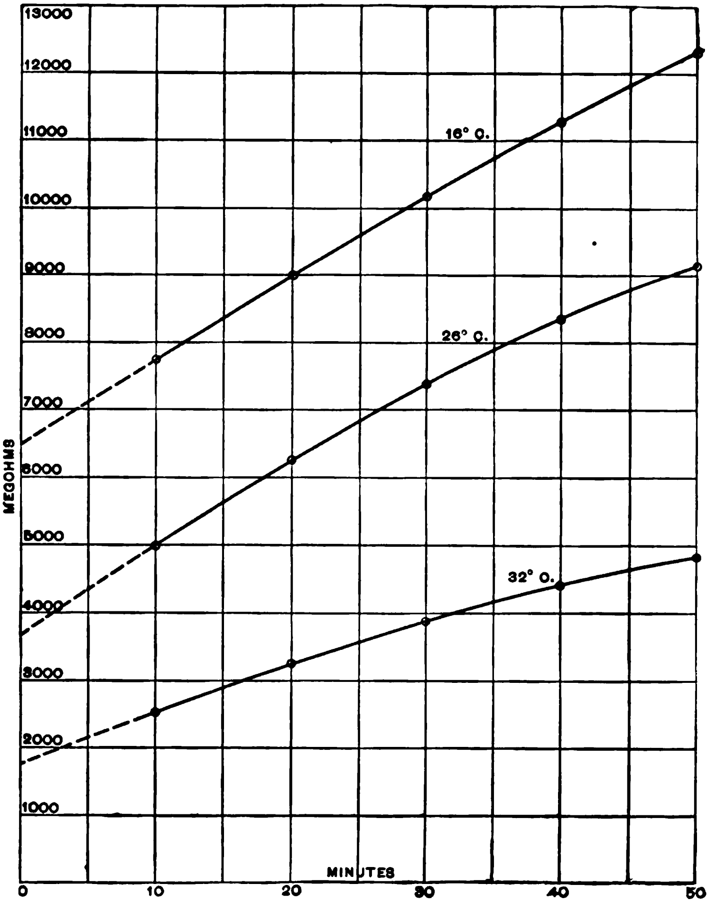

Fig. 20.

Condenser Resistance after Different Times of Electrification at Different Temperatures.

The condenser resistance at different temperatures, after three minutes of electrification,[1] is given in Fig. 21, and is in form quite similar to the corresponding curve for vacuum oil (Fig. 16). The sudden bend in a curve after 40 degrees corresponds to the rapid changes in capacity at these temperatures. Between 42 degrees and 50 degrees the capacity increases very rapidly. When the capacity becomes infinite, the resistance will, of course, be zero.

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[1] See preceding note.

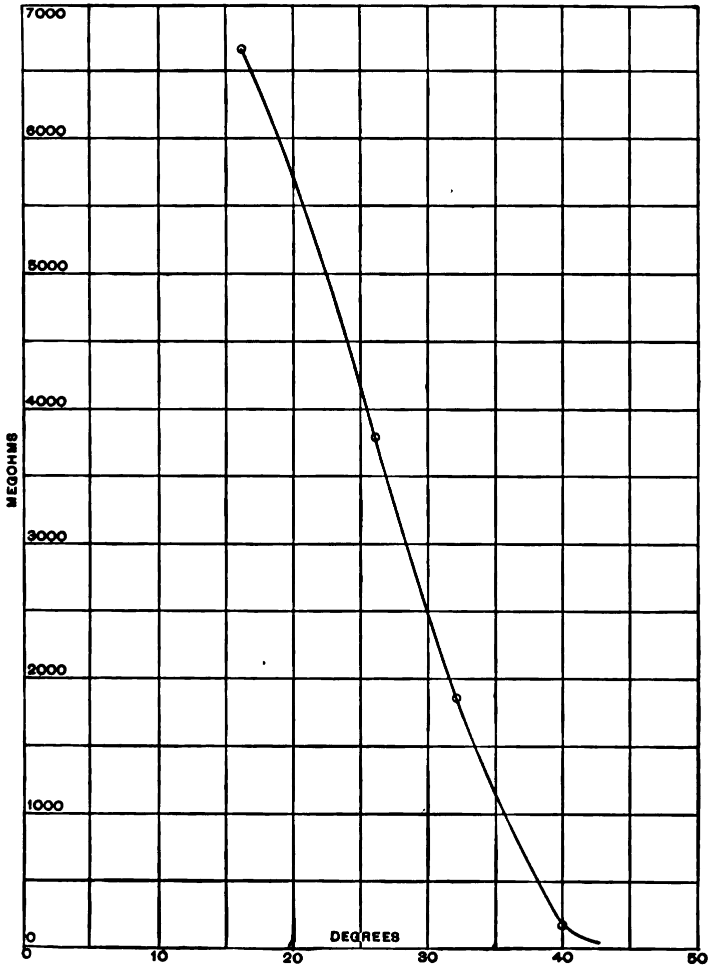

Fig. 21.

Condenser Resistance for Different Temperatures after Three Minutes' Electrification.

Mr. M. A. Hess read a paper before La Société Française de Physique, a review of which appeared in *L'Électricien*, January, 1894, in which he says, in speaking of the phenomena of absorption: "If the product of the capacity and resistance of different portions of the dielectric sheet are equal, there will be no residual discharges. . . . It has been observed that the insulation diminishes when the temperature rises. . . . From this it is seen that there is a temperature at which the phenomena of the residual charge disappears."

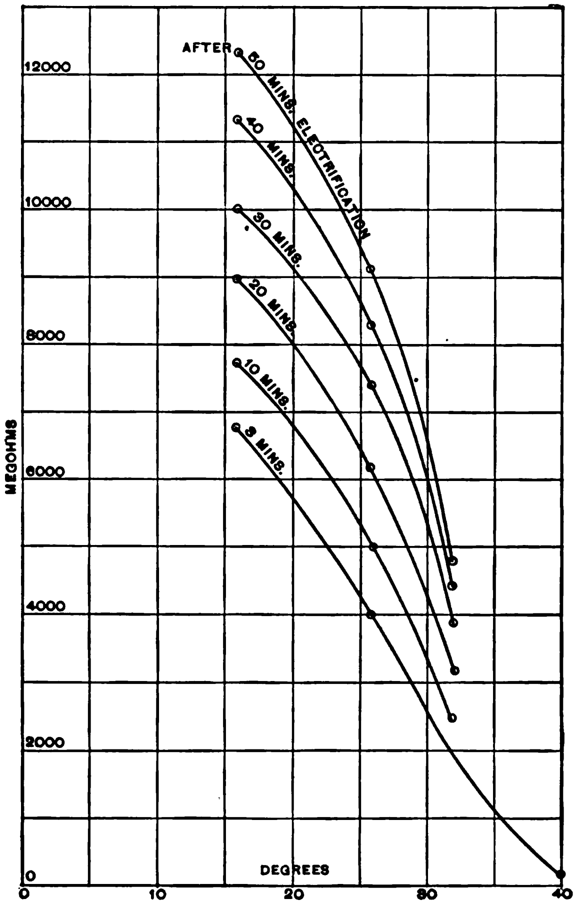

Fig. 22.

Condenser Resistance for Different Temperatures after Different Times of Electrification.

In Fig. 22 curves are drawn after different times of electrification similar to the curve in Fig. 21. These emphasize the results already discussed. These curves come nearer and nearer together at high temperatures as absorption is eliminated, and apparently would coincide at some temperature between 40° C. and 50° C., the limiting temperature the condenser could stand.

The experiments given above would indicate that in the case of the paraffin condenser, absorption will never be eliminated until the capacity becomes infinite. The temperature was raised until the resistance became only one-twentieth of its normal value. The absorption, though diminishing, was very definite in amount, and the condenser would evidently break down before the residual discharges entirely cease.

Conclusion.

The action of a condenser is dependent, in a marked degree, upon its previous history; so much so, in fact, that its previous charges may be of more importance in determining its action under certain circumstances than charges received later. In that case the previous absorption, after neutralizing the absorption due to the final charge, may give rise to residual discharges which increase with time up to their previous initial value.

Absorption gives rise to the phenomena of residual charges, and causes the condenser to depart from the exponential law in discharging through its own dielectric. The increase in apparent resistance in the condenser during discharge is associated with this effect of absorption. Previous charges modify these results, increasing or decreasing them according to whether the previous charges were in the same or opposite sense.

The resistance of pure oils is constant at any one temperature, but falls off rapidly with the temperature. There are, accordingly, no absorption or residual effects in pure oils.

In solid dielectrics the effects of absorption are diminished as the temperature is increased, as shown by the residual discharges and by the changes in the insulation resistance.

This investigation can in no wise be considered as comprehensive. Many of the results here described are already known, but, as far as the writers are aware, they have not before been presented together so as to show the relation between them. The fact that the previous condition of the condenser has such an influence upon its action, causes considerable embarrassment in an investigation of this sort, inasmuch as a condenser used once under certain conditions may be practically useless in the same investigation for further experiments. One of the condensers experimented upon retained its past charges after being short-circuited for a month, and it was necessary to employ another condenser, which was neutral, for further experiments.

A GENERAL THEORY OF THE GLOW-LAMP. II.[1]

By H. S. Weber.

III.

THE definitive quantity in the case of the glow-lamp is the energy which the electric current develops in the filament in a unit of time. By means of this the temperature of the filament is determined when the surface is known, and thus finally the quantity of light which the filament radiates is given. One of the fundamental questions in the domain of glow-lamp illumination is that of the relation between the amount of light radiated and the expenditure of energy necessary to its production. Voit has attempted to find a simple expression for this quantity based upon experiments with incandescent lamps made at the Munich electrical exhibition. As has already been pointed out he found that the light radiated was neither proportional to the square nor to the fourth power, but approximately proportional to the cube of the energy consumed. "The formula $H=qE^3$ may be regarded as an empirical rule which for glow-lamps is between about 6 c.p. and 120 c.p." For the constant q, Voit found, from the Munich measurements: —

Edison lamp	(16 c.p.) $q =$	37.6×10^{-6}	Siemens lamp	(16 c.p.) $q=$	22.5×10^{-6}
Edison lamp	(8 c.p.) $=$	110.6×10^{-6}	Small Müller lamp	(20 c.p.) $=$	21.3×10^{-6}
Small Swan lamp	(10 c.p.) $=$	84.8×10^{-6}	Medium Müller lamp	(50 c.p.) $=$	6.7×10^{-6}
Large Swan lamp	(40 c.p.) $=$	9.6	Large Müller lamp	(100 c.p.) $=$	2.1×10^{-6}
Maxim lamp	(28 c.p.) $=$	14.8	Cruto lamp	(10 c.p.) $=$	25.0×10^{-6}

The constant q according to these measurements is very different in the case of different lamps, and as Voit has pointed out, it is small in proportion as the useful light energy of the lamp increases. No further investigation concerning the factors which are involved in this coefficient was attempted by Voit, and nothing has since been done in this direction. In my measurements of thirty-three types of glow-lamps, I have found for every lamp Voit's empirical relation, according to which the intensity of light is proportional to the cube of the energy consumed, fairly well established. It appeared, however, that the size of Voit's coefficient is not quite a constant, but that it increases as the light increases from small intensities, then more rapidly with increasing brightness to a maximum, and finally diminishes again.

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[1] A paper read at the Frankfort Congress for Electrotechnics. Translated for the Physical Review by E. L. N.

A glance at the column marked $q = \frac{H}{E^3}$ of the last four tables shows the change in this quantity with increasing HE as just described. At the same time it appeared from measurements, as Voit had shown, that the size of the coefficient for different lamps is enormously different. As a mean value of q we find from the above tables:—

For the Woodhouse lamp	41.0×10^{-6} and 32.0×10^{-6}
For the Sunbeam lamp	2.12×10^{-6} and 1.49×10^{-6}
For the lamp of the Allgemeinen Electricitäts-Gesellschaft	87.9×10^{-6} and 70.4×10^{-6}
For the Cruto lamp	75.7×10^{-6} and 55.8×10^{-6}

where H is the mean horizontal intensity or the mean spherical intensity respectively. It is easily seen that all these phases are represented in a formula for radiation already presented. The intensity of any given visible homogeneous ray of wave-length λ is given through the formula:—

$$s = I \frac{c}{\lambda^2} \pi F e^{aT - \frac{1}{b^2 \lambda^2 T^2}}.$$

The brightness of this radiation is proportional to that quantity. In order to obtain an expression for the total brightness, we may take advantage of the fact that the candle-power of a solid glowing body increases in proportion to the brightness of the homogeneous ray, of which the wave-length is about 0.54μ. If we set in this last equation in place of λ this value, then the expression gives us the relation between the total brightness and the temperature.

The energy which is consumed in the glowing filament when the same has been warmed to a stationary temperature T is, according to the above equation,

$$E = ICFTe^{aT}.$$

If we take cognizance of the fact that the constant of total radiation has a value of

$$\tfrac{1}{2}\pi\sqrt{\pi}bc,$$

we obtain as an expression for the quotient H/E^3 the following formula:—

$$\frac{H}{E^3} = \frac{m}{C^2F^2} \cdot \frac{1}{T^3 \cdot e^{2aT + \frac{1}{b^2\lambda^2T^2}}} = \frac{m}{C^2F^2}\phi(T), \qquad (6)$$

in which small m is a constant. In order to determine the function $\phi(T)$, we must know, aside from the quantities $a = 0.0043$ and $\lambda = 0.54$, the constant b^2 for carbon. From Langley's observations concerning the distribution of energy in the spectrum of black carbon at a temperature of 178°, I find the quantity $b^2 = 0.192 \times 10^{-6}$, where the wave-lengths were measured in microns. From my own measurements upon the change in brightness of homogeneous rays from the gray carbons of the Sunbeam lamp, I determined for the temperature interval $T = 1450° \cdots T = 1650°$, $b^2 = 0.190 \times 10^{-6}$. If we assume for b^2 the latter value, then the quantity $b^2\lambda^2$ is equal to 0.0052×10^{-6}.

In general, the function depends in a complicated way upon the temperature. If, however, we have to consider only the temperature interval from 1400° to 1600°, within which range the temperature of the glow-lamp lies, it may be shown that the character of this function for the interval in question is very simple. The following table shows the change of $\phi(T)$ for successive intervals of 10°:—

T	lg $\Phi(T)$	$\Phi(T)$	T	lg $\Phi(T)$	$\Phi(T)$
1440°	−42,959	0.953 × A	1520°	−42,913	0.998 × A
1450	−42,945	0.966	1530	−42,916	0.995
1460	−42,934	0.977	1540	−42,920	0.991
1470	−42,924	0.987	1550	−42,927	0.984
1480	−42,918	0.993	1560	−42,936	0.975
1490	−42,914	0.997	1570	−42,947	0.965
1500	−42,912	0.999	1580	−42,959	0.953
1510	−42,911	1.000			

The last vertical column of this table indicates that the function $\phi(T)$ between $T=1400°$ and $T=1580°$ goes through precisely the same changes as the factor q already described. It increases slowly from 1400° on, reaches a maximum of 1510°, the value of which is expressed by the equation

$$3+2aT-\frac{2}{b^2\lambda^2T^2}=0,$$

and then diminishes slowly. Between 1475° and 1542° the value of $\phi(T)$ changes only 1 per cent; between 1463° and 1555°, 2 per cent; from 1554° to 1565°, 3 per cent.

Observations with the various individual lamps are in entire accord with this result; viz. the maximum for q in the case of the Woodhouse lamp occurs in the neighborhood of 1415°; for the Sunbeam lamp, in the neighborhood of 1498°; for the Berlin lamp, 1503°; and for the new Cruto lamp, finally, at 1514°.

We may safely assume, then, that if the value of the function $\phi(T)$ for a temperature of 1510° is A, the quantity $q=HE^3$, for the entire range of temperature between 1452° and 1465°, will change at most 3 per cent from the constant value mAC^2F^2. Further, since the maximum value of q is

$$q_{\max}=\frac{mA}{C^2F^2},$$

the value of $q_{\max}C^2F^2$ must be constant for all lamps. This relationship is found to be true in fact, as the following tables for the four lamps will show: —

	$q_{\max}$	F^2	C^2	$q_{\max}C^2F^2$
Woodhouse lamp .	33.10×10^{-6}	0.790 cm.^{4}	0.285×10^{-9}	7.3×10^{-15}
Sunbeam lamp . .	1.57	25.400	0.172	6.9
Berlin lamp . . .	72.30	0.577	0.167	7.3
New Cruto lamp. .	56.50	0.399	0.292	6.6

The small variation for the value given in the last vertical column of this table is easily accounted for, since these quantities involve the square surface of the filament, which is difficult to determine, and the square of the constant of total radiation.

The mean value of this quantity ($q_{\max}C^2F^2$) I found for all the carbons investigated to be 6.9×10^{-15}.

Since q changes only about 1 per cent between 1460° and 1560°, we may assume with a high degree of approximation for this interval, for all lamps, the relation

$$H = \frac{6.8 \times 10^{-15}}{C^2F^2} \cdot E^3;$$

where H is the mean spherical candle-power expressed in British candles.

IV.

In its application to the glow-lamp, the relationship between the energy consumed in the filament in a unit of time, and the resulting brightness of radiation, is the most important quantity. Ever since the first measurements upon lamps were made it has been known that with increasing temperature of the filament this quantity diminishes very rapidly. But the law of the relationship between this quantity and the temperature has not been accurately known. The formulas which we have already developed from our observations enable us to express this relation in a simple manner.

Since the energy expended in a unit of time is

$$E = ICFTe^{aT}, \text{ where } C = \tfrac{1}{2}\,\pi\sqrt{\pi}\,cb,$$

and since the brightness expressed in any units — for example, British candles — is given by the expression

$$H = \alpha\frac{c}{\lambda^2} \cdot \pi F e^{aT - \frac{1}{b^2\lambda^2T^2}},$$

in which α is a factor of proportionality, and λ is the value 0.54, it follows that the quantity $E_1 = \frac{E}{H}$ is given by the equation

$$E_1 = \frac{I\pi\lambda\sqrt{\pi}}{2\alpha} \cdot b\lambda T . e^{\frac{1}{b^2\lambda^2T^2}}. \tag{7}$$

The energy necessary to develop a unit of light in a unit of time is consequently proportional to the temperature of the function, and depends upon the value of

$$\Psi(b\lambda T) = b\lambda T e^{\frac{1}{b^2\lambda^2T^2}}.$$

The change of this temperature of the function for the given value of $b^2\lambda^2=0.0552\times 10^{-6}$ for the interval of temperature applicable to the glow-lamp, 1400° to 1650°, is given in the following tables for intervals of ten degrees. At the same time values of this function for the higher temperatures, 1700°, 1800°, 1900°, and 2000°, are given, in order to indicate the further progress of this function at higher temperatures.

T	$\Psi(b\lambda T)$	T	$\Psi(b\lambda T)$	T	$\Psi(b\lambda T)$
1400	3501	1500	1135	1600	455
1410	3093	1510	1027	1610	419
1420	2740	1520	931	1620	387
1430	2433	1530	846	1630	358
1440	2166	1540	770	1640	332
1450	1934	1550	702	1650	308
1460	1731	1560	642	1700	214
1470	1552	1570	588	1800	115
1480	1395	1580	539	1900	68
1490	1257	1590	495	2000	44

On account of the great importance of the quantity E, an experimental proof that E is in fact such a function of the temperature, as it is represented to be by Equation 7, is of considerable value. For the purposes of this proof we may make use of the contents of the tables which have already been given to illustrate the properties of the four different glow-lamps. If we seek by means of these tables the values at the temperatures indicated, and divide these values by the value of $\Psi(b\lambda T)$, for the same temperatures we should obtain a constant quotient in case E really is of the form given in Equation 7.

The numbers given in the last columns of the following four tables are these quotients. It will be seen that for each of the four lamps the fluctuations are such as to size and direction that they may well be ascribed to errors of observation.

LAMP OF THE BERLIN CO. (ALLGEMEINE ELEK.-GESELLSCHAFT).

E_1	T	$\Psi(b\lambda T)$	$\frac{\Psi(b\lambda T)}{E_1}$
10.87 W.	1464°	1659	153
8.67	1483	1354	156
7.07	1503	1103	156
5.88	1522	914	155
4.91	1541	763	155
4.18	1557	640	158
3.57	1574	568	158
3.09	1591	491	158
2.71	1607	430	158
2.41	1621	384	159

SUNBEAM LAMP.

E_1	T	$\Psi(b\lambda T)$	$\frac{\Psi(b\lambda T)}{E_1}$
9.64 W.	1463°	1677	174
8.26	1478	1426	173
6.70	1498	1159	173
5.20	1524	897	173
4.11	1549	709	172
3.27	1573	573	176
2.68	1595	475	177
2.25	1615	403	179
1.94	1636	342	176
1.69	1654	299	177

NEW CRUTO LAMP.

E_1	T	$\Psi(b\lambda T)$	$\frac{\Psi(b\lambda T)}{E_1}$
14.03 W.	1434°	2326	166
10.63	1460	1731	163
8.41	1482	1367	163
6.08	1514	989	163
5.09	1532	831	163
4.32	1550	702	163
3.70	1566	610	165
3.14	1582	530	169
2.66	1600	455	171
2.31	1618	393	170

LAMP OF WOODHOUSE AND RAWSON.

E_1	T	$\Psi(b\lambda T)$	$\frac{\Psi(b\lambda T)}{E_1}$
20.60 W.	1400°	3501	175
12.37	1441	2141	173
8.20	1479	1411	172
5.77	1514	989	172
4.21	1548	716	170
3.63	1561	637	175
3.17	1578	549	173
2.76	1592	487	176
2.55	1606	433	170
2.23	1620	387	173

The contents of these four tables can also be used to demonstrate that the four different filaments to which they apply have the same or nearly the same values for the constant b. If b^2 is the same for these four carbons and equal to 19×10^{-6}, then the quotients must be the same for all four lamps. In order to compare these quotients among themselves, however, it is first necessary to take cognizance of the question of spherical intensity, since these lamps have very different relations between the mean spherical and the mean horizontal candle-power, as is shown by the following tables: —

	$\Psi(b\lambda T) : E_1$	R	$R\Psi(b\lambda T) : E_1$
Berlin lamp	157	0.801	126
Sunbeam lamp	175	0.711	124
New Cruto lamp	165	0.734	121
Woodhouse lamp	173	0.780	135

If we apply to each lamp its proper reduction factor, we find that the correct value of the quotient given in the last column is nearly the same for the four types. It follows from the equation for E_1, already given, that

$$-\frac{dE_1}{E_1}=\frac{db}{b}\cdot\left(\frac{2}{b^2\lambda^2T^2}-1\right),$$

a result which shows that the percentage of decrease in E_1, which is caused by a given percentage of diminution in b, is as much larger than the factor

$$\left(\frac{2}{b^2\lambda^2T^2}-1\right)$$

as the latter is larger than 1. This factor, however, has the following values (when $b^2\lambda^2=0.0552\times10^{-6}$); viz.

17.48 when $T = 1400^\circ$
16.23 when $T = 1450^\circ$
15.10 when $T = 1500^\circ$
14.08 when $T = 1550^\circ$
13.15 when $T = 1600^\circ$
12.31 when $T = 1650^\circ$

The change in the constant b of n per cent of its value would also produce a change in the value of E_1, which change would be from 12 to 17 times as great.

The tables given can also be made to furnish an answer to the question of the percentage of decrease in E_1 when the temperature rises one degree.

From Equation 7 it follows that

$$-\frac{dE_1}{E_1}=\frac{dT}{T}\left(\frac{2}{b^2\lambda^2T^2}\right)-1,$$

from which we can calculate the diminution of E_1 for one degree rise of temperature. The result is as follows: —

$\frac{1}{14}\times 17.48\% = 1.24\%$ for 1400°
$\frac{1}{14.5}\times 16.23 = 1.12$ for 1450°
$\frac{1}{15}\times 15.10 = 1.01$ for 1500°
$\frac{1}{15.5}\times 14.08\% = 0.91\%$ for 1550°
$\frac{1}{16}\times 13.15 = 0.82\%$ for 1600°
$\frac{1}{16.5}\times 12.31 = 0.75\%$ for 16.50°

Within the interval of temperature of 1400° to 1650° a rise in temperature of one degree produces, very nearly, a diminution of one per cent in the amount of energy necessary for the production of a unit of light. For this interval of temperature, therefore, a rise of n degrees in temperature will improve the economy of light production by nearly n per cent.

V.

The result of the discussion in Section III., according to which the total brightness of the glow-lamp may be expressed by means of the constant, $\alpha=6.8\times10^{-15}$, the square of the radiating surface, the square of the constant C, the total radiation, and the cube of the expenditure of energy, gives us the means of determining the dimensions which a carbon filament must have in order to give a certain intensity of light under prescribed conditions. In order to calculate the length and the radius of the filament, which we will assume is of circular cross-section, we must know the difference of potential at which the lamp is to be used, the candle-power H which is to be produced, the temperature at which this candle-power is to be delivered, and finally the nature of the carbon, with required specific resistance, and the constant C of total radiation. Since the consumption of energy per unit of total brightness is determined by the temperature, we can set in place of the temperature a given value for E_1. From these six quantities α, ΔP, H, E_1, w, and C, the dimensions of l and ρ may be determined in the following manner: —

$$H=\frac{\alpha}{C^2F^2}\cdot E^3\text{; or since } E=E_1\cdot H,$$

$$H=\frac{\alpha}{C^2F^2}E_1^3H^3 \text{ and } F_2=\frac{\alpha}{C^2}E_1^3H^2,$$

or

$$l^2 4\pi^2\rho^2=\frac{\alpha}{C^2}\cdot E_1^3H^2.$$

The resistance of the carbon is

$$\frac{lw}{\pi\rho^2}=\frac{\Delta P^2}{E}=\frac{\Delta P^2}{E_1H}.$$

From these two equations it follows that

$$l=\sqrt[3]{\frac{\alpha H\cdot E_1^2\cdot\Delta P^2}{4\pi w C^2}};$$

$$\rho=\sqrt[6]{\frac{\alpha w^2\cdot H^4\cdot E_1^5}{4\pi^4\cdot C^2\cdot\Delta P^4}}.$$

If the cross-section of the filament be rectangular, with one side a and the other b, and if $a=bn$, where b is a given number, the equations for the determinations of b and a are as follows:—

$$l=\sqrt[3]{\frac{\alpha H E_1{}^3 \cdot \Delta P^2}{\frac{4}{n}(1+n)^2 w C^2}};$$

$$a=\sqrt[6]{\frac{\alpha w^2 \cdot H^4 E_1{}^5}{4(n+n^2)^2 \cdot C^2 \Delta P^4}}.$$

In making use of these operations it is to be noted that the assumed value of the constant α, 6.8×10^{-15}, applies to mean spherical candle-power, and that the specific resistance of the carbon is to be taken as the resistance per cubic centimeter, the dimensions of the filament being expressed in centimeters.

As an example of the usefulness of this formula the following illustration may serve: A Siemens lamp, the carbon of which had a specific resistance $w=0.00209$ at its normal temperature, and a radiation coefficient $C=0.0000129$, gave 16 mean horizontal candles when the potential difference was 98 volts. The efficiency was 3.4 watts per candle. The reduction factor of this lamp to mean spherical candle-power was 0.71. In this case, then, H was equal to 11.36, and E_1 to 4.79. The measured length of the filament was found to be 15.58 centimeters, and the radius, measured at nine points equally distant along the thread, was 0.00756 cm. According to the formulas just given for l and ρ, l should be 15.74 cm., and ρ should be 0.0077 cm.

The measured dimensions were somewhat smaller than the calculated, which may result from the fact that the measurements were made at ordinary temperatures.

VI.

The general expression for radiation affords an example of the formula for the optical efficiency of the glow-lamp (the relation between the visible energy of radiation and the total radiation). The total energy of the radiation is

$$S'=\int_{\lambda_1}^{\lambda_2} \frac{c}{\lambda^2} \cdot \pi F \cdot e^{aT-\frac{1}{b^2\lambda^2T^2}} \cdot d\lambda,$$

where λ_1 and λ_2 are the wave-lengths of the borders of the visible spectrum. A further development of this form gives: —

$$S' = c\pi F b T e^{aT} \left\{ \int_{\frac{1}{b\lambda_2 T}}^{\infty} e^{-x^2} \cdot dx - \int_{\frac{1}{b\lambda_1 T}}^{\infty} e^{-x^2} dx \right\}.$$

If we take as wave-length boundaries of the visible spectrum the values $0.38\,\mu$ and $0.78\,\mu$; and if we set $b^2 = 0.19 \times 10^{-6}$, and assume as our interval of temperature 1400° to 1650°, the lower limit of the second integral will fall between 4.07 and 3.46, and as a consequence the value of the second integral for all these values will be smaller than 10^{-7}. S' may then be reduced to the following form: —

$$S' = c\pi F b T e^{aT} \int_{\frac{1}{b\lambda_2 T}}^{\infty} e^{-x^2} dx.$$

The energy of the total radiation, according to Equation 3, is

$$S = c\pi F b T \frac{\sqrt{\pi}}{2} \cdot e^{aT},$$

so that the expression for the optical efficiency is

$$N = \frac{S_1}{S} = \frac{2}{\sqrt{\pi}} \cdot \int_{\frac{1}{b\lambda_2 T}}^{\infty} e^{-x^2} dx.$$

For $T = 1500°$ the value of this integral is 0.0049, and $N = 0.55\%$.
For $T = 1550°$ the value of this integral is 0.0065, and $N = 0.73\%$.
For $T = 1600°$ the value of this integral is 0.0083, and $N = 0.94\%$.
For $T = 1650°$ the value of this integral is 0.0104, and $N = 1.17\%$.

For normal temperatures of glow-lamps, therefore, the efficiency is in the neighborhood of 1 per cent.

An experimental determination of this kind gives from 4 per cent to 5 per cent. (Measurements made by Blattner under my direction in 1885 gave about 5 per cent. Those made later by Merritt gave about 4 per cent.) These, however, were dependent upon the assumption that an aqueous solution of alum absorbed every part of the dark heat and let through all the visible rays. Since, however, this is not quite true, and these liquids let through some of the dark rays which lie near the boundary of the visible spectrum, the values experimentally obtained must be too large. If these solutions, for example, absorbed only those dark rays the wave-lengths of which are greater than (1.0 μ), the calculated value for the efficiency would be

$$N = \frac{2}{\sqrt{\pi}} \cdot \int_{\frac{1}{bT}}^{\infty} e^{-x^2} \cdot dx,$$

and the numerical values of the efficiency as follows:—

$T = 1500°$; $N = 3.0\%$.
$T = 1550°$; $N = 3.7\%$.
$T = 1600°$; $N = 4.3\%$.
$T = 1650°$; $N = 4.9\%$.

VII.

It has long been known that the rapid diminution in the life of glow-lamps with rising economy is to be explained in the fact that carbon at a certain temperature begins to suffer notable vaporization, which with further rise of temperature increases in intensity. After having found it possible to determine the temperature of carbon filaments for every condition of incandescence, it seemed to me important to fix the temperature at which carbon begins to show evaporation, and to show whether this temperature is the same for all sorts of filaments. The investigations which I have made in this direction are based upon the observations of the resistance of filaments. The resistance of the carbon filament remains constant at a given temperature as long as its cross-section is maintained. When the temperature is reached, however, at which appreciable quantities of carbon evaporate from the surface of the filament, then the resistance suffers a slow increase with time. If we raise the temperature of the filament, maintaining it constant at each step for a considerable time, and notice during this time the change of resistance, we may obtain an estimate of the temperature at which a considerable increase of resistance occurs.

As time intervals, during which the chosen temperatures were to be held constant, I selected thirty hours. By means of accumulators of large capacity, it is not at all difficult to regulate the strength of the current, so that the energy developed in the filament will remain constant to within one-thousandth of its value. From time to time during the test the current strength and the difference of potential were read, and the resistance was calculated from these values. The observations were thus continued until a temperature was found at which a sensible steady increase of resistance with the time became apparent. As an example of these measurements the following list of observations may be taken. They refer to the Woodhouse lamp in which F=0.895 cm.^{2}, R= 0.780, and C=0.0000169.

Series I.	Series II.	Series III.
ι = 1.4458 amp.	1.5107 amp.	1.5615 amp.
ΔP = 60.52 v.	62.80 v.	64.74 v.
W = 41.86 ohms	41.57 ohms	41.46 ohms
E = 87.50 w.	94.87 w.	101.09 w.
H = 28.01 c.p.	35.03 c.p.	42.26 c.p.
E_1 = 3.12 w.	2.70 w.	2.39 w.
T = 1580°	1596°	1610°

The resistances were as follows : —

Hours.	Ohms.	Ohms.	Ohms.
0	41.86	41.57	41.76
3	41.86	41.59	41.47
7	41.87	41.56	41.49
10	41.85	41.58	41.52
20	41.86	41.60	41.57
26	41.85	41.59	41.59
30	41.87	41.59	41.62

In a similar manner a lamp of the Berlin Company was tested, in 1888 at 1590°. It was found that the resistance of this lamp remained constant for thirty hours. At 1603°, on the contrary, it increased decidedly, rising to larger and larger values. It appears, therefore, that the temperatures at which various specimens of carbon begin to vaporize are somewhat different.

MINOR CONTRIBUTIONS.

ON THE THERMAL CONDUCTIVITY OF CAST IRON AS A FUNCTION OF TEMPERATURE.

BY I. THORNTON OSMOND.

IT is well known, from the experiments of Forbes and of Tait, that the thermal conductivity of wrought iron diminishes rapidly as the temperature increases, until the temperature of about 150° C. is reached, after which the conductivity changes less rapidly and becomes a minimum at about red heat.

Having reason to believe that this was not true for cast iron, and being unable to find any investigation on the subject, I had Messrs. Jenkins and Lingle, of Bellefonte, Pa., cast some bars and pieces from the No. 1 foundry pig iron made at the Bellefonte furnaces from Centre Co., Pa., ores: a soft gray cast iron, low in carbon, and high in silicon. Mr. George W. Kessler, a candidate for the master's degree, undertook the investigation, and presented the details of the work and tables of the observations very fully in a thesis, from which the observational data of this article are drawn; that is, Tables I., II., and III.

The method employed was the classic one used by Forbes for wrought iron; but to avoid cutting large cavities for thermometers, temperatures were measured thermo-electrically.

The principal apparatus, and its arrangement in use, are sufficiently indicated in Fig. 1. The galvanometer used was of the Thomson, astatic, mirror pattern; coils 5.84 ohms.

The experimental work embraced four principal operations, viz.: —

(1) To determine the size, density, and specific heat of the bar of cast iron.

(2) To calibrate the set of apparatus.

(3) To find the temperatures along the bar.

(4) To find the rate of cooling of a short piece of the same cast iron as the bar, and same diameter.

1. *The Bar.*

From fragments of pieces cast from the same iron, at the same time that the bar was cast, the density was found to be 7.140, and the specific heat 0.1155.

The bar principally used was 66 cm. long, with a projection at right angles at one end 6 cm. long, and was turned to 3.28 cm. diameter.

On the upper side of this bar, at distances 6, 8, 10, 12, 14, 16, 19, 22, 25, 29, 33, 37, 42, 47, 52, 58, and 64 cm. from the furnace end, holes 3 mm. diameter were drilled half-way through, for the insertion of the thermo-couple. These holes, the points of observation along the bar, are numbered from 1 to 17 in the order of their distances given above. See abscissæ of Curve II.

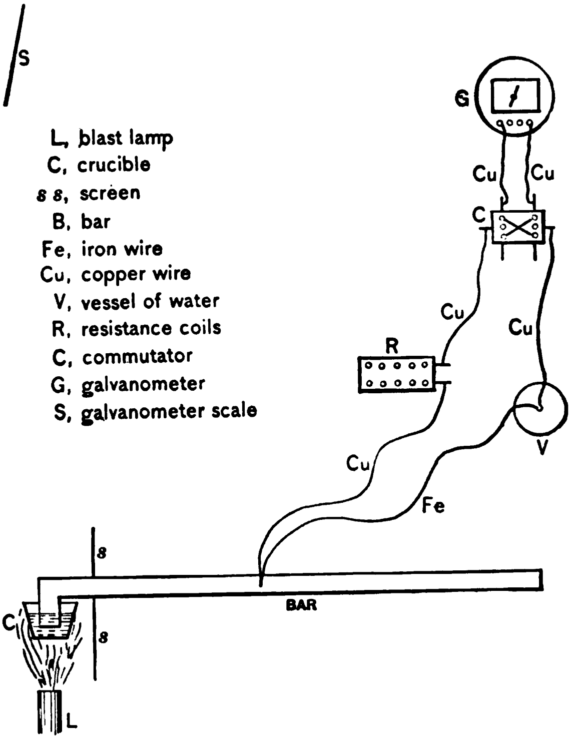

Fig. 1.

2. *Calibration of Apparatus.*

After some preliminary trials with the apparatus arranged as in Fig. 1, by which the resistance to be used, position of the controlling magnet of the galvanometer, etc., were determined, the junction of the thermo-couple was removed from the bar and put in a vessel of mercury at over 200° C., the cool junction being in a vessel of water at 21° C. The thermometers used were without errors that need be considered in this work, shown by recent testing. As the mercury cooled, the observations given in Table I. were made by simultaneous readings of temperature and deflection.

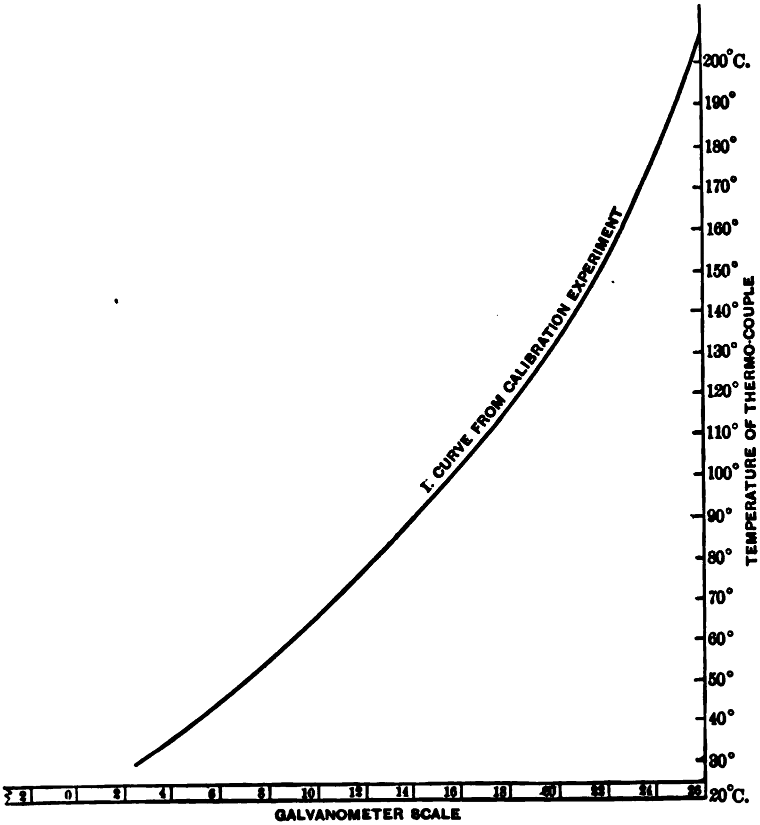

Fig. 2.

This calibration is shown in Fig. 2. Observations giving a curve almost indistinguishable from one here shown were taken at another time, serving both to test the accuracy of this set, and to show the unchanged condition of the thermo-couple.

TABLE I. (See *Curve I.*)

Temperatures.		**Deflections.**
Cool junction.	**Hot junction.**	**Scale readings.**
21° C.	200° C.	25.7
21° C.	176° C.	24.2
21° C.	153° C.	22.2
21° C.	131° C.	19.7
21° C.	110° C.	16.7
21° C.	93° C.	13.7
21° C.	77° C.	11.2
21° C.	64° C.	8.7
21° C.	51° C.	6.2
21° C.	40° C.	3.9

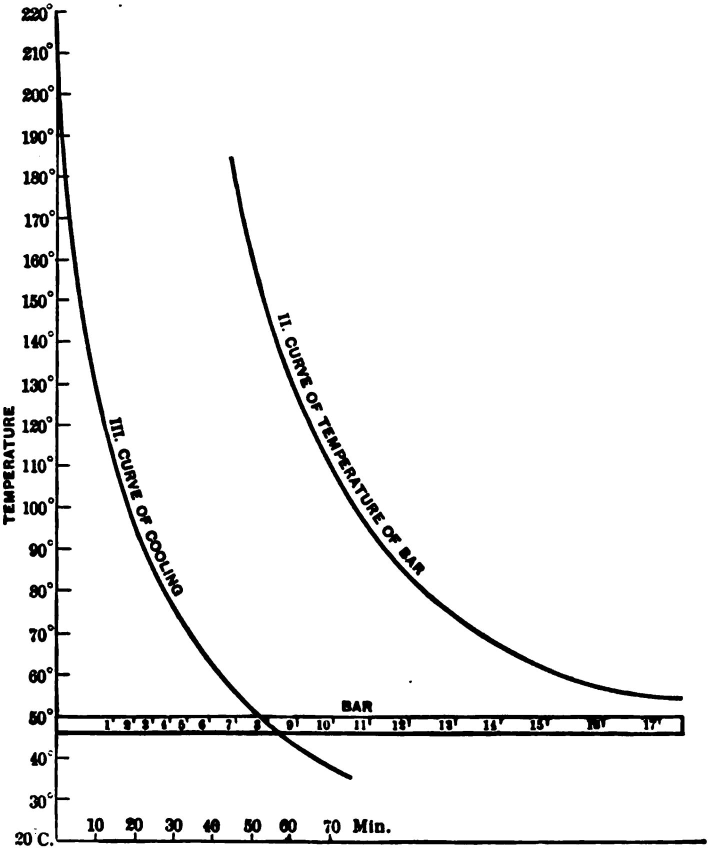

Fig. 3.

3. *Temperatures along the Bar.*

The right-angled projection at one end of the bar was put in a small crucible (set in asbestos and brick) in which by a strong and very steady blast lamp aluminum was melted. To insure good thermal contact, mercury was placed in the holes along the bar; but finding that it vaporized from some of them (during the long heating necessary), careful trial showed that its presence or absence gave no observable effect.

After a number of preliminary experiments, to get everything in order and to ascertain the best adjustment, the bar was heated, on May 25, for many hours, until the deflection obtained from any one point ceased to vary, showing a steady condition of temperature at each point.

Four sets of observations were then made along the bar, as given in Table II. The cool junction of the thermo-couple was in a vessel of water at 21° C., and there were 14 ohms unplugged in the resistance box, as in the calibraton experiment. The other junction was inserted successively in several holes along the bar, and the deflections read as soon as they were steady. These results are shown in curve II., Fig. 3.

TABLE II. (See *Curve II.*)

Point of bar.	Obs. I. Scale reading.	Obs. II. Scale reading.	Obs. III. Scale reading.	Obs. IV.[1] Scale reading.	Mean.
1	25.0	24.1	—	—	6.65
2	26.4	26.1	24.25	24.3	7.26
3	26.0	27.3	26.05	26.4	8.39
4	25.4	25.7	25.45	—	9.51
5	26.0	26.6	26.3	26.2	11.50
6	26.8	26.2	26.7	—	13.92
7	25.7	25.1	25.95	26.8	16.00
8	23.4	23.2	23.75	—	18.65
9	21.1	21.5	22.05	22.3	21.74
10	18.4	18.7	18.85	—	23.45
11	15.9	16.15	16.35	15.6	25.64
12	13.6	14.1	14.05	—	26.50
13	11.3	11.55	11.55	11.6	26.27
14	9.1	9.8	9.85	9.3	25.71
15	8.0	8.4	8.45	8.7	26.44
16	6.9	7.35	7.2	7.6	25.26
17	5.9	6.9	6.9	6.9	24.55

---

[1] The exhaustion of the blast pressure made it necessary to go rapidly through part of Obs. IV., omitting some points.

4. *Curve of Cooling.*

A separate short bar of the same iron, cast at the same time as the principal bar and turned to the same diameter, after a long heating to a high temperature, had one junction of the same thermo-couple previously used with the main bar inserted in the same manner as in the latter: the same apparatus and conditions being in circuit.

The observations made while this piece cooled are given in Table III., and shown by curve III. in Fig. 3.

TABLE III. (See *Curve III.*)

Minutes . .	0	1	2	3	4	5	6	7	8	9
Deflection .	26.7	26.1	25.4	24.7	24.0	23.3	22.6	21.9	21.2	20.6
Minutes . .	10	11	12	13	14	15	16	17	18	20
Deflection .	20.0	19.4	18.7	18.2	17.6	17.0	16.6	16.1	15.5	14.6
Minutes . .	22	25	30	35	40	50	60	70		
Deflection .	13.7	12.4	10.6	9.2	8.1	6.0	4.5	3.5		

The inclinations of the tangents to curve III. represent rates of cooling per minute in degrees centigrade.

All these curves lie so closely upon the points platted that in the reduced drawing herewith the points, except two or three of curve II., are inseparable from the curves.

5. *Results. Table IV.*

In Table II. it will be noticed that the deflections at the first six points of observation are irregular and anomalous. On converting the deflections into temperatures, by use of curve I., the reason of this is evident. Point 6 of the bar is nearly at the temperature of inversion of the copper-iron thermo-couple. Hence, the following *Table of Results* begins with the 18th centimeter of the bar, lying between points 6 and 7. Column 2 is obtained from curve II., and gives the temperature at the middle of the centimeter in column 1. Column 3 is obtained from curve III. (dividing by 60, to bring to seconds), and gives the rate of cooling per second at the middle of the centimeter in column 1. Column 4 is the product of the density, the specific heat, and the rate of cooling. By not using the area of cross-section of the bar, the numbers in this column represent the heat lost per centimeter length per second by a bar of unit cross-section for the centimeter indicated in column 1. Column 5 gives the flow of heat per second across unit area at the end of the centimeter given in column 1: found by summing the loss of heat per second from all subsequent centimeters of the bar. Column 6 gives the conductivity, in C. G. S. units, at the end of the centimeter given in column 1: found by dividing the difference of temperature at the middle of two centimeters into the flow of heat per second at the cross section between them.

TABLE IV.

RESULTS.

Centimeter of bar.	Mean temperature. (Middle of centimeter.)	Mean rate of cooling. (Middle of centimeter.)	Heat lost per second per unit cross-section.	Flow per second across unit area. (Outer end of centimeter.)	Conductivity. C. G. S. units. (Outer end of centimeter.)
18th	221.0	—	—	1.6154	0.1130
19th	206.7	0.250	0.206	1.4094	0.1060
20th	193.4	0.200	0.165	1.2444	0.1037
21st	181.4	0.162	0.134	1.1104	0.1181
22d	172.0	0.125	0.103	1.0074	0.1259
23d	164.0	0.105	0.0866	0.9208	0.1196
24th	156.3	0.097	0.0800	0.8408	0.1184
25th	149.2	0.085	0.0699	0.7709	0.1264
26th	143.1	0.077	0.0635	0.7074	0.1141
27th	136.9	0.065	0.0536	0.6538	0.1168
28th	131.3	0.058	0.0478	0.6060	0.0962
29th	125.0	0.052	0.0429	0.5631	0.1251
30th	120.5	0.047	0.0388	0.5243	0.1219
31st	116.2	0.045	0.0371	0.4872	0.1037
32d	111.5	0.042	0.0346	0.4526	0.1131
33d	107.5	0.040	0.0330	0.4196	0.0999
34th	103.3	0.035	0.0289	0.3907	0.1184
35th	100.0	0.032	0.0264	0.3643	0.1214
36th	97.0	0.029	0.0239	0.3404	0.1031
37th	93.7	0.027	0.0223	0.3181	0.1060
38th	90.7	0.026	0.0214	0.2967	0.1099
39th	88.0	0.024	0.0198	0.2769	0.0989
40th	85.2	0.022	0.0182	0.2587	0.0958
41st	82.5	0.020	0.0165	0.2422	0.0969
42d	80.0	0.019	0.0157	0.2265	0.0906
43d	77.5	0.018	0.0148	0.2117	0.0960
44th	75.3	0.017	0.0140	0.1977	0.0941
45th	73.2	0.016	0.0132	0.1845	0.0923

TABLE IV. (*continued*).

Centimeter of bar.	Mean temperature. (Middle of centimeter.)	Mean rate of cooling. (Middle of centimeter.)	Heat lost per second per unit cross-section.	Flow per second across unit area. (Outer end of centimeter.)	Conductivity. C. G. S. units. (Outer end of centimeter.)
46th	71.2	0.015	0.0124	0.1721	0.0820
47th	69.1	0.014	0.0116	0.1605	0.0892
48th	67.3	0.013	0.0107	0.1498	0.1152
49th	66.0	0.013	0.0107	0.1391	0.1159
50th	64.8	0.0127	0.0105	0.1286	0.0989
51st	63.5	0.0122	0.0101	0.1185	0.0911
52d	62.2	0.0116	0.0096	0.1089	0.0907
53d	61.0	0.0112	0.0092	0.0997	0.0997
54th	60.0	0.0107	0.0089	0.0908	0.1009
55th	59.1	0.0104	0.0086	0.0822	0.1028
56th	58.3	0.0102	0.0084	0.0738	0.1056
57th	57.6	0.0098	0.0081	0.0657	0.1095
58th	57.0	0.0095	0.0078	0.0579	0.0965
59th	56.4	0.0095	0.0078	0.0501	0.0835
60th	55.8	0.0093	0.0077	0.0424	0.0707
61st	55.2	0.0090	0.0074	0.0350	0.0700
62d	54.7	0.0088	0.0072	0.0278	0.0556
63d	54.2	0.0086	0.0071	0.0207	0.0518
64th	53.8	0.0085	0.0070	0.0137	0.0685
65th	53.6	0.0084	0.0069	0.0068	0.0680
66th	53.5	0.0983	0.0068	—	—

It appears that the results from the last six or seven centimeters of the bar must be rejected. The outer end of the bar (which was too short for so large a diameter) was at a temperature considerably above that of the surrounding air. It seems more probable that this produced an effect along several centimeters of the bar than that there is such a sudden and great change in conductivity between 60° and 50° temperature.

Conclusion.

For such cast iron as was used in this investigation, the thermal conductivity is about fifteen per cent greater from a little below 100° C. to 200° C. than it is between about 60° C. and 90° C.

With a longer bar of less diameter, now in preparation, to be electrically heated, and with a thermo-couple of other materials (perhaps aluminum and silver), I hope to carry this investigation of the thermal conductivity of cast iron to both lower and higher temperatures.

PHYSICAL LABORATORY, PENNSYLVANIA STATE COLLEGE,
June 25, 1894.

On Some Magnetic Characteristics of Iridium.

By S. H. Brackett.

THIS paper does not claim to present all the characteristics of iridium in its magnetic relations, or to discuss the facts to which attention is called, but merely to state some points of interest which seem not to have been noted before, and which suggest reason for further investigation. The work here alluded to was done in intervals of busy elementary science teaching, and may be made more complete when further opportunity occurs.

Iridium was not left out of the list of substances so carefully examined by Faraday, and is mentioned by him as being very slightly diamagnetic. Iridium, as at present manufactured, may be presumed to differ from that used by Faraday, and in the absence of analysis of the specimens here tested there is to be allowed a presumption that they are not pure; but the manufacturer, Mr. John Holland, is authority for saying they are more than 98 per cent pure, with some platinum, a trace of phosphorus and no iron, and no iron has been employed in manipulation.

The first bar tested with such results as to excite further interest in the subject was 13.3 mm. long, 3.2 mm. wide, and 0.9 mm. thick. It was one of several different substances being tested for small amounts of magnetism or diamagnetism. A large electromagnet of portative force of about 5000 grams per square centimeter of surface was furnished with soft iron pole-pieces, shaped so as to furnish a very strong field of force. When the bar of iridium was brought near the poles it was strongly attracted sidewise, and could not be made to stand radially. It acquired a permanent *transverse* polarity so strong as to appear to be a permanent diamagnet. Its extremely small thickness rendered test of attraction and repulsion inoperative.

In the field of any ordinary magnet it everywhere set itself at right angles to the lines of force, and when suspended by a fiber under a glass, it readily assumed the east and west position. In a place where H is determined as 0.158, its period of oscillation was 18.7 seconds, as compared with a steel bar of the same length and weight, whose period in the same place was 5.3 seconds, so that the ratio of its intensity of magnetism to that of steel was nearly as 1 to 12.

It was very difficult to magnetize it longitudinally by the ordinary stroking methods, which were likely to leave it in a very heterogeneous condition, so that it might take a position oblique to the lines of force.

To make further investigation and to determine some of its properties quantitatively, a bar was procured of Mr. Holland, 25.7 mm. long and

3.3 mm. square, weighing 3.57 grams. Apparatus was constructed suitable to the size of this bar, and careful tests were made of its permeability, which was found to be practically unity. Whether the bar had been previously magnetized or had had nearly all magnetism removed by heating, no change of strength of current produced any perceptible change of the lines of force. Even striking it forcibly while it was in the coil failed to produce any magnetism by induction, and it could only be magnetized by actual contact with a magnet and then jarring it.

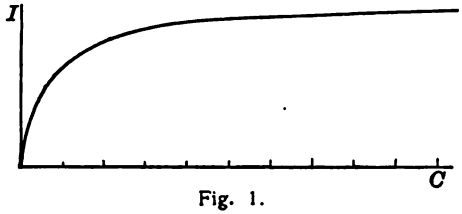

Fig. 1.

On placing it axially between the soft iron pole-pieces of the magnet, and using successively currents of different strength, striking the pole-pieces each time, different degrees of magnetic intensity were given to it, and on plotting the magnetization as ordinates and the current as abscissas, a curve, shown in Fig. 1, was obtained similar to those usually made of permeability.

This bar, as well as the other, is much more readily magnetized transversely than longitudinally. It has only to be placed between flat pole-pieces, so as to be in a uniform field of force, and jarred vigorously. A steel bar of the same size, tempered glass hard, cannot be magnetized transversely when subjected to the same conditions.

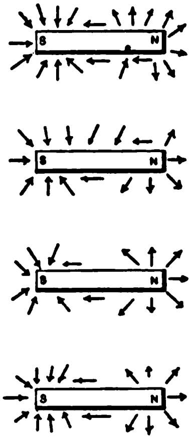

Fig. 2.

When attempts had been made to magnetize it longitudinally, as mentioned above, discrepancies between its magnetic moment at different trials and that obtained by transverse polarity led to further examination of the distribution of its magnetism. The field around it was plotted by means of a little steel magnet 2.5 mm. long, suspended by a cocoon fiber, and the distribution was found to be very irregular. To see if a more uniform distribution could be secured, a method was devised which might be called octuple touch. Stroking was done on all four sides of the bar at once, from the middle to each end. The iridium was held by a small clamp in the center of the space between the poles of the magnet, held only by its center. Four soft iron levers were hinged so that their ends would grasp the iridium near the clamp on one side of it, and four similar levers did the same on the other side of the clamp. These sets of levers were laid on the poles of the magnet, so that when the latter was excited the levers became the real poles of the magnet. When they were applied the circuit was closed, and the sets of levers were held tightly against the magnet, and pulled off in opposite directions at the same time. By this means much better results were obtained, but the distribution was still somewhat irregular, as shown in Fig. 2, representing the four sides of the magnet. The arrows show the exact place of the little exploring magnet.

When magnetized transversely the field of force is much more regular, and it was feasible to delineate this field by sifting filings through a handkerchief on very thin paraffined paper, fixing them by warming and then photographing. It will be seen (Fig. 3) how plainly the whole side of the bar (*a*) is one pole of the magnet. In (*b*) lines of force are shown crossing the bar, and the usual curves are seen at the ends, which are in this case the real sides of the magnet.

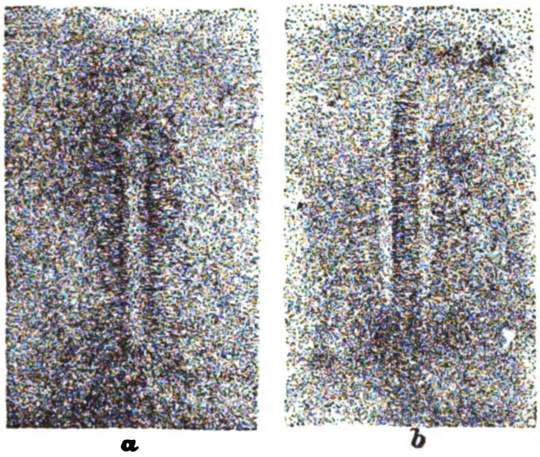

Fig. 3.

Very low permeability and great coercitive force, with a high intensity of magnetization, are plainly exhibited by these specimens of iridium, and these characteristics are consistent with the known physical properties of the substance.

NOTES.

Hermann von Helmholtz.—On September 8, 1894, at noonday, in Charlottenburg, near Berlin, died Hermann Ludwig Ferdinand von Helmholtz. Few men of his time have been more gifted; few men of science, certainly, have been granted a more universal and generous recognition of their powers. Like Lord Kelvin in Great Britain, Professor von Helmholtz has been for more than a quarter of a century the acknowledged leader of the physicists of his nation.

The fame of Helmholtz rests upon an uncommonly broad foundation. In speaking of him some years ago, the mathematician, W. K. Clifford, who was himself a man peculiarly qualified to form an opinion, said:—

"In the first place he began by studying physiology, dissecting the eye and the ear, and finding out how they acted, and what was their precise constitution; but he found that it was impossible to study the proper action of the eye and ear without studying also the nature of light and sound, which led him to the study of physics. He had already become one of the most accomplished physiologists of this century when he commenced the study of physics, and he is now one of the greatest physicists of this century. He then found it was impossible to study physics without knowing mathematics; and accordingly he took to studying mathematics and he is now one of the most accomplished mathematicians of this century."—*Seeing and Thinking*, p. 18.

Helmholtz, however, was more than physiologist of the first rank, mathematician of the first rank, and physicist of the first rank; he was likewise a philosopher in the best sense of the word, and a writer whose style, considered from the literary standpoint, is remarkable for its clearness and beauty.

One of the main qualities which characterize the scientific work of Helmholtz is a certain completeness and maturity; a completeness which was the natural product of clearness of thought, of deepest insight, and of extraordinary analytical power; a maturity which was the legitimate fruit of the long-continued preparation for his life-work. He was born in Potsdam on the 31st of August, 1821, in which Prussian town his father was a gymnasial master. It was in 1871, as nearly as one may venture to draw the lines which divide a lifetime into its epochs, when, at the age of fifty years, he definitely abandoned physiology for the field of labor with which his name will ever be primarily associated.

Throughout this extraordinarily long preliminary period, during which Helmholtz was successively student of medicine at the University of Berlin (1838–42), assistant at the charity hospital (1842–43), military surgeon in his native city of Potsdam (1843–47), teacher of anatomy in the art academy of Berlin and assistant in the anatomical museum of that city (1848), professor of physiology and general pathology at Königsberg (1848–55), professor of anatomy and physiology at Bonn (1855–58), and professor of physiology at Heidelberg (1858–71), there is abundant evidence from his written work that he was primarily a physiologist, and only incidently engaged in physical investigation. Of the eighty-five papers assigned to him in the Royal Society's catalogue, the greater number are devoted to subjects purely physiological, and in the case of most of the others it is easy to discover that the inquiry was made in connection with some question of physiological importance, or was the outcome of suggestions arising from researches in that science.

Thus in the opening paragraph of the first paper that he devoted specifically to an electrical subject (upon the "Duration and Course of the Electrical Currents induced by Changes in Current Strength," *Poggendorff's Annalen*, Vol. 83, 1851), he says: "The method suggested by Pouillet for the measurement of small time-intervals by means of the action which an electric current exerts during that time upon a magnet, I have used for physiological purposes. In the course of those measurements I found myself compelled to undertake investigations of the duration of the currents induced by sudden changes in the strength of the currents which I had used in the excitation of anatomical parts, and also upon the manner in which the currents used for the measurement of time began, under the influences of the opposed induced currents due to the closing of the circuit, to flow through the coils of the galvanometer. "By means of these experiments I have been able to secure certain confirmations of a very broad mathematical principle, which deals with the determination of the development of such currents. The purely physical features of these investigations are gathered together in the following paragraphs."

Then follows the important monograph familiar to all physicists, in which the limitations of the law of Ohm are indicated, and the case of circuits with fluctuating electromotive forces is treated in the broadest and most fundamental manner.

Even in the case of the essay upon the conservation of energy we note two subordinate articles which would seem to indicate that this great problem was approached from the point of view of the physiologist, and was worked out in the hope of gaining some insight into the mysteries of the vital processes. These two papers, which were among the earliest of the published works of their author, deal respectively with the *consumption of material* and with the *development of heat* in muscular action.

How glorious, even when considered solely with reference to physics, were the fruits of this first period during which physics was subordinated to physiology.

It is marked by a succession of productions, any one of which taken singly would have sufficed to secure its author a lasting place in the annals of the science.

First of all, we have the paper on energy ("Ueber die Erhaltung der Kraft," Berlin, 1847), the strange fate of which it was to be refused admission to the pages of *Poggendorff's Annalen*, to lie well-nigh unnoticed for several years after its appearance in pamphlet form; then to be attacked in vigorous manner by Clausius (1853), and later to subject its author to virulent attacks on the part of Dühring and others, as an imitator of and dishonest borrower from his forerunner, Robert Mayer. At last it came to be recognized as chief of the early attempts to give expression to the principles upon which, later, the doctrine of energy was to be built, and it was reprinted in sumptuous manner as a classic.

This famous essay appears the more remarkable when we consider the year in which it was written, and that it was the performance of a military surgeon in an obscure garrison town, banished for the time from the scientific atmosphere of the universities, and unable, as appears from the reply which Helmholtz made to criticisms of Clausius (*Poggendorff's Annalen*, Vol. 91, 1854), to perform the experiments necessary to the justification of some of his assumptions.

In 1851 followed the description of the ophthalmoscope ("Beschreibung eines Augenspiegels zur Untersuchung der Netzhaut im Lebenden Auge," Berlin, 1851), a discovery which, because of its great utility, served to bring Helmholtz into immediate prominence.

The year 1852 saw the development of the Young-Helmholtz theory of color-sensation. In 1856 appeared the first part of the *Handbuch der Physiologischen Optik*, a work which was to occupy its author for ten years.

The method in which this great work was executed was characteristic of the man. In the preface to the first edition he says, translated freely: —

"It has been my chief endeavor in working out this book to convince myself by the use of my own eyes and by my own experience of the truth of all the facts, even of such as are of minor importance." To carry out such a task in the thorough manner which distinguishes the handbook was in itself a good decade's work, but Helmholtz's mind continued to busy itself with other important problems. In the same year (1856) he published his first acoustical paper, on combination tones; two years later appeared the important monograph upon the hydrodynamical equations which express vortex motion (*Crelle's Journal*, Vol. 55). This was the first of Helmholtz's researches that was clearly free from all physiological origin, motive, or suggestion. It formed the starting-point of his remarkable series of studies of fluid motion.

In 1859 there was published in the *Münchener Gelehrter Anzeiger*, Vol. 48, also in *Poggendorff's Annalen*, a paper entitled "Die Klangfarbe der Vocale," which contained the germ of what was to be an important chapter in Helmholtz's best known work, the *Tonempfindungen* (1862).

During the years which followed, up to 1871, which year closes the epoch which we are now considering, we find a variety of physiological contributions, the completion of the handbook of physiological optics, and various papers dealing with the topics of electro-dynamics, acoustics, and hydrodynamics, all domains in which the work of Helmholtz had become familiar to readers, and authoritative. Noteworthy also is the opening of a new line of thought. The discussion of the foundations of the Euclidian axioms ("Ueber die thatsächlichen Grundlagen der Geometrie," *Heidelberg Verh. Nat. Med.*, IV., 1868) was a theme to which the philosophic mind of Helmholtz was peculiarly fitted, and his contribution to the theory of the nature of space, which, through the efforts of Grassmann, Riemann, and others, was being developed and established, was of the greatest service.

Such were the principal physical labors of Helmholtz during this long first period — a lifetime in itself, whether measured in years or in achievement. Two other periods followed, perforce shorter, but scarcely less important in their results. One of these may be counted from 1871, in which year Helmholtz went from Heidelberg to Berlin, as successor to Magnus in the chair of physics and as the director of the physical laboratory. It ends with the establishment of the Imperial Institute in 1888. The other comprises the closing years of his life.

The period of his Berlin directorship was one of increased activity in the publication of monographs relating to physics, but it was chiefly notable for the building up of the most important teaching center for original work in Europe.

When Helmholtz was called to Berlin, the laboratory consisted of a suite of rooms in one wing of the main university building. These were abandoned in 1877 for the new laboratory upon the bank of the Spree, a structure unsurpassed, probably, among physical laboratories of that time.

In his relations to the average undergraduate, Helmholtz's attitude was unlike that of some of the other great teachers of science of his day, whose care in the direction of the labors of the youngest inmate of their laboratories was as elaborate as their pains in conducting reseach. Some found him cold and distant, perhaps; but these were, for the most part, of a class to whom he could have imparted but little. To foreigners who came to study with him he was proverbially kind. That he was ever quick to recognize latent powers and alert to rouse those who possessed them into activity, there is abundant evidence.

Of the long list of important researches which emanated from his laboratory during his directorship, but a small proportion was published under his own name, but those whose privilege it was to make their beginnings in research under his guidance know how great was his skill in selecting a topic adapted to the individual, in outlining a method, and in directing the investigation to a successful ending.

Often his influence upon those who came under him was far reaching in its results. Thus Heinrich Hertz, whose untimely death the world of physics is now mourning, relates in the preface to the work upon which his fame will chiefly rest, that the theme was developed from a problem offered by the Berlin Academy while he was a student in Helmholtz's laboratory; and Helmholtz, on his side, in the introduction which he wrote to Hertz's remarkable posthumous volume on mechanics, recounts the early evidences of power which he discerned in the latter. "Having occasion," he adds, "to propose a theme for a prize essay in physics, I selected a problem in electrodynamics, with a conviction, afterwards confirmed, that Hertz would interest himself in it, and would attack it with success."

This period of double activity, as investigator, with physics as the prime subject of attention, and as director of the most important of university laboratories, drew to a close, as has already been indicated, with the establishment of the Imperial Institute for Physical and Technical Research. To the presidency of this institute at the time of its opening in 1888 Helmholtz was called, and to the direction of investigations, for the pursuance of which it had been founded, he devoted the few remaining years of his life. He had already reached the pinnacle of scientific renown. All the honors — scientific, academic, national, royal, and imperial — which the world had to bestow, had been showered upon him. In 1877 he was rector of the University of Berlin; in 1883, the Emperor William I. bestowed upon him hereditary nobility; in 1891, the celebration of his seventieth birthday, which was held in common with that of his colleague Virchhow, was the occasion of international scientific applause and gratulation.

Up to the time of the stroke which was the near precursor of his earthly end, the powers of Helmholtz showed no signs of waning. Paper after paper in *Weidemann's Annalen* bore witness to his continued scientific productiveness. The number of that periodical for September, 1894, the very month of his death, contained a paper on "Deductions from Maxwell's Theory of the Movements of the Pure Æther"; a memoir quite in his best style, and one the closing lines of which indicated that more might be expected upon the same subject.

The program of the annual meeting which was held in Vienna September 24–30, 1894, contained no less than three papers and addresses which he was to deliver in general session. The second stroke, which came upon him in the first week of September, when he seemed to be recovering from the former attack, cut all his earthly activity short forever.

Much uncompleted work appears to have been interrupted by Helmholtz's sudden death, among other things the completion of the new edition of the *Handbook of Physiological Optics*, and a compendious treatise on Mathematical Physics. The latter book, which is to include the development of optical theory along lines already indicated by the work of Maxwell and of Hertz, will, it is reported, be brought to completion by Dr. Arthur König.

Of Helmholtz's more important monographs, only a few of the earlier have been mentioned, the earlier having been selected, not as more important than his later work, but because they are of especial interest as showing the beginnings in the various lines of thought which were to occupy him throughout the remainder of a long career. Ninety-nine of these were gathered into the first two volumes of his *Wissenschaftliche Ahhandlungen* (1882–83). The third volume is announced to contain all papers subsequent to 1883. The numerous addresses in popular form, delivered by Helmholtz on various public occasions, have likewise been gathered together and constitute the well-known *Populäre Wissenschaftliche Vorträge.*

In 1893, Helmholtz undertook for the first time a journey to America, upon which trip he was accompanied by Mrs. von Helmholtz and by four assistants from the Reichsanstalt. The party visited Chicago, and Helmholtz attended the sittings of the chamber of delegates of the international congress of electricians, of which body he had been elected honorary president. He took an active part in the deliberations of the chamber, to whose service he brought his fundamental knowledge of the subjects under consideration, and likewise a wide experience from the sittings of previous congresses. This was his last important public appearance. On the return voyage it was his misfortune to fall down the companion way of the ship, sustaining injuries from which, at his age, the most serious consequences were to be feared. Scarcely had he made complete recovery when the two paralytic shocks occurred to which his death was due. Thus passed from earth, at the age of seventy-three years, the first physicist of our time.

E. L. N.

The American Association for the Advancement of Science. — The forty-third meeting was held in the Polytechnic and Packer Institute, Brooklyn, New York, August 15–24, 1894. The evening addresses and receptions were held in the Academy of Music and Art Building. The meetings as a whole were well attended, and none of the sections failed to secure an interesting program. The usual meetings of the various affiliated societies were held in the same buildings, and also the meetings of the Society for the Promotion of Engineering Education; but whether such tend to increase the interest in the meeting of the Association proper or not, is a question. In the intervals between the meetings of the sections, the members of the Association were well entertained by excursions, carefully planned by the local committees, on the Sound, up the Hudson, and down the Harbor to Long Branch; and those interested in the animal life of the deep sea were taken out over night to the Gulf Stream on the steamer *Fish Hawk* of the United States Fish Commission.

Before Section B, Professor William A. Rogers delivered his vice-presidential address upon "Obscure Heat as an Agency in Producing the Expansion of Metals under Air Contract," in addition to which twenty-four papers were presented as follows: —

"A Problem in Stereoscopic Perspective," by W. LeConte Stevens. — "On the Directed Velocity of the Particles in the Electric Arc," by Benjamin W. Snow. — "Aluminum Violins," by Alfred Springer. — "The Photography of Manometric Flames," by William Hallock. — "A New Form of Spectrophotometer," by Edward L. Nichols. — "The Thermal Conductivity of Cast Iron," by I. T. Osmond. — "Thermal Conductivity of Copper at Low Temperature," by R. W. Quick and B. S. Lanphear. — "The Influence of Heat and Electricity upon Young's Modulus for a Piano Wire," by Mary C. Noyes. — "The Influence of Temperature upon the Transparency of Solutions," by Edward L. Nichols and Mary C. Spencer. — "Studies of the Lime Light," by Edward L. Nichols and Mary L. Crehore. — "A Calorific Effect of the Velocity of Migration of Hydrogen Ions," by Samuel Sheldon. — "On the Radiation of Obscure Heat by a Metallic Bar," by Benjamin W. Snow. — "The Infra-Red Spectra of Metals," by E. P. Lewis and E. S. Ferry. — "On Magnetic Potential," by Frederick Bedell. — "On Electric Strength," by George W. Pierce. — "A Phonographic Method for Recording the Alternating Current Curve," by C. J. Rollefson. — "On the Solution-Tension of Metals," by Harry C. Jones. — "A New Recording Thermometer for Closed Spaces," by W. H. Bristol. — "On Some Magnetic Characteristics of Iridium," by S. H. Brackett. — "Lighting as a Case of the Dissipation of Energy," by N. D. C. Hodges. — "Some Peculiar Lightning Flashes," by Alexander McAdie. — "The Attributes of Matter," by J. W. Powell. — "Definition of Motion," by J. W. Powell. — "Device for Comparing Natural with Tempered Musical Scales," by Morris Loeb.

The next meeting will probably be held in San Francisco.

NEW BOOKS.

A Treatise on Hydrostatics. By ALFRED GEORGE GREENHILL, Professor of Mathematics in the Artillery College, Woolwich. pp. 536; 108 figs. London and New York, Macmillan & Co., 1894.

To the engineer, the spectacle of a text-book of more than five hundred pages devoted to the subject of Hydrostatics, by him considered as one of the smaller subdivisions of the Mechanics of Fluids, is something of a surprise. To the mathematician and physicist, however, the case is doubtless different, as Hydrostatics offers a most fruitful and pleasant field for the application of mathematics. That the author of this book shares the latter opinion, and has consequently labored *con amore* in its production, is to be inferred from the following passage taken from p. 3 of the work: —

"Hydrostatics is a subject which, growing originally out of a number of isolated practical problems, satisfies the requirements of perfect accuracy in its application to the largest and smallest phenomena of the behavior of fluids; and at the same time delights the pure theorist by the simplicity of the logic with which the fundamental theorems may be established, and by the elegance of its mathematical operations; so that the subject may be considered as the 'Euclidean Pure Geometry of the Mechanical Sciences.'"

Professor Greenhill is the well-known author of several mathematical text-books and of a portion of the article on Hydromechanics in the *Encyclopædia Britannica* and presents us in the present work with a very full development and discussion of the principles of Hydrostatics and their applications, with some digressions into the field of Hydrodynamics, and a brief presentation of the main elements of Thermodynamics. The usual topics are treated, viz.: Hydrostatic Thrust, Specific Gravity, Floating Bodies, Liquids in a Bent Tube, Pneumatics, Stress in Containing Vessels, Capillarity, Pressure of Liquid in Moving Vessels; and a chapter from Hydrodynamics, called Hydraulics, or Motion of Fluids. While it is evident that the work is designed chiefly for the student of Mathematics and Physics, who will find a wealth of problems (some three hundred or more) of various degrees of difficulty on which to exercise his skill and patience, the author has not neglected the needs of the student of engineering, who will meet with numerous applications to engineering appliances and apparatus, especially in matters relating to Naval Architecture; with, in some cases, detailed descriptions of a most practical and useful character. As instances in point may be mentioned problems, descriptions, and practical matters relating to gauges, safety-valves, barometers, thermometers, caissons, ships, retaining walls, diving-bells, gasholders, siphons, balloons, etc. In many cases certain refinements of treatment are given, not ordinarily presented, such as the stability of a ship when aground, stability of a ship with liquid cargo, barometer with vitiated vacuum, law of increase of the velocity of liquid in a siphon just beginning to operate, etc. The bulk of the book is, therefore, not to be wondered at when we consider its range of treatment and the variety of wants that are met.

Though the use of the Calculus is infrequent, the student is supposed to have a command of its principles and operations, the author remarking in the preface, in quotation marks, "'It is easier to learn the Differential Calculus than to follow a demonstration that attempts to avoid its use.'" In the matter of illustrations, while they are excellent in execution, the reader will find them less numerous than might have been desired and in some instances very inconveniently placed, figure and corresponding text being sometimes several pages apart. Occasionally a single figure, crowded with the necessary detail, is made to serve for several different demonstrations or references. It must be said that the letterpress has a rather monotonous appearance, which would have been much relieved, and the reader aided, by the use of heavy type for paragraph headings and leading substantives. Numerous references, scattered throughout the work, form quite an ample bibliography of the subject.

Various commendable features of the book, as seen from the standpoint of the engineer, are the following: (1) the frequent insertion of numerical examples and illustrations; (2) the use of the gravitation unit of force, as pounds, kilograms, and tons; (3) the definite expression of the nature of a quantity denoted by a symbol, or combination of symbols, as secured by adding the full statement of its character in some system of units, such as P pounds, v feet per second, A square feet, p pounds per square inch, D pounds per cubic foot; these expressions, however, being abbreviated, respectively, to P lbs., v ft./sec., A ft.^{2}, p lbs./in.^{2}, D lbs./ft.^{3}, which are just as effective; (4) dealing with the specific gravity of a substance chiefly by employing the weight of a unit of volume, *i.e.* its "heaviness" or "density." In this connection it is worthy of note that although the author occasionally asks the student to determine the whole normal fluid pressure on a *curved* surface, he makes prominent the fact that this sum total has no mechanical significance (p. 82). The obtaining of this total normal pressure is, of course, little more than an exercise in mathematics, but in some books, from failure of their authors to mention the above fact, the student is led to think that the total normal pressure is of some practical importance.

Further comment is suggested by a few incidental points. On p. 8, where stress is defined as composed of two "equal and opposite balancing forces," the word "balancing" seems of doubtful propriety, since two or more forces can hardly be said to balance each other unless they all act on the same body, and that body is in equilibrium under their action; and even then the phrase does not seem strictly appropriate, since force is the name given to actions of bodies on bodies, we having no conception of an action of force on force. Again, on p. 8, as to the quotation from Newton's third law of motion that action and reaction "are equal and opposite," it has always seemed unfortunate to the present writer that the word "simultaneous" is not usually added, in order to forestall misconception, since in popular and historical language the "reaction" is generally subsequent to the "action." On p. 21, the "Keeley Motor in America" is referred to in a way that seems to lend it more dignity than it deserves.

The comparison suggested in Ex. 11 of p. 185 evidently refers to the Principle of Least Work used in dealing with structures having redundant elastic members. The solution of the problem of finding the stresses in a thick hollow cylinder (p. 395) has the same unsatisfactory character as the treatment in Rankin's *Applied Mechanics*, the student being left in doubt as to the certainty of the result; however, a final test of its correctness could not well be supplied, as is done in Cotterill's *Applied Mechanics*, without some groundwork of the theory of elasticity.

The "water ram," or increased pressure in a pipe in which water is flowing, occasioned by the shutting of the escape valve in a given time, is mentioned on p. 428, and an average value (a time-average) determined. Such an average value, however, is of little practical value, as the maximum pressure occurring during the closing of the valve may be enormously greater than the average, this maximum depending not simply on the total time of closing, but upon the rate of closing at different stages of the operation, as well as, to some extent, on the compressibility of the liquid, and the extensibility of the enclosing pipe.

In the treatment of the hydraulic ram (p. 428) it is difficult to understand why the author should have resorted to the very roughly approximate method there given, instead of using the more satisfactory one already employed in the case of the siphon (when first set into operation [unsteady flow], see p. 274). According to the expression derived on p. 428, the efficiency is greater the greater the height to which the water is lifted, with a given working head, or fall; just the reverse of actual fact, as shown by Eytelwein's experiments.

The hint, on p. 438, that a certain preceding investigation might serve to determine, to some extent, the tendency to produce sea-sickness at a given point of a rolling ship or swinging body, suggests to the present writer the propriety of calling attention to a somewhat similar case where the motion is rectilinear, as in a railroad car on a straight horizontal track. So long as the velocity is uniform, whatever its amount, a person standing in the car, having once adjusted his feet for relative stability of his body, feels no need of further adjustment. If now the velocity, v, begins to increase, and continues to increase with a *constant* acceleration, p, after a single readjustment of the feet at the instant of this change of acceleration (which was also *constant*, =zero, while the velocity was uniform), no necessity is felt for further adjustment. From this we may conclude that the tendency to sea-sickness — that is, the frequent necessity of readjustment of support to preserve relative equilibrium — depends, not on velocity, nor even on acceleration, but on the time rate of *change* of the acceleration; that is, on the value of $\frac{dp}{dt}$, or $\frac{d^2v}{dt^2}$, or $\frac{d^3s}{dt^3}$; where s and t denote distance and time respectively. The more abrupt and frequent the changes in the acceleration, the greater the disturbance of relative equilibrium and the disquieting physiological effect dependent thereon. The occurrence of a normal acceleration, due to the passing of the car from a straight to a curved portion of the track, calls for similar readjustments of support laterally, and the railroad engineer makes use of a "transition curve" (or curve of gradual change of curvature) to secure a gradual instead of an abrupt change of normal acceleration.

In the chapter on Hydraulics, or Motion of Fluids (p. 461), by a remarkable oversight the author makes no mention of steady, as distinguished from unsteady, flow; cases previously considered of liquids in motion (hydraulic ram, and the siphon in initial stages) belonging to the latter category, while those of this chapter are comprehended in the former. The chapter contains about twenty pages and is from necessity very tersely written, the topics considered being beyond the strict scope of the book; but contains frequent numerical illustrations. Occasionally a theoretical result is announced without sufficient mention of practical facts or limitations. For example (p. 464), it is stated of a Pitot tube (a "J-shaped tube"), that if the extremity of its shorter upturned branch receive the impact of the falling water at the base of Niagara, the height of the fall being 162 feet, a column of stationary water of double this height, *i.e.* 324 feet, will be balanced in the other branch. It would have been well to give some experimental results in this connection. With a Pitot tube of small bore and sharp edge at the point presented to the current, Darcy found the height, h, of still water maintained in the vertical tube almost exactly equal to the height due to the velocity, v, of the stream; *i.e.*

$h=v^2/2g$. However, with one of his own instruments, probably furnished with a very blunt point at the extremity, Weisbach obtained $h=\frac{1.56\,v^2}{2g}$, with velocities ranging from 0.32 to 1.24 meters per second. In the Civil Engineering laboratories at Cornell University, results for this height, h, have been found to differ but little for Pitot tubes of three different forms of tip: one sharp-edged like that of a pencil case; another with the end faced off square, the wall of the tube having a thickness equal to the diameter of the bore; the third having a flaring or trumpet-shaped extremity. The bore of the tube was the same in all three cases; viz., 0.042 in., while the velocities ranged from 6 to 18 feet per second. The formula obtained was $h=\frac{1.2\,v^2}{2g}$, nearly.

To the three laws of fluid friction (p. 479) might have been added a fourth: that it is proportional to the "heaviness" of the fluid; and on p. 480 attention might have been called to the fact that of the two coefficients there used, f is an abstract number, while k is not. The assertion on p. 480, as to loss of energy due to fluid friction in a pipe (with steady flow), that if $\frac{d}{l}=4f$, all the energy due to the head $\frac{v^2}{2g}$ is wasted by frictional drag, seems of little importance, since without further data no means are at hand for determining the velocity, v.

At the foot of p. 480 (though attendant circumstances are not made very specific), we find a brief proof of a fact mentioned by Professor Cotterill (*Applied Mechanics*, § 259), communicated to him by Mr. Hearson, that in the case of a long supply pipe and an unlimited water supply, the speed of a motor at the extremity should be regulated to such a value, for a maximum efficiency, that the loss of head from fluid friction in the pipe is one-third of the total head. On p. 463 we find a definition given of the "Miner's Inch," as if it were a fixed module; whereas different values are used in different localities in the mining states of America.

I. P. CHURCH.

Schneekrystalle Beobachtungen und Studien. Von Prof. Dr. G. HELLMANN. Mit elf Abbildungen im Text und acht Tafeln in Heliogravure und Lichtdruck nach mikrophotographischen Aufnahmen von Dr. R. Neuhauss in Berlin. Large 8vo, 66 pp. Berlin, R. Mückenberger, 1893.

It is rare to find a work which even in its outward form bespeaks the beauty of the subject of which it treats, as does this dainty volume on snow crystals by Professor Hellmann. With its silver-lettered, pale gray binding, its many beautiful illustrations, and its excellent typographical work, it makes a most attractive appearance.

As will be seen from the title, this work is the result of observations and studies of snow crystals by the author. For many years he had busied himself with the examination and study of these dainty crystallizations. Again and again he attempted to draw by hand the ever-varying forms as he beheld them through a microscope, only to find, however, that an exact reproduction in this way was utterly impossible, and that all pictures of snow crystals that had ever been given, as, for instance, we find them in our text-books of physics and meteorology, were more or less conventional. Finally he was so fortunate as to secure, in the winter of 1892 and 1893, the co-operation of Dr. Neuhauss, whose achievements in photography are well known. With Dr. Neuhauss's aid, Professor Hellman has been able to give to the world the first microphotographs of snow crystals ever printed, and hence this book brings us the first true pictures of these evanescent, flimsy structures, as they are found on the surface of the earth. It is needless to say that this is a most praiseworthy achievement. The eight plates given, of which one is photogravure and the others heliotype, have been reproduced from Dr. Neuhauss's microphotographs, the magnification being twelve and twenty times the natural size of the crystals. The plates permit the use of a lens, so that even minute details may be examined with ease.

The book is divided into four chapters, the first, historical, being entitled: The Study of Snow Crystals from the Time of Albertus Magnus to James Glaisher. This chapter contains the most noteworthy cuts of snow crystals found in the literature of this epoch (1555 to 1855), and thus affords the opportunity for a most instructive examination. The last ones, those of Glaisher in 1855, are those usually found in our text-books. When these are compared with the microphotographs, the first feeling is likely to be of disappointment; for the beautiful and wonderful symmetry which we were accustomed to see, has disappeared, or very nearly so, and instead appear asymmetrical structures that are but rarely complete in form. But it is this very fact which gives value to these photographic reproductions. Here we have not what the eye has not seen and the hand been tempted to add. Instead of cold, ideal pictures, we have warm, living structures, whose very mutilations reveal the life within them. The hexagonal formation underlying all snow crystals was first pointed out by Kepler in 1611.

The second chapter deals with the Morphology of Snow Crystals. But brief mention can be made here of the salient points. Of the sixty snow figures given on the eight plates but four were found upon accurate measurement to be perfectly regular, one of them being a six-sided laminar one with a diameter of but 0.3 mm. The deformation in outward form is probably due to air motions and to the resistances which the flake encounters on its downward journey. When these disturbing conditions are a minimum, — that is, on a perfectly quiet day, — the most regular forms occur.

These deformations can be classified as imperfect formations, displacements, and asymmetries, all revealed by the plates, and even capable of measurement. With respect to the inner structure of the snow crystal, the most remarkable circumstance is that in all forms appear pointed capillary tubes filled with air or water. This fact serves to differentiate the snow crystals from all other ice crystals, such, for example, as occur in window-pane frosting, where the author failed to find these tubes. Nordenskiöld even observed these tubes filled with water when the temperature was −8° C. As to the size of the crystal, it was found that the star-shaped forms are always larger than the laminar ones. The mean diameter of the former was 2.4 mm., of the latter 1.3 mm. The largest occurring was 7–8 mm.; the smallest, 0.3 mm. The average size of the more rarely occurring six-sided prismatic forms could not so well be ascertained. The dependence of size upon temperature is as follows: The mean diameter of radiating, star-shaped crystals at −6° C., was 3.4 mm.; at −8°, 2.2 mm.; and at −12°, 1.2 mm., decrease in size thus accompanying decrease in temperature. This is due to the fact that with decrease of temperature the amount of aqueous vapor in the atmosphere decreases, and that, too, almost in the same proportion exhibited by the decrease in the size of the crystals. The number of star-shaped forms decreases as the temperature decreases, while the thin laminar ones increase in number.

In the third chapter, New Division of Snow Crystals, the following classification on a crystallographic basis is proposed: 1. Tabular snow crystals, as, for example, those where the main axis or thickness of the crystals is very small in comparison with the other and secondary axes, such as star-shaped forms, plates, and combinations of these two; 2. Snow crystals columnar in form, *i.e.* where the development has been nearly even in the direction of each axis. About 75 per cent of all snow crystals are to be referred to the first class.

The last chapter, headed Origin of Snow Crystals, is a very short one, and reveals how little we know about these interesting formations. It is generally believed now that they are formed directly from the aqueous vapor in the atmosphere without first going through the state of liquefaction. They begin probably as microscopical nuclei of snow which occur frequently in the upper regions of the atmosphere and in the polar regions. Around these nuclei as a center the snow crystal is gradually evolved by new crystallizations. But such questions as to the nature of the inner molecular forces which come into play, or the cause of the different hexagonal shapes which we meet with in the same snow-fall, or the conditions under which Class I. or Class II. are formed, have as yet received no answer. Thus to the physicist and meteorologist are suggested interesting questions for solution.

Professor Hellmann closes his valuable contribution to science with a bibliographical appendix containing notes and extracts from the works cited.

L. A. BAUER.

Recent Text-Books in Heat.[1]

Of the four books reviewed below, the most extensive is the one by Mark R. Wright. This book is of an intermediate grade, between beginning work and advanced work. According to the preface it "is intended for those who have read the elementary parts of the subject as treated in 'Sound, Light, and Heat,' or who are able at once to attack a more advanced work."

The author has attempted to cover all branches of the subject generally treated under that head, including a chapter on Thermodynamics, and one on Climate. The subject of the heat produced during chemical combination has been entirely omitted. The omission is probably intentional, but it would seem that there is no good reason why this subject should be entirely neglected in a work of this character.

Such a large portion of all the important work in heat has been described, or at least mentioned, that one is tempted to look for omissions. With this in view, we find that although Langley's bolometer is described, no mention is made of Boy's radio-micrometer. No mention is made of that very promising advance in calorimetry, known as Joly's steam calorimeter. Siemen's "electrical pyrometer" is described, but nothing is said of the interesting work of Callender and Griffiths on platinum resistance thermometers.

We are a little surprised to find Rowland's researches on Joule's equivalent passed with a sentence. The statement is made that Rowland "obtained numbers which indicate that J is not constant." We should expect this statement to be followed by an explanation of the effect of these researches on our knowledge of the specific heat of water at different temperatures.

It is unfortunate that the number 772 should be retained for Joule's equivalent, although it has been proved that Joule's own work, when properly corrected for the specific heat of water, gives a considerably larger number.

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[1] Heat. By Mark R. Wright. pp. 336. New York, Longmans, Green & Co., 1893.

Practical Work in Heat. By W. G. Woollcombe. pp. 61. Oxford, Clarendon Press, 1893.

Heat: an Elementary Text-book, Theoretical and Practical. By R. T. Glazebrook. pp. 230. Cambridge, University Press, 1894.

Thermodynamics of Reversible Cycles in Gases and Saturated Vapors. Lectures by M. I. Pupin. Edited by Max Osterberg. pp. 114. New York, John Wiley & Sons, 1894.

In speaking of the heat absorbed by the atmosphere, Poulet's work is referred to, but nothing is said of Langley's Mt. Whitney researches. On p. 95, we are told that "of all known substances water has the greatest specific heat." This is clearly an inadvertence, as the specific heat of hydrogen is correctly given in another place.

The book is written from the standpoint of a teacher. It is divided into thirteen chapters, and they are subdivided into paragraphs with headings in full-faced type. The presswork is excellent, as are the numerous illustrations, quite a number of which are new.

The description of experiments is good, and the explanation of principles is unusually satisfactory. The derivation of formulas is very complete, sometimes superfluously so. The author seems to have made an especial effort to help the student over his greatest difficulty, namely, the translation of physical laws and principles into formulas and numerical relations. To this end one finds everywhere general laws and principles, as well as experimental facts, put into the form of equations in a manner that no student ought to find abrupt. Accompanying this work are numerical illustrations and solved problems in profusion. The completeness of this part of the work makes it especially desirable as a reference book, or for students reading the subject without a teacher. As a text-book for the class-room, teachers will be disposed to think that it leaves too little for the student to do for himself.

There is one thing, however, in this numerical work that ought to be protested against. Numbers and results are often given to the fourth, fifth, and sometimes to the sixth and seventh significant figure. Any one who has done experimental work in heat knows that the fourth figure is seldom reached even by the skilled experimenter. It is surely then a waste of energy, as well as positively misleading, to carry numerical calculations so far. One of the hardest things to instill into the practices of laboratory students is the proper degree of refinement to be used in their computations.

In the preface the author states that, "A fair selection of the experiments described should be performed by the student." The book is, nevertheless, designed for the class-room, and experiments are described more as work that has been or may be done, than as work to be done.

Woollcombe's *Practical Work in Heat* is a small volume devoted wholly to laboratory work. In its 60 pages, 50 experiments are described under the heads Thermometry, Expansion, Calorimetry, Evaporation, and Radiation. The work has grown out of lectures and manuscript directions to a class of boys in an English High School. Although most of the experiments are "within the range of boys of fourteen," they are not so elementary as that would make it appear. All the experiments are quantitative, *i.e.* none are purely illustrative.

The directions for performing experiments are concise, though full enough for the purpose. The formulas are derived in full, leaving nothing for the student to do but to substitute his observed quantities in them.

This course of experiments is taken consecutively by the whole class, which necessitates that the apparatus be simple and inexpensive. Much of it we are told "may be readily obtained or easily made from the ordinary stock of a chemical laboratory."

The author very properly emphasizes the necessity of having the students tabulate their data and results. To this end forms are given to show the student how best to arrange his work. It would have added to the value of these forms if they had contained typical data of no greater degree of accuracy than the best students might attain under favorable circumstances.

Very little is said of sources of error or how to avoid them.

The author seems to have lost sight of the fact that small errors of observation may lead to a relatively much greater error in a result, when the latter is obtained by elimination between two equations. In Experiment 39, for example, both the heat of fusion and the specific heat of paraffin are determined by elimination between two equations involving observed quantities and the two unknowns. As each of these equations is already a function of difference of temperatures, the result obtained will frequently be of little value. Another example, on p. 59, is one in which a bulb tube is calibrated by weighing it empty, and twice when containing mercury to different heights in the tube. It would be very much more accurate to weigh directly an amount of mercury which will fill a given length of the tube, and also the bulb full of mercury. It is difficult for a student to understand why results obtained as in the examples given are often so far wrong, even when the observations have been carefully made, and the mathematical work is correct.

Glazebrook's elementary text-book on Heat belongs to the "Cambridge Natural Science Manuals." It is at once a laboratory manual and a text-book for the class-room. The book is of quite an elementary character, although it covers a great deal of ground, and one who thoroughly grasps what it contains knows a good deal about heat. Quite a number of the 55 experiments to be performed by the student are purely illustrative, although the greater portion of the work is quantitative.

The experiments are performed by the student in connection with a text-book course of study which is taken at the same time. The object of the experimental work is merely to give the student a real knowledge at first hand of the science he is studying, and not to make him a skilled experimenter. Each student performs a large number of experiments with simple apparatus, and is not expected to attain any considerable degree of accuracy.

In this connection one thing to be highly commended is the specific pointing out of sources of error under the head "Defects of the Experiment," or "Sources of Error." It seems to the reviewer, however, that the author has left out the most prolific source of error in calorimeter work; namely, the errors affecting temperature determinations.

Both Woollcombe and Glazebrook have been eminently successful in arranging a series of quantitative experiments in heat which do not require expensive apparatus or skillful manipulation. However, it must be remembered that the results attained cannot be *very* much better than qualitative ones. The probable error will, in most cases, be very large.

Teachers will find in both these books some new experiments, old experiments performed with simple apparatus, and many valuable suggestions on the construction and setting up of such apparatus.

Browne's translation of Clausius's work on Thermodynamics has now been out of print for some time. This fact lends an especial interest to Dr. Pupin's lectures, which have been edited by a student in Columbia College. These lectures were delivered in a ten weeks' course to a class of undergraduates. "The course forms a theoretical introduction to a practical course on heat engines." On account of the brevity of the course, the author confines himself to reversible cycles in gases and saturated vapors. The methods and notation of Clausius are frankly adopted. Indeed, the lectures consist almost wholly of judicious selections from Clausius of those parts most interesting to engineering students. We have first a statement of the first law of thermodynamics and its application to perfect gases. Next the second law of thermodynamics is derived from the "Axiom of Clausius," and finally the two laws are applied to determine the behavior of saturated vapor during reversible operations.

Much of what we find in Clausius's classical work is omitted, as unsuitable to the purpose in view. On the other hand, some of the mathematical expressions which Clausius has left to the student to interpret we find here quite fully explained. This is notably true in regard to the meaning and derivation of the differential equation $\frac{dQ}{T} = dS$, from the integral equation for cyclical operations, $\int\frac{dQ}{T} = 0$. This is a point which has been in great need of explanation, students generally finding Clausius's treatment inadequate. I think Dr. Pupin's treatment capable of considerable improvement, although it is quite clear as it stands. In this connection, I would call attention to a misstatement. On p. 48 we find this statement: "In other words, dQ_{12} [an infinitesimal quantity of heat passing in or out of the working substance] depends on the volume, pressure, and temperature, and the method of their variation during the operation represented by the element 12. Now all these things are completely defined by the co-ordinates of the extremities of the element, *hence* dQ_{12} *is a function of these co-ordinates.* The same is true of the infinitely small quantity $\frac{dQ_{12}}{T_{12}}$." If dQ_{12} is strictly a function of the initial and final conditions of the working substance, so must it be when the quantity of heat becomes finite, and this is not true. Probably it is meant that this is true within an infinitesimal of the second order. Even in this light, the above reasoning is at fault, for the quantity $\frac{dQ}{T}$ is solely a function of the initial and final conditions, no matter whether the quantity of heat is finite or infinitesimal.

Another case in which the author explains quite fully what is sometimes a little obscure, is the specific heat of a saturated vapor. For this purpose, he invents a "definitional process" which the student cannot fail to find a great help in understanding what is meant by the specific heat of a saturated vapor.

On the whole, these lectures contain, in small compass, a very satisfactory treatment of the thermodynamics of gases and saturated vapors. One cannot help wishing, however, that the author had edited his own lectures.

F. J. Rogers.

A Laboratory Manual of Physics and Applied Electricity. Arranged and edited by Edward L. Nichols. Two volumes. New York, Macmillan & Co., 1894.

The first volume of this Manual has been prepared by Ernest Merritt and Frederick J. Rogers, and constitutes a Junior course in general Physics. The work is arranged as a series of separate experiments, arranged in groups for student instruction.

The second volume consists of Senior courses and outlines of advanced work arranged in four parts as follows: Part I., "Experiments with Direct Current Apparatus," by G. S. Moler, H. J. Hotchkiss, and C. P. Matthews; Part II., "Experiments with Alternating Currents," by Frederick Bedell; Part III., "Senior Courses in Heat and Photometry," by C. P. Matthews; Part IV., "Outlines of Advanced Work in General Physics," by Edward L. Nichols.

The Incandescent Lamp and its Manufacture. By Gilbert S. Ram. London, The Electrician Publishing Company, 1894.

In the review of this work which appeared in the Physical Review, Vol. I., No. 2, the name of the author was incorrectly given as S. T. Ram.

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THE
PHYSICAL REVIEW.

THE APPARENT FORCES BETWEEN FINE SOLID PARTICLES TOTALLY IMMERSED IN LIQUIDS. I.

By W. J. A. Bliss.

THIS investigation was undertaken at the suggestion of Professor Milton Whitney, of the United States Agricultural Department, in order to throw light on the physical bearings of certain points in his theory of the effect of fertilizers upon the retentiveness of soils for moisture. I desire to express my obligations to him for his continual interest and valuable suggestions, and for much information and many references upon the subject of flocculation.

I would also acknowledge my indebtedness to Professor Rowland for his courtesy and assistance at all times; and to Dr. Joseph S. Ames for his continual guidance and suggestions in both theory and experiment.

§ 1. The experiments described in the first part of this paper were made in order to reproduce, if possible, some of the phenomena of flocculation under more definite and controllable conditions, while in the second part (to be published in a subsequent number of the Physical Review) an attempt is made to give a fuller theory of these phenomena than has hitherto appeared. The facts of flocculation are well known and may be summarized as follows: —

When a few drops of various solutions are added to a jar of pure water, in which clay or finely divided sand is suspended, certain marked effects occur: —

(1) If acids, lime, common salt, or almost any acid or neutral salt are added, the clay particles gather themselves into groups, — they become flocculated.

(2) When a very small trace of ammonia, caustic potash or soda, carbonate of potash or soda, or borax is added, the particles seem to attain a more finely subdivided condition than in pure water and may thus be held in suspension much longer.

(3) When an excess of these last substances is added, the same effect is produced as by the addition of acids, — flocculation is caused.

PART I. EXPERIMENTS.

§ 2. Observations on flocculation have, for the most part, been made only incidentally in experimenting upon the rate of settling of sediments in water and solutions. Professor Whitney has, however, placed a little clay-water under a powerful microscope, and upon adding a drop of very dilute hydrochloric acid has seen the particles form into closer groups. Violent currents are, of course, set up by the entry of the drop, so that individual grains can hardly be watched and little can be told of the process which has taken place except that, after the currents cease, the grains are in groups instead of separate.

§ 3. It therefore seemed interesting to try with larger bodies, whose movements could be more closely watched, whether the film of liquid, which we must suppose to be adherent to each of their surfaces — even when in so-called contact under water — could be made to change its thickness considerably by adding a slight trace of salt, and whether a sufficient force could be so called into play to move the bodies. This is, of course, a totally distinct phenomenon from the well-known interactions of floating solids, such as corks, though both are due to the same molecular forces. Experiments were accordingly begun in November, 1892, by a method suggested by Dr. Ames. A pair of large lenses, used for projecting Newton's rings in lecture experiments, were placed one on top of the other in a vessel of pure water, and the reflected system of rings obtained. Several grams of 20 per cent salt solution were then put in. A simultaneous expansion of the rings would have meant that the glasses had moved nearer than in pure water, a contraction the reverse, but the lenses were far too heavy, and no motion could be observed even with a microscope. After several months of trial of more and more delicate modifications of this apparatus, the following method was found to answer: —

Method.

§ 4. *ABC* (Fig. 1) is a large evaporating crystal which was filled with pure water and placed under the microscope. *L* is an ordinary spectacle lens and *F* a very small fragment of a microscope cover-glass, the thinnest obtainable. A mirror reflected a beam of sodium light *RF* on this fragment, and the Newton's rings formed between its lower surface and the lens were observed in a microscope *M*. The latter was fitted with a three-inch objective and a micrometer, whose scale was 1 mm. = 675 divisions. An average size of the fragment of cover-glass used was 400 × 600 divisions, or about one-half a square millimeter in area, weighing ¼ mg. The appearance of the rings was now carefully noted,

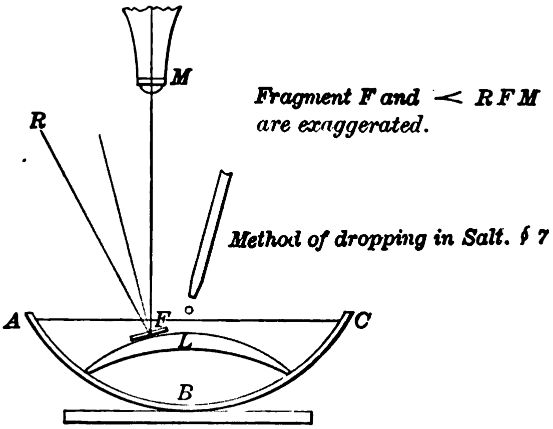

M
R
Fragment F and ∠ RFM are exaggerated.
Method of dropping in Salt. § 7
A
F
C
L
B

Fig. 1.

and their position determined with the micrometer cross-hairs. After watching carefully for half an hour or more to make sure that no accidental change was taking place in the rings, a little solution of the salt to be tried was introduced, the manner varying as the experiments gave more experience. Whether the light fragment had been attracted closer to the surface of the lens or repelled from it was now shown by the way the rings changed. The amount even of motion away from the lens could be roughly determined by the number of times the central spot changed color, the surrounding rings meanwhile shrinking. On the other hand, an expansion of the rings denoted a drawing down of the fragment, and the distance could be approximated in the same way. Further details will be found in §§ 7 to 10 below, and a consideration of the effect of change in the index of refraction and other sources of error in § 11 and § 12.

Results.

§ 5. The substances tested were:—

(1) NaCl and HCl (chiefly the former), as types of substances which tend to produce flocculation.

(2) KOH and NH_4OH, as types of those which tend to prevent it.

§ 6. The results are briefly:—

1. KOH. (*a*) The first effect of the change from pure water to very dilute KOH is to raise the fragment, off the lens, very roughly a distance of 7×10^{-6} cm. on the average.

(*b*) As the solution around the fragment grows in strength, the direction of motion is reversed, and the fragment settles back partly. This return motion did not always take place, and when it did, was usually less than the direct, so that the fragment was left further from the lens than in pure water.

(*c*) By increasing considerably the amount of KOH by further additions, the fragment was sometimes brought back to its position in pure water, and, more rarely, seemed to be drawn closer to the lens.

(*d*) By Method I., which allowed the solution about the particle of cover-glass to decrease in density as the salt first added settled to the bottom, the fragment would be drawn down when a second drop was added, would rise again as this in turn settled, and could be made to repeat this process several times, approaching, however, a position closer to the lens as the repeated drops began to affect the strength of the solution throughout the vessel.

(*e*) By Method II., which allowed solution of definite density gradually to replace pure water, .0007 gr. per cubic centimeter was shown to be too much KOH to produce the greatest separation of lens and fragment. .0024 seems to be enough to bring them closer together than in pure water, though this limit is more uncertain.

To sum up: A trace of KOH — less than .0007 gr. per cubic centimeter — added to pure water tends to increase the thickness of the glass-liquid surface film and so separate two glass surfaces immersed in the liquid. An excess beyond this trace tends to reverse this phenomenon. (Compare effects of excess of alkali on suspended clay or sand particles § 1.)

2. NH_4OH. Few experiments were tried with this substance on account of the fumes. Of four experiments, one gave no result, one a drawing down of the fragment (flocculation), two (one of them very accurate) a result precisely like KOH.

3. NaCl. A large number of trials were made with this, most of which showed absolutely no motion of the rings. A few of the earlier experiments gave contradictory motions one way or the other. The later, more careful ones showed either no change whatever or, in one or two instances, a motion of as much as 1×10^{-6} cm. towards the lens, so small a distance that it may easily have been imagined or due to accident.

4. HCl. Tried only a very few times, usually without effect. Once there seemed to be undoubted raising of the fragment, once a slight drawing down, and once a raising followed, with increasing strength of solution, by a greater drawing down.

In a number of experiments after repeated failure with drops of NaCl and HCl, a drop of KOH on being added at once produced its characteristic effect. This indicates conclusively that the effects of KOH are not due to currents in the liquid, since the solutions were all dropped in in the same way and were, moreover, very closely of the same density and index of refraction.

Borax was tried once without result.

Details.

§ 7. The chief difficulty to be overcome was in making the change from pure water to dilute solution. At first a much larger vessel than that described in § 4 was used, and the salt was added by pouring in a few grams of 20 per cent solution at a distance from the fragment. The time taken for the solution to reach any given strength around the fragment was indeterminate, and therefore the position of the rings had to be observed with great care from the moment the salt was put in until all possibility of effect was past. This was done by making, during this time, repeated settings of the cross-hairs upon the middles of three or four successive bands, which could be done to within $\frac{1}{20}$ of the width of a band. In the course of from five minutes to an hour or more from the time of putting in the salt, there were at times considerable changes in the positions of the rings, *i.e.* a shifting in or out of $\frac{1}{4}$ or $\frac{1}{2}$ the width of a ring. There was, however, never any certainty that the changes were coincident with the arrival of the salt in the neighborhood of the fragment, and the results could never be repeated with any certainty by trying to reproduce the same conditions. It may, however, be said that observations kept up for an hour before any salt was put in rarely showed a small fraction of the change which followed its addition. The results as a whole were, however, contradictory and showed nothing.

The shallow crystal shown in Fig. 1 was now substituted for the larger vessel used before, so that a drop of solution could be put in with a rod or dropper, as in that figure, and settling to the surface of the lens, pass over the spot occupied by the fragment. It was then easy to ensure the presence of salt in the surface films between the two within a few minutes after the drop was put in by giving the lens a very slight slope and dropping in the solution a little higher up than where the small fragment F lay. The whole process was now as follows: —

Method I. — The lens, dish, and a number of fragments were soaked for a day in strong bichromate solution. They were then carefully washed in running water and left to soak in it over night. With a final rinsing in the morning in water filtered to remove grit, the fragments were picked up on a glass rod and placed on the lens, care being taken to touch the latter only on the edges. The lens was then put in the dish and a final rinse given. This was sure to wash the light fragments off the lens if the surfaces were not so clean as to make very good contact, and therefore furnished a very useful though often most provoking test of the efficiency of the cleaning process. Enough filtered tap-water was now put in to cover the lens entirely, and the whole put under the microscope. The reflector was then placed so as to make the angle *RFM* as small as possible. All the fragments were then brought into the field in turn, and the one which gave the most perfect rings chosen. The position of one or more of the edges of this were then fixed with the micrometer as a check upon motion of the fragment as a whole along the lens. A rough sketch of the fragment and a few of the central rings being made, and also a note descriptive of the general appearance of the whole system of rings, the latter were carefully watched for a half hour, and, if any change occurred in their position or in the point of closest contact between the fragment and lens, the former was discarded as not trustworthy, and others successively were tried until one was found that stood the test. A drop of solution — 20 per cent in the earlier and 5 per cent in the later experiments — was now put in, as described above, and in the course of a few minutes the changes in the rings took place, whenever there were any at all. These were carefully noted, together with the time at which they took place, and were afterwards tabulated, so that the whole evidence could be seen together. Extracts from these tables are given herewith.

TABLE I.

METHOD I.

(+ means motion of rings *out*, or glass *down*.)

No.	Solution and number of drops.	Motion.		Remarks.
		Rings (number of changes).	Fragment (10^{-5} cm., up and down).	
XXIII.	KOH 20% few drops	−10	−30	Rings very good and complete. Size of fragment, 3 sq. mm. Both same in XXV. and XXVI.
		then reversed		
		+2	+6	
	NaCl 20% repeated drops	No effect at all		Nos. XXIII.-XXVIII. contrast the effect of acid and alkali, no change whatever being made between the two parts of the experiment.
XXV.	HCl several	No effect		See No. XXIII.
	KOH 20% 1 or 2	Marked contraction		
XXVI.	several each of NaCl & HCl	No effect with either		See No. XXIII.
	KOH 20% 1 or 2	Marked contraction		
XXVIII.	NaCl 20% several	+½ −5	+1.5 −20	Very small fragment, .2 mm. in area, triangular in shape, with point of contact in one corner. About 60 of rings showing. See, also, XXIII.
	KOH 20% 3 or 4	−5	−15	
XXXV.	KOH Very dilute few drops	−1 followed by	−3 *greater* +	Washed in HCl instead of bichromate. One of very few experiments where reverse motion was greater than the original.

TABLE I. (*continued*).

METHOD I.

(+ means motion of the rings *out*, or glass *down*.)

No.	Solution and number of drops.	Motion.		Remarks.
		Rings (number of changes).	Fragment (10^{-5} cm. up and down).	
XXXI.	NH_4OH dilute 1 or 2	−1	−3	Showing effect of excess of alkali, as do also XXXVII. and XXXVIII. Rings clear and complete. Fragment large, about 4 sq. mm. Each time a drop of KOH entered, it drew down the fragment, which rose as the salt settled or became diffused, but not to so great a distance from the lens as before. Hence, as the solution throughout the vessel strengthened, the change each time became less, and the fragment drew continually nearer to the lens.
		followed by		
		+½	+1.5	
	more	−1¼	−3.7	
		(no reverse motion)		
	KOH 20% 1 or 2	+6	+18	
		followed by		
		−5	−15	
	1 or 2 more repeated	smaller + smaller −	followed by each time	
XXXVII.	KOH 20% 1 drop	−1½	−4.5	Washed in very strong bichromate solution for 3 hours and then in running water for 2, and obtained very clear and steady rings.
	several more	no effect		
	more yet	+1.5	+4.5	
		followed by		
		−¼	−3.8	
	twice again	each time same as the last growing less		
XXXVIII.	KOH 5% 1 drop	−2.5	−7.5	First experiment after four months' absence. Very much like the last, except that the fragment does not seem to have risen between the last drops.
		followed by		
		+1.7	+5.0	
	more	+½	+1.5	
	more	no effect		

§ 8. After the preliminary methods had been given up, all the experiments, except six with KOH, were made substantially as above. Not one single instance of a contraction of the rings following the addition of NaCl was observed by this method, and most of these experiments were never even tabulated. Of six that are included in the table, three show no motion whatever, and three a very slight spreading of the rings, the greatest amount being one-half the width of the central spot, *i.e.* corresponding to an approximation of the two surfaces by 1.5×10^{-6} cm. This lack of effect is easily explained on the theory that NaCl would, if possible, produce a drawing together of the surfaces, as it flocculates clay; but they could actually move closer only in a few accidental cases, since the weight of the fragment, of itself, pressed it into contact with the lens, owing to the care which was taken to have the surfaces perfectly clean. With KOH, on the other hand, within a very few minutes after the drop went in, the rings would almost invariably draw in towards the center, the latter changing color about four times on the average. This change occupied about one-half minute, and was then often followed by an immediate reversal of the motion of the rings. This reverse motion was sometimes rapid, at times quite gradual, but very rarely did it indicate a motion beyond the point from which the fragment originally started. Sometimes it took a dozen drops to bring the fragment as close to the lens as it would go, and in the meanwhile each drop, as it entered, would draw it down, and in five or ten minutes it would commence to rise once more as the drop flowed past and solution around the fragment weakened, only to be again drawn down a little further by the entry of the next drop, thus indicating that for each degree of the strength of solution there was a tendency to a film of liquid of definite thickness between the glasses.

The trials with HCl and NH_4OH need not be dwelt upon, as, where any results followed, they were like those with NaCl and KOH. The most interesting experiments by this method were those where a number of drops of NaCl (or in two experiments HCl) were added and produced no effect, or, in a few instances, a slight approximation of the glasses; and then, without in any manner changing the position of the little fragment or any other condition of the experiment, a drop of KOH was put in, and found at once to produce its characteristic separation of the glasses. It was thus shown very conclusively that the effects with this salt were in no way accidental, but depended on a specific difference between it and the two others, since the drops were all put in in the same manner, and were carefully made of very nearly the same refractive index. Moreover, by reversing the process and adding the alkali first, the same difference was shown.

§ 9. METHOD II. — To show more conclusively that the effect of KOH was not due to disturbance on account of currents in the liquid, and to gain, if possible, a more definite idea of the density of the solution which accompanied the phenomena, the following more elaborate method was used a few times, only with KOH :—

The lens was supported near the top of a much deeper vessel, into the bottom of which liquid could be introduced by a siphon from a glass reservoir. The latter was supported on a stand of variable height, so that the rapidity of the flow through the siphon could be regulated or even reversed if necessary. The vessel containing the lens being placed under the microscope, the siphon was started and the vessel filled. The flow was then stopped, and the best fragment, as to rings and stability, was selected by trial as before, and the flow of water being again started, was watched for half an hour or more to test the effect of the mere current upwards due to the siphon, which was run at a considerably greater rate than that afterwards used. If, as was usually the case, the fragment was found to be so stable that the rings were in no degree affected by this much more rapid flow, the reservoir was filled with water to a known mark and a measured quantity of the KOH solution stirred into it. Now starting the flow at a very slow rate, the salt solution of known concentration very gradually rose in the vessel under the microscope, the pure water flowing off at the top. The fragment was always so placed that it would be directly under the point of overflow, so that the alkalinity of the liquid in which it lay could be easily determined by touching test paper to this overflow. In this manner the moment when the liquid at the point in question changed from pure water to solution of known strength could be ascertained very closely. Its very remarkable coincidence with the time of the changes in the rings in the few trials made by this method, served to preclude any doubt that one was the cause of the other, while the almost imperceptible strength of current prevented the phenomena from being ascribed to such mechanical disturbance. These experiments gave, further, the rough values of the respective strengths of solution which caused the greatest separation of the two surfaces and a closer approximation than in pure water. These were given in § 6.

TABLE II.

METHOD II. (BY SIPHON.)

(+ means motion of the rings *out*, or fragment *down*.)

No.	Grams KOH per cu. cm. siphoned in.	Motion.		Remarks.
		Rings (number of changes).	Fragment (10^{-5} cm. up and down).	
XLII.	.0024	+2.5	+7.5	No contraction at all; fragment seems suddenly to have passed from the pure water into solution dense enough to show effects of excess of alkali. See section 10 and Fig. 2.
XLVI.	.0008	−2	−6	Washed in fresh bichromate. Center of rings shifted a very little, otherwise experiment was very good. After the first part of the experiment the .0008 solution was run off and siphon filled with the stronger solution.
		followed by		
		+1.5	+4.5	
	.0015	slight further +		
XLVII.	.0007	−1.5	−4.5	Made to test the effect of acidity or alkalinity of surface before experiment. Everything precisely the same as in XLII. except that between the two the lens and fragments were soaked in strong KOH and then in running water.
		followed by		
		+5	+1.5	

§ 10. The way in which observations of changes in the rings were usually noted is shown by the sketches in Fig. 2, which are copied from my note-book, Experiment No. XLII. The shape of the fragment was a trifle more irregular, but roughly a triangle in which $AB = .89$ mm., $AC = 1.42$, $BC = 1.54$. The same ring is made heavy throughout and serves to fix the amount and direction of the change in the whole system. The siphon ran at varying rates with pure water (Method II. was used) from 2.45 P.M. to 3.25, and was then started with KOH solution whose strength was .0024 grams per cubic centimeter. The change noted in sketch 2 began at 4.25. An expansion from the center of two rings is shown, as also that the stability of the fragment was not very good, since it tilted over a very little towards AC during the experiment.

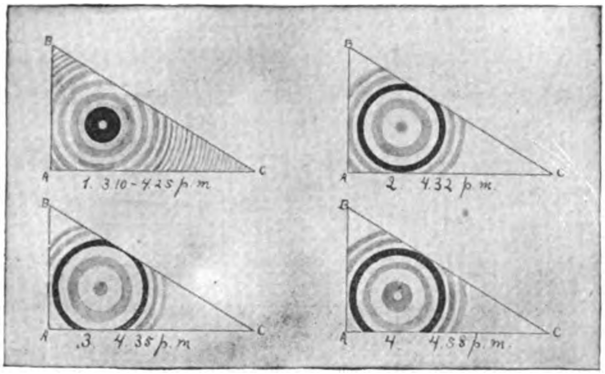


Fig. 2.

In the earlier experiments distilled water was used, but it was found that that available was a little greasy, and in all the later trials filtered tap-water was used for the final washing and the experiment itself.

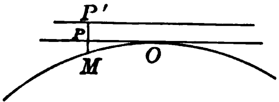
Sources of Error.

§ 11. CHANGE IN THE INDEX OF REFRACTION. — Let the point P (Fig. 3), in a plane tangent at O to a sphere of radius R, be at a distance $MP = e$, from the sphere, measured perpendicular to the plane; and let $OP = \rho$. Then if P' is a point in a parallel plane at a distance p above the tangent one, $t = p + e = MP'$.

Hence $$\rho^2 = 2Re = 2R(t - p) = \left(\frac{n}{2\mu}\sec. r - 2m\right)\lambda R, \qquad (1)$$

where λ is the wave-length in air; r the angle of refraction through the film of liquid, and μ the index of refraction of the latter; and where $m = \frac{p}{\lambda}$, *i.e.* m is the distance p expressed in wave-lengths in air. n is the number of the ring of radius ρ from the center and is, of course, even for a dark band and odd for a light one.

The point observed in the experiments was the number of times the central spot ($\rho = 0$ approximately) changed through a complete cycle, *i.e.* from n to $n - 2$, n being always counted from the ultimate black spot which would occupy the center were the lens and fragment in perfect contact. If, then, n changes from n to $n + \delta$ when salt is put in and the index of the dilute solution thus formed is μ_1, then the condition that it is the central spot which is being observed gives



$OP = p.$ $MP = e$ $PP' = p.$
$MP' = t.$
Fig. 3.

$$\frac{n}{2\mu}\sec r - 2m = \frac{n + \delta}{2\mu_1}\sec r - 2m_1 = 0,$$

or $$m_1 = \frac{\mu}{\mu_1}m + \frac{\sec r \cdot \delta}{4\mu_1}.$$

Therefore, if

$$\frac{\mu}{\mu_1}m + \frac{\sec r \cdot \delta}{4\mu_1} > m; \ \text{i.e.}\ \frac{\sec r \cdot \delta}{4} > m(\mu_1 - \mu),$$

m_1 is greater than m, *i.e.* the particle has moved away from the lens. In order, then, that the change δ shall be wholly accounted for by the change in the index of refraction, m, the distance of the fragment from the lens at starting must be at least as great as

$$m = \frac{\sec r \cdot \delta}{4(\mu_1 - \mu)}.$$

I give below the requisite values of m for various values of δ, the number of times the center changed color. The data are for 5 per cent and 20 per cent solutions, the strength of the *drops* put in, which was very much diluted before reaching the fragment, and for $r = 0$, which makes m a minimum.

	μ_1.	$\mu_1 - \mu$.	m		
			$\delta = 1$.	$\delta = 2$.	$\delta = 4$.
NaCl 5 %	1.340	0.006	41	83	167
20 %	1.369	0.035	7	14	29
KOH 5 %	1.341	0.007	35	71	142
20 %	1.360	0.026	9	19	39
NH_4OH	—	negative	∞	∞	∞

The average change in the experiments due to a few drops of 5 per cent KOH was a contraction of two rings ($\delta = 4$), and to account for it wholly we should have to suppose a distance of 142 wave-lengths or $\frac{1}{10}$ mm. to separate the two glass surfaces supposed to be in contact at starting. Further, the change in the rings with NaCl should have been greater than with KOH; and NH_4OH should have produced the opposite change to what it did. The indices for such dilute solutions are not given in Landolt and Börnstein's tables, but were calculated from refraction equivalents. They are at 18° and μ for water is taken as 1.334.

§ 12. Osmotic Pressure. — A more important source of error was suggested by Dr. Ames, who thought there might be a very strong force tending to lift the fragment due to imperfect diffusion of the salt into the interstice between the two glasses. A method of attacking the question was found in an extension of an equation given by J. J. Thomson.[1] The Lagrangian function of the molecules of salt (supposed to behave like those of a perfect gas) in the volume of a solution and also in the surface films bounding it, is there given. Here two similar terms were added for the volume of liquid between the glasses and the two glass surfaces bounding thereon, the density of the salt being supposed to differ there from its value in the open liquid immediately around. The glass surfaces are for simplicity assumed plane and parallel and separated by a distance x. If, then, H is the Lagrangian function of the salt molecules in the interstice and the liquid immediately around the fragment, $\frac{\delta H}{\delta x}=F$ is the force tending to separate the two. To determine F completely would require a knowledge of the rate at which the solutions become equalized in strength as x increases under the action of the force. A minimum value, not far from the true one, may be found by assuming that the only change is that due to the flow into the interstice, as the particle rises, of enough strong solution from without to fill the increase of volume.

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[1] Application of Dynamics to Physics and Chemistry, pp. 190, ff.

Let v_1 represent the volume of liquid between the fragment and the lens; $\frac{1}{2}S_1$, the area of the lower face of the fragment, *i.e.* S_1 the total area of glass surface bounding the volume v_1; t_1 the thickness of the film on this surface and T_1 its surface tension, ρ_1 and σ_1 the densities of salt in v_1 and S_1 respectively; and $\eta_1=v_1\rho_1$ and $\xi_1=\sigma_1 S_1 t_1$ the total weights of salt in each. Let the same characters without subscripts denote similar quantities in the liquid immediately around. J. J. Thomson's equation for H is (adding two necessary terms in the subscript quantities):—

$$\begin{aligned}(1)\qquad H=\;&\eta R\theta\log\frac{v\rho_0}{\eta}+\eta\{f(\theta)-\omega_1\}-ST\\ &+\eta_1 R\theta\log\frac{v_1\rho_0}{\eta_1}+\eta_1\{f(\theta)-\omega_1\}-S_1T_1\\ &+\xi R\theta\log\frac{St\rho_0}{\xi}+\xi\{f(\theta)-\omega_1\}\\ &+\xi_1 R\theta\log\frac{S_1t_1\rho_0}{\xi_1}+\xi_1\{f(\theta)-\omega_1\};\end{aligned}$$

where θ is the temperature and R an analogous quantity to the same symbol in Thermodynamics, *i.e.* $R\theta\rho=p$, the osmotic pressure; ω_1 and ρ_0 are constants of integration not depending on x. Remembering that

$$(2)\qquad \frac{\delta\eta}{\delta x}+\frac{\delta\eta_1}{\delta x}+\frac{\delta\xi}{\delta x}+\frac{\delta\xi_1}{\delta x}=0;$$

since the total quantity of salt is not a function of x, we get

$$F=\frac{\delta H}{\delta x}=R\theta\left[\left\{\frac{\delta\eta_1}{\delta x}+\frac{\delta\xi_1}{\delta x}\right\}\log\frac{\rho}{\rho_1}-\tfrac{1}{2}S_1(\rho-\rho_1)\right].$$

For complete solution we need an equation giving

$$\frac{\delta\eta_1}{\delta x}+\frac{\delta\xi_1}{dx},$$

the rate of inflow of salt as the fragment rises, but we can find a minimum by assuming that no diffusion takes place during the very short time occupied by the change in the rings, and no weak solution flows out to be replaced by stronger. Then

$$\frac{\delta\eta_1}{\delta x}+\frac{\delta\xi_1}{\delta x}=\rho\frac{\delta v_1}{\delta x}=\tfrac{1}{2}\rho S_1,$$

since $\frac{1}{2}S_1$ is the area and x the height of the space v_1. Substituting we get the minimum value of the force,

$$F=\frac{R\theta}{2}\left[S_1\left\{\rho\left(\log\frac{\rho}{\rho_1}-1\right)+\rho_1\right\}\right].$$

Remembering that $p=R\theta\rho$, we have

$$F'=\frac{S_1}{2}\left\{p\left(\log\frac{\rho}{\rho_1}-1\right)+p_1\right\},$$

where S_1 is the area of glass bounding on the interstice, *i.e.* twice the area of the lower surface of the little piece of cover-slip; ρ_1 is the density of salt in the interstice and ρ in the liquid outside; and p_1 and p are the pressures of a number of hydrogen molecules per unit volume equal to the number of salt molecules in densities given. The assumption that the salt in dilute solution follows Avogadro's law is to be noted. From data in Everett's "C. G. S. System of Units," $p=4\times 10^8\rho$ and $p_1=4\times 10^8\rho_1$. Hence

$$F'=2\times 10^8 S_1\left\{\rho\left(\log\frac{\rho}{\rho_1}-1\right)+p_1\right\}\text{ dynes,}$$

or

$$4\times 10^6\rho\left\{\frac{\rho_1}{\rho}-\left(1+\log\frac{\rho_1}{\rho}\right)\right\}$$

dynes per square millimeter of the under surface of the glass fragment. In the following table F' is given for various ratios of the density in the interstice to that in the liquid around. ρ is taken as .00075, an average value in the experiments by Method II., where alone the density was definitely known.

F' FOR KOH.

$\frac{\rho_1}{\rho}$.	F'.	$\frac{\rho_1}{\rho}$.	F'.	$\frac{\rho_1}{\rho}$.	F'.
0.00	∞	0.10	4200	0.70	180
0.01	10830	0.30	1500	0.90	15.7
0.05	6150	0.50	520	1.00	00

In order to compare this with the force downward due to the weight of the fragment, a whole cover-slip was weighed and its diameter measured, and was found to weigh .46 mg. per square millimeter, or to be subject to a downward force of 2.94 dynes per square millimeter in water, which is insignificant compared to F'.

Considering the first effect with KOH alone, *i.e.* the rise of the fragment as the drop of solution reaches it, this force would abundantly explain the action, since the fragment ought to rise until the densities above and below it became equal, and then, the force having ceased to exist, drop back. The explanation is insufficient because: —

(1) Were this force alone acting, a second drop should again cause the fragment to rise, since the solution beneath it would again be more dilute than that outside, but in the majority of cases the reverse was the case and a second drop, if it produced any effect, usually caused a fall of the fragment, which is quite in accord with the fact that excess of alkali flocculates.

(2) So far as could be found,[1] NaCl, in ordinary cases, diffuses less rapidly than KOH, and if this holds for very dilute solutions and capillary openings, we should expect a greater disparity in the strength inside the interstice and without in the case of NaCl, and consequently a greater lifting force, whereas no lifting at all was observed. The lifting force for equal ratio of the strength inside and out is with NaCl 95 per cent of that with KOH.

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[1] Ostwald's Solutions, trans. by Muir, p. 134.

While, in the absence of sufficient data with regard to diffusion in capillary spaces, some may prefer to consider the experiments as largely explained by the osmotic force, it seems to the writer more natural to suppose, considering both (1) and (2), that the effects are the result of two sets of forces both tending to raise the fragment in the case of dilute KOH, and, on the other hand, almost exactly counterbalancing each other in the case of NaCl and stronger KOH, the first set being the molecular forces which act in flocculation, and the other those which bring about diffusion.

§ 13. To sum up: The effects summarized in § 6 are a reproduction of those observed in the flocculation of clay particles, as may be seen from the following comparison: —

Results of the Experiments.	*Flocculation.*
1. The effect of very dilute alkalies, represented by KOH, is to raise the glass fragment away from the lens.	1. Fine solid particles are held further apart in very dilute alkaline solutions than in pure water.
2. As the KOH solution became stronger the fragment returned to its original position, and at times seemed drawn closer to the lens.	2. More concentrated alkaline solutions reverse this effect and cause flocculation.
3. If the glass surfaces were closely in contact at starting, very little effect should have been expected from substances tending to draw them closer. They could only act by bending the fragment. This lack of effect was found with NaCl, in marked contrast with the strong effect of KOH.	3. The effect of NaCl and the other substances of its class is to produce a closer grouping of the particles of clay than in pure water.

The writer hopes to extend these qualitative results to a wider range of substances and, if possible, obtain quantitative observations connecting the thickness of the adherent films separating the glasses with the density of the solution.

THE DISTRIBUTION OF ENERGY IN THE SPECTRUM OF THE GLOW-LAMP.

(PRELIMINARY MEASUREMENTS.)

BY EDWARD L. NICHOLS.

I.

THE glow-lamp affords an interesting subject of study. In the first place, it is possible by varying the current in the filament to secure a very wide range in the temperature of the radiating surface, extending upwards to temperatures considerably higher than those of ordinary luminous gas flames, and to maintain more constant the condition of incandescence during the progress of each set of measurements than is possible in the case of any other source of light. In the second place, the study of the spectrum of the incandescent lamp affords an answer to certain questions of considerable practical importance.

In 1885, Messrs. Siemens and Halske, of Berlin published the results of measurements for the purpose of showing the superiority of the silver-gray surface obtained by treating filaments of glow-lamps by bringing the same to incandescence in an atmosphere consisting of volatile hydro-carbon. In the following year Mr. Mortimer Evans[1] described comparisons of the radiation from bright and black incandescent lamp filaments in which the superiority of the former was very clearly demonstrated. Evans selected two filaments of similar size and structure, one of which he subjected to a surface deposition obtained by flashing in an atmosphere of ordinary coal gas; the result being a hard coating of carbon of the color of lampblack. The other filament was flashed in an atmosphere of volatilized hydro-carbons, by which means the usual silver-gray surface was obtained. The two filaments were then mounted and exhausted to the ten-thousandth of an atmosphere, and characteristic curves were made, showing, (1) the relationship between the efficiency of the lamps, expressed in watts per candle, and candle power (see Fig. 1); (2) the relation between the total watts consumed in each lamp and the candle power. These curves show a very much greater amount of light for the energy expended in the case of the filament with bright surface.

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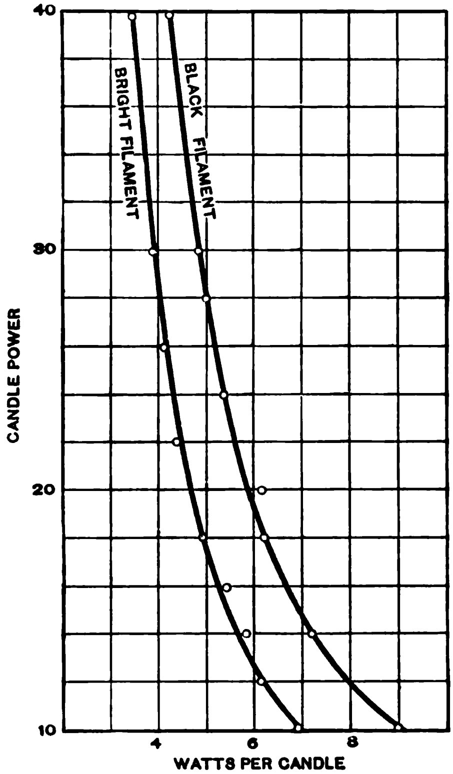
[1] Evans, Proceedings of the Royal Society, Feb. 18, 1886.

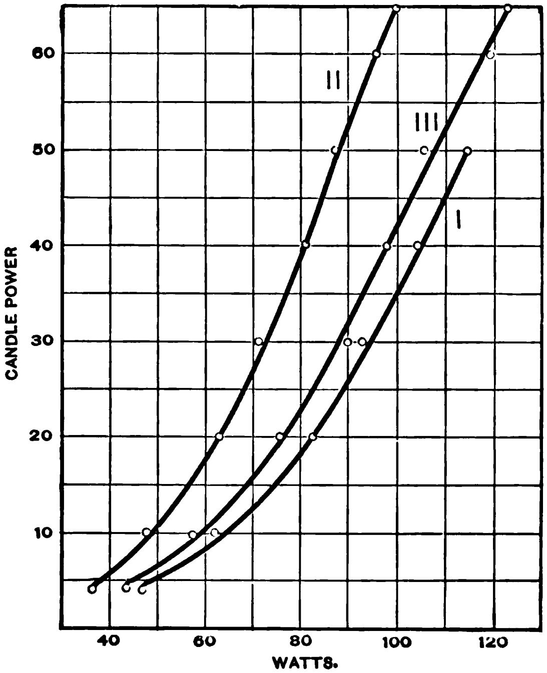


Efficiency of Black and Bright Filaments (after Evans).

Fig. 1.

Subsequently, two filaments of close firm structure and of equal size were obtained which possessed the usual dull, dead black color resulting from carbonization without treatment. The characteristic curve of one of these was obtained after mounting the same in the usual manner and securing a good vacuum, after which it was removed from the lamp and was flashed in hydro-carbon vapor. After exhaustion it was again tested, measurements showing marked increase in efficiency; then it was removed from the bulb for a second time and flashed in coal gas, by which process a new surface of the color of lampblack was obtained. Having been remounted in this final condition and thoroughly exhausted for the third time, another characteristic curve was obtained. This curve was found to coincide quite closely with that of the original untreated filament. These three curves are given in Fig. 2.

Mr. Evans' experiments seem to leave no question as to the fact that the bright surface obtained by flashing in hydro-carbon vapor possesses properties which fit it peculiarly for the production of light. Evans noted the fact that the lamp bulbs containing the black filaments were much hotter than those which surrounded filaments with a silver-gray surface, and he remarks: "I have little doubt that the loss of efficiency when black was due to the energy supplied being radiated in large quantities as heat waves from the blackened surfaces, which these surfaces when bright would not emit."



I. An Untreated Filament.
II. Same, after treatment (bright).
III. Same, after treatment (black).
(After Evans.)

Fig. 2.

That the *total* radiating power of carbon in the form of lamp-black is very much greater than that of the silver-gray graphitic form obtained by flashing in hydro-carbons has been shown by

H. S. Weber in a paper read before the Electrical Congress in Frankfort in 1891.[1] Weber, who tested thirty-three varieties of glow-lamp filaments, found for the black carbon a coefficient of total emission lying between 0.0000169 and 0.0000174, and for treated filaments with bright surface, coefficients lying between 0.0000127 and 0.0000132.

During the winter of 1891–92, Mr. John C. Shedd[2] made a large number of comparisons of commercial lamps with treated and untreated filaments, for the purpose of further testing the conclusions reached by Evans. His method consisted in studying the visible spectrum of such lamps by means of the horizontal-slit photometer. The lamps were maintained for this purpose at various degrees of incandescence.

Mr. Shedd's results were in the main confirmatory of the higher efficiency of bright carbons, but they indicate clearly that the efficiency of the incandescent lamp is dependent upon other factors than the character and radiating power of the surface of the filament. He found, for example, that the vacuum especially has much to do with the relationship between the energy developed in the lamp and the candle-power.

His comparisons between glow-lamps, at different stages of incandescence, and the gas-flame, bring out two facts which are of interest in the comparative study of treated and untreated filaments.

(1) Spectro-photometric measurements with lamps of untreated filaments of vegetable fiber showed that the light was of the same color as that from gas (in the argand burner) when the lamp was being operated at an efficiency of 6.8 watts per candle. All efficiencies lower than this showed relatively higher intensities in the red, while all efficiencies above 6.8 showed a light which was bluer than that of the gas. The relative intensities of the various wave-lengths in the light emanating from such filaments were found to vary quite rapidly as the incandescence rose. Such a lamp, for which the ratio $\frac{\text{glow-lamp}}{\text{gas-flame}}$ for the region of the D line was unity, gave for wave-length $0.4\,\mu$ about 1.6.

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[1] Weber, Verhandlungen des Internationalen Elektrotechniker-Congresses zu Frankfurt am Main, 1891, II., p. 49. See also Physical Review, Vol. II.

[2] Thesis: in Library of Cornell University, 1892.

(2) In the case of treated filaments with bright surface, on the other hand, equality with gas-light as regards whiteness was reached only at somewhat higher efficiencies, viz. between 4 and 6 watts per candle. The changes in the color of the light from such lamps appeared, moreover, to be much less marked than in the case of filaments with black surface. The variation in quality, indeed, was frequently scarcely noticeable when the lamp was brought up from 7 to 4 watts per candle. It would appear from Mr. Shedd's measurements that in the case of black filaments the increase in the intensity of short wave-lengths with temperature was much more rapid than in the case of the longer wave-lengths; whereas with bright filaments all the wave-lengths of the visible spectrum gained in intensity at very nearly the same rate.

II.

A study of the results obtained by Mr. Shedd made it evident to me that the influence of the vacuum of the glow-lamp was so great as to render all comparisons unsatisfactory between different lamps in which the pressure was an unknown factor. In the hope of overcoming this difficulty I had some lamps constructed, through the kindness of a friend engaged in the manufacture of glow-lamps, by means of which this difficulty could be obviated.

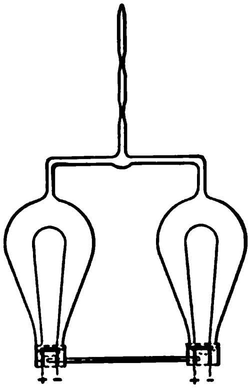
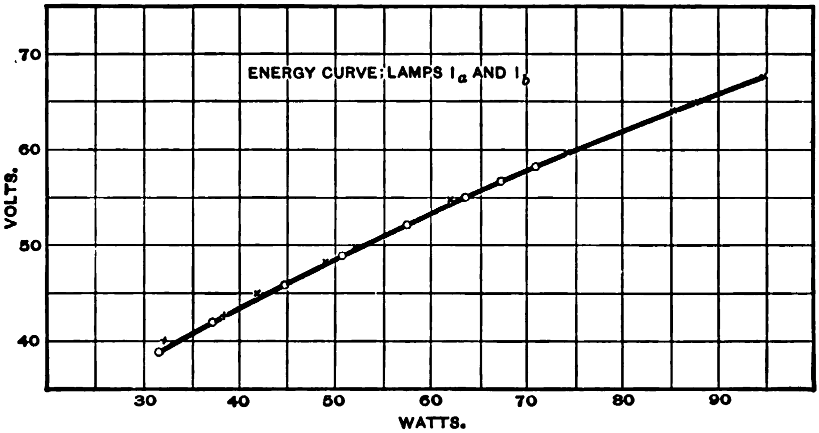


Fig. 3.

Two lamp filaments as nearly identical as possible were taken. These were brought to identity as regards resistance by the usual method of flashing in hydro-carbon vapor; one of them was then given a coat of black carbon by smoking. They were mounted in bulbs of the usual form, the two being connected by a glass tube as shown in Fig. 3. The pair of lamps thus connected were placed upon a pump together, and were exhausted in the usual manner, after which they were sealed. These lamps could be brought to incandescence separately; but since there was permanent communication between the interiors of the two bulbs, whatever changes in vacuum occurred in the one would be shared by the other lamp.

Upon measuring the candle-power, voltage, and current, in the case of these two lamps throughout a wide range of incandescence and platting the characteristic curves, it was found that the difference in efficiency as expressed by watts per candle was very great indeed. The efficiency of the smoked filament was extraordinarily low. Inspection of the results obtained made the cause of this difference obvious, the fact being that the mantle of carbon obtained by smoking was of extremely high electrical resistance. The result was that when the same current was sent through the two lamps, the total watts generated in each were identical (see Fig. 4); but since the radiating surface of the smoked filament was very largely increased, the temperature of the lamp was greatly lowered. The consequence was that the black lamp required a higher voltage and a correspondingly greater amount of energy to bring it to a given candle-power than did the bright filament, which was without the superficial non-conducting layer of carbon. It seems probable that something of the same kind may have occurred to vitiate the experiments made by Mr. Evans, although, doubtless, the non-conducting film in the case of his lamps was much thinner than in the pair of lamps now



Crosses refer to 1_a, Circles to 1_b.

Fig. 4.

under consideration. Although it was impracticable to compare these lamps with one another by the mere determination of the candle-power and the watts per candle, they afforded excellent material for the study of the laws of radiation of these two widely different carbon surfaces. I accordingly subjected them to detailed measurements as regards the distribution of energy in the infra-red of the spectrum through a very wide range of temperatures, making use, for this purpose, of a method nearly the same as that described by E. F. Nichols in the first volume of this Review.[1] Indeed, in many of these measurements Mr. Nichols did me great service, bringing to bear upon what was in many respects an operation of unusual delicacy the skill attained by long practice in a similar research.

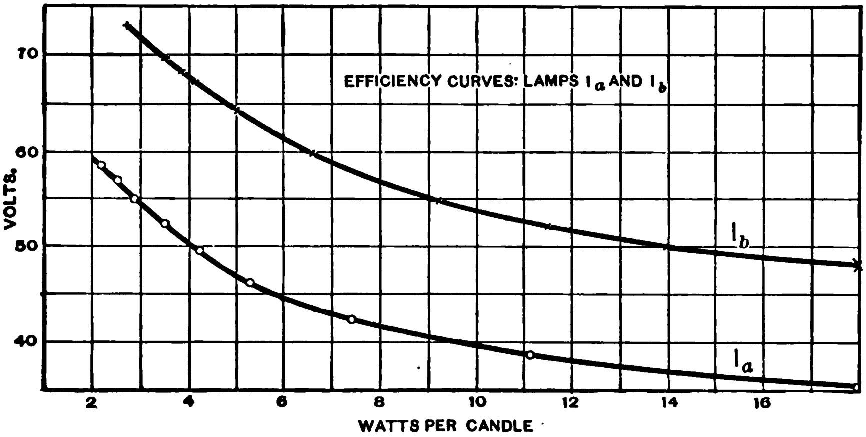


Fig. 5.

III.

The apparatus used in the study of the spectrum of these lamps was that described by Mr. Nichols in the paper just cited. It consists of a spectrometer, in place of the eyepiece of which is inserted a linear thermopile of antimony and bismuth, the edge of which had been filed down until only a very narrow strip was exposed to radiation. Dispersion was obtained by means of a prism of carbon bi-sulphide contained between glass walls. This substance was selected because there is reason to believe that it obeys more closely than any other available material Cauchy's law for dispersion.

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[1] Nichols, PHYSICAL REVIEW, Vol. I., p. 1, 1893.

The use of the spectrometer with glass lenses, and of a prism the walls of which are of glass, in the study of the distribution of energy in the spectrum, is open to the serious objection that both glass and carbon bi-sulphide exert selective absorption upon the rays of light which pass through them. The only substitute for a prism and lenses would be a concave grating, or a plane grating used in connection with a concave mirror. The feebleness of the spectra produced by means of the grating offers in itself a serious hindrance to its use in bolometric work. The theory of the diffraction grating, as given by Rowland, is, moreover, conclusive as regards the unfitness of ruled gratings for the determination of energy in the spectrum, and it had been previously shown experimentally by Paschen that the exploration of the spectrum of incandescent lamps by means of the grating and bolometer led to results which were full of the most serious errors. No more striking verification of the theory can be given than the curves exhibited in Paschen's paper. So serious, indeed, are the vagaries of the diffraction grating in this respect that it would seem as though all work involving absolute measurements of the distribution of energy in the spectrum, in which diffraction has been made use of, must be discarded and the work done over by other methods.

As regards the fitness of the material used in the apparatus with which the experiments to be described were made, it may be noted that Ångstrom has studied the absorption of carbon bi-sulphide in the infra-red and has found it to be transparent as far as wave-length 3.0μ, beyond which lie two strong absorption bands. Since, however, my measurements do not extend beyond 3.0μ, there seems to be every reason to assume that no serious errors will be introduced by the passage of the rays under investigation through this medium.

Ernest Nichols, in the article just cited, has studied the diathermancy of glass. His curve for the percentage of light transmitted by a plate of glass (see Physical Review, Vol. I, p. 8) indicates great uniformity in this material, up to the longest wave-lengths to be considered. The diminution of light due to the interposition of the specimen of glass which he tested, is scarcely greater than the losses by reflection at the two faces of the plate, and while there is evidence of very slight decrease in the transmitting power as we pass from 2.0μ to 3.0μ, the change cannot amount to more than one or two per cent. Under these circumstances, it seems quite safe to assume that no serious errors will result from selective absorption of the rays of the infra-red spectrum of glow-lamps, either by the glass of the lenses and the walls of the prism, or by the contents of the prism itself.

The most serious objection to the use of the bi-sulphide prism lies in the fact that the dispersion is very small for the longer wave-lengths. The shortening of the spectrum in the infra-red is such that measurements beyond wave-lengths $3\ \mu$ are of questionable value.

The choice of the thermopile, the width of the face of which was something more than half a millimeter, instead of a more strictly linear instrument such as the bolometer, is open to criticism. In point of fact, the thermopile showed itself to be so much more sensitive than any bolometer which Ernest Nichols and the writer were able to construct, that we were led to adopt it in spite of this objection. It seemed quite certain, moreover, that there was but little of interest to be found in the spectrum of glow-lamps beyond the regions which it was possible to explore with the thermopile, so that it did not seem worth while to expend a great deal of time in the construction of a bolometer.

Our experience appears to differ from that of others who have worked in this field; but it may not be without interest to state that we both tried bolometers constructed of the platinum cross-hair wire, used by Snow in his exploration of the bright line spectra of the alkalies, — an investigation which would seem to demand the very highest delicacy; also bolometric strips of iron, nickel, aluminium, etc. We were not able within the time at our disposal to construct a bolometer, the sensitiveness of which for the purposes of our measurements was greater than one-twentieth of that of the linear thermopile.

The galvanometer used in these studies was the one briefly described by Ernest Nichols in his article already cited. It was built in the instrument shop of the department by F. C. Fowler, but the moving parts were made by Mr. Nichols, to whom also I owe the very careful study of the bi-sulphide prism.

IV.

Mr. Nichols, in his paper, has described briefly the calibration of the bi-sulphide prism. Since, in work of the kind under consideration, the accuracy of this calibration is of the utmost importance for the determination of wave-lengths in the infra-red, it may be well to give some additional details. The containing vessel of the prism consisted of a block of cast iron bored through so as to give a cylindrical opening about 4 cm. in diameter. The glass faces of the prism were cemented over this opening, forming an angle of almost exactly 60°. Since the refractive index of carbon bi-sulphide changes greatly with the temperature, it was necessary to have an accurate knowledge of the temperature of the contents of the prism at the time when each measurement was made. The method finally adopted for this purpose was as follows: —

At the back of the prism a small pocket was constructed, one side of which was formed by the iron wall of the prism itself. Within this a delicate thermometer was inserted, the bulb of which was surrounded by water.

The dark room, within which the spectrometer was mounted, was an inside room of the laboratory, containing only one opening, that, viz., through which the rays from the incandescent lamp which it was our purpose to study reached the collimator lens of the spectrometer. The temperature changes in this room were very slow, and a comparison of the readings of the thermometer with those of a similar instrument, the bulb of which was placed within the liquid of the prism itself, showed that the difference of temperature within and without, through the very slight range to which both would be subjected, was negligible. Having satisfied ourselves that the indications of the thermometer in the water pocket at the back of the prism might be taken, the other thermometer was withdrawn and the prism was sealed up. Readings of the remaining thermometer were made before and after each set of readings.

The prism was standardized for a temperature of 23.5° C., a reading which represented very nearly the mean temperature of the room, and all observations were reduced to this temperature. The entire range to which the prism was subjected in the course of the measurements was about 3°. The indices of refraction of the prism were determined by Mr. Nichols with great care for temperatures lying within the above-mentioned range, the wave-lengths selected for this purpose being those of the C and F lines of Fraunhofer. The results of this calibration, which are given in Table I. of Mr. Nichols' paper, are in close agreement with similar determinations made by Wüllner.

The method of calibration consisted in determining with all possible accuracy the Cauchy equation of the prism from the indices of refraction at the standard temperature for wave-lengths corresponding to the lines A, B, C, and *b* of Fraunhofer. By means of the equation thus obtained a curve was platted extending to the wave-lengths in the infra-red which we wished to measure. This method of determining the longer waves is less satisfactory in some respects than that made use of by Mouton and afterwards by Rubens, but it was believed that the errors thus introduced would not be important in comparison with those which result from other sources.

V.

Briefly stated, the method of experimenting was as follows : —

The thermopile already described was placed in closed circuit with the galvanometer, the latter instrument having been sensitized to a degree found by previous trials to suffice for the exploration of the region to be tested. The spectrometer was then set to a circle reading corresponding, at normal temperature, to the wave-length in question. The thermometer was then read, the time noted, and the inner room was closed, not to be entered again until the measurements for that region of the spectrum had been completed.

The thermal disturbances produced in the course of these preliminary adjustments lasted in uncontrollable intensity for about thirty minutes, after which time it became possible to bring the galvanometer needle to a position such as to render the scale visible to an observer at the telescope. The drift, still very rapid, would then diminish in the course of ten minutes more to a state of comparative repose, such that readings for the ballistic throw could be obtained. The next step was to close the lamp circuit and to read volts and amperes. The galvanometer was then brought to the proper part of the scale, and preliminary calibrating throws were taken. For the purpose of calibration a subsidiary coil was used. This coil was mounted permanently about twenty centimeters behind the needles. It could be thrown into shunt circuit with a compensated resistance through which flowed a very steady current from a storage battery. The current was measured from time to time by means of an accurate mil-ammeter. The swing due to the closing of the circuit of the subsidiary coil gave the relative sensitiveness of the galvanometer. Such calibrations were made before and after every reading of the thermopile throws, and thus all the determinations were rendered reducible to a common scale, although the figure of merit of the galvanometer varied greatly in the course of the experiments.

After the completion of the preliminary calibration, the radiation measurements were begun, sets of five throws being taken for each condition of the lamp, which was changed stepwise between each set until the entire range to be covered had been included.

When all the readings to be made for the region of the spectrum to which the thermopile was exposed had been completed, the inner room was entered, the thermometer was read, and the time noted. The spectroscope was then set to a new wave-length, and after thermal equilibrium had again established itself, the operations above described were repeated. In this way the entire infrared spectrum between 0.8 μ and 3.0 μ was explored.

The data thus obtained were reduced in the following manner:—

(1) All deflections were brought to a common scale by reference to the continually repeated calibration throws.

(2) Wave-lengths were determined by correcting the values cor-

responding to the circle readings of the spectroscope for temperature. These corrections were obtained graphically from calibration curves platted to a large scale.

(3) The deflections were corrected for the width of the face of the thermopile. This was a most important correction, since, owing to the fact that the spectrum was prismatic instead of being normal, the surface of the pile received radiation from a larger and larger number of waves as the wave-lengths increased.

To be comparable one with another, in different parts of the spectrum, the deflections should be those obtained by exposure of a surface varying in width with its position, in such a manner as always to receive radiation from a region of unit breadth, measured in wave-lengths. This correction it was possible to apply with satisfactory approximation up to wave-length 3 μ. How important this correction was will appear from Fig. 6, in which a corrected and uncorrected curve are platted to a common scale. In this diagram the width of the thermopile is taken as normal for the region of the D line.

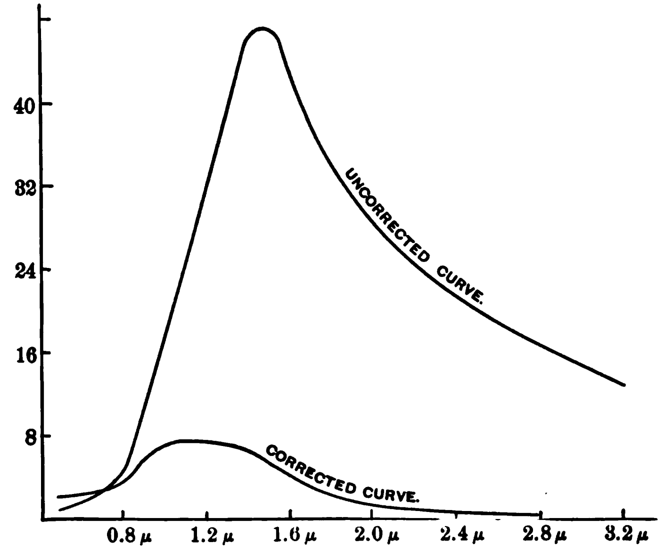

Fig. 6.

The two lamps described in the second section of this paper were measured throughout a range of temperatures from the red heat upwards to a condition of incandescence beyond which they could not be carried without rapid deterioration. The corrected results are given in the following tables: —

TABLE I.

SPECTRUM OF GLOW-LAMP 1_b (BLACK FILAMENT).

Wave lengths.	Intensities at					
	35 watts.	45 watts.	55 watts.	65 watts.	75 watts.	85 watts.
0.8 μ	0.360	0.744	1.140	1.490	1.820	2.220
1.0	0.859	1.450	2.050	2.700	3.410	4.180
1.2	1.560	2.350	3.170	4.040	5.010	6.090
1.4	2.280	3.150	3.970	4.730	5.460	6.160
1.6	1.560	2.050	2.590	3.050	3.490	3.830
1.8	0.859	1.140	1.360	1.630	1.860	2.080
2.0	0.490	0.656	0.793	0.923	1.050	1.160
2.4	0.148	0.195	0.240	0.284	0.325	0.362
2.8	0.0465	0.0613	0.0768	0.0910	0.106	0.194

TABLE II.

SPECTRUM OF GLOW-LAMP 1_a (BRIGHT FILAMENT).

Wave-lengths.	Intensities at								
	75 watts.	65 watts.	55 watts.	45 watts.	35 watts.	25 watts.	15 watts.	10 watts.	5 watts.
0.7 μ	2.22	1.66	1.08	0.626	0.240	0.107	—	—	—
0.8 μ	3.30	2.62	1.90	1.260	0.744	0.464	0.248	0.224	—
0.9	5.45	4.22	3.13	2.270	1.480	0.900	0.372	0.306	—
1.0	6.43	5.16	3.17	2.970	2.040	1.300	0.461	0.312	0.167
1.2	6.84	5.84	4.75	3.790	2.790	1.74	0.732	0.372	0.313
1.4	6.27	5.50	4.72	3.880	3.050	2.06	0.799	0.599	0.419
1.6	4.40	3.86	3.27	2.680	2.030	1.33	0.572	0.418	0.257
1.8	2.20	2.02	1.78	1.480	—	0.813	0.361	0.281	0.179
2.0	1.31	1.20	1.07	0.908	—	0.523	0.255	0.199	0.115
2.4	0.64	0.527	0.467	0.400	0.333	0.258	0.147	0.125	0.118
2.8	0.28	0.259	0.233	0.204	0.169	0.133	0.077	0.0633	0.0469

Of these two sets of measurements, those upon the lamp with the bright filament were the more complete, being carried down to temperatures much below the red heat. The results may be treated graphically in either of two ways: —

(1) By the construction of curves with intensities as ordinates and electrical activity expressed in watts as abscissas, each curve representing a separate wave-length.

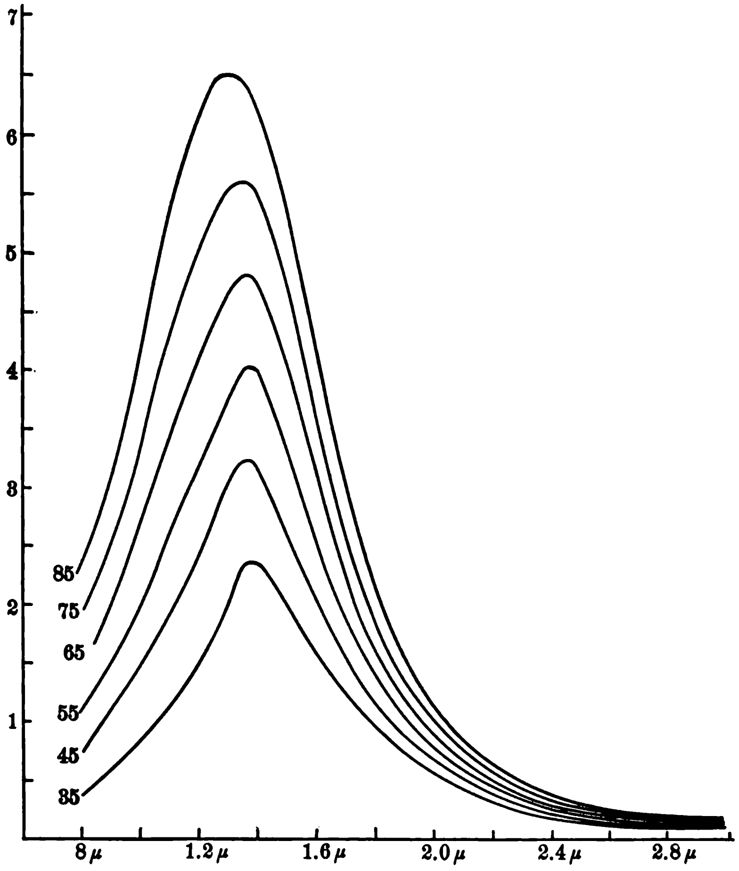

Fig. 7.

(2) By platting intensities as ordinates, and wave-lengths as abscissas.

Curves obtained by the former method show the rise in intensity for each wave-length with increase in the energy expended in the lamp. These curves are very similar in form to those for the radiation of platinum as a function of the temperature, as measured by the writer[1] and also by Violle.

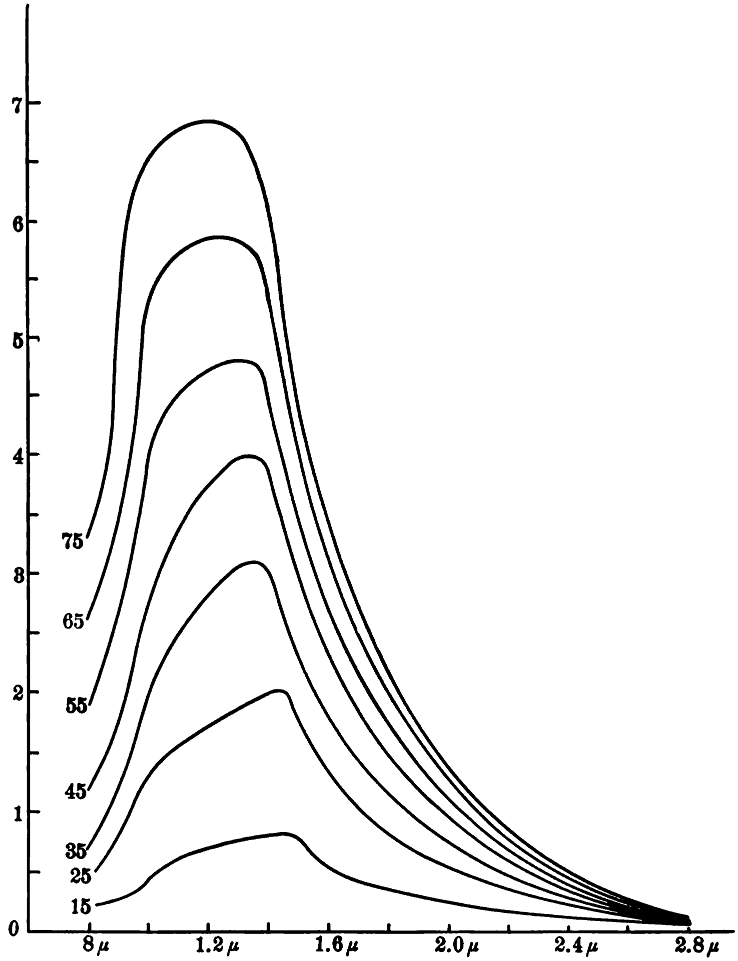

Fig. 8.

The curves of the second type are shown in Fig. 7, which shows the distribution of energy in the spectrum of the lamp with lamp-black-coated filament (1_b), and in Fig. 8, which gives similar curves for the lamp with bright filament (1_a).[1] It will be seen that the distribution is very different in the two cases, the maximum in the case of the latter being nearer the visible spectrum. Were both sets of curves extended to the shorter wave-lengths, it would unquestionably be found that the area of the portions lying within the range of the luminous rays is larger in the case of bright filaments, a result which would afford a complete explanation of the superiority of such filaments in the production of light, as shown by Evans in the experiments already described.

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[1] Nichols, Ueber das von glühendem Platinum ausgestrahlte Licht. Göttingen, 1879.

Similar measurements to the above were made upon a commercial lamp with untreated filament bamboo. The lamp in question is the one used as a reference light in the experiments of E. F. Nichols upon absorption in the infra-red.[2] It will be seen, by reference to his curves, that the distribution in the spectrum of this lamp was in every respect similar to that in the spectrum of lamp 1_b.

The complete investigation of the problem with which this present paper deals would involve the direct measurement of the temperatures of the radiating surface. In such an investigation, to which the experiments here described are preliminary, the writer is now engaged. The results are to be given in a subsequent article.

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[1] The only measurements in any way comparable with these, so far as the writer is aware, are those of Abney and Festing. Their curves are, however, platted with scale readings instead of wave-lengths as abscissas, so that it is difficult to make any exact comparison.

[2] E. F. Nichols, *l.c.*

THE INFLUENCE OF HEAT AND THE ELECTRIC CURRENT UPON YOUNG'S MODULUS FOR A PIANO WIRE.

By Mary Chilton Noyes.

Object of the Investigation.

MOST of those who have investigated experimentally the effect of change of temperature upon the elasticity of metals have found that the elasticity decreases as the temperature rises. Wertheim's[1] results, however, indicate an increase of elasticity for iron and steel with rise of temperature, the elasticity reaching a maximum somewhere between 100° and 200° C. He was such a careful experimenter, and the increase observed was in some cases so considerable, that it seems improbable that these results are altogether in error. One object of the present investigation was to see if any indication could be found of a maximum value for the modulus between 100° and 200°.

Many of those who have experimented on this subject have used only a few temperatures, and few have gone to temperatures beyond 100°. There have also been considerable variations in the results. It seemed desirable to obtain the modulus for a series of temperatures extending nearly to 200°, and to secure the various temperatures by a method not previously employed in investigating the subject.

General Plan of the Experiments.

The different temperatures desired have been obtained by means of an electric current from a storage battery. The wire to be tested was mounted horizontally on a comparator, and carefully adjusted so as to be parallel with the lines of the comparator bed. It was placed inside a glass tube of about 2½ cm. diameter for protection against air currents. On the outside of this tube was wound a helix of German silver wire. In some of the determinations the test wire was maintained at the desired temperature by means of a current through this helix, while in others a current was passed through the piano wire itself. It was thought that if the magnetization had any effect upon the elasticity, it would be different in the two cases. If the two methods of heating gave similar results, as it was expected they would, it was supposed that the magnetic effects could be regarded as negligible. It will be seen later that the results obtained did not agree with this supposition.

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[1] Ann. de Chim. and Phys., Ser. 3, 12, 385.

Two openings were made in the glass tube, through which the positions of two marks on the wire were observed by microscopes. When the heating was accomplished by means of a current through the helix, the temperature was determined by a thermometer placed inside of the glass tube, but so arranged as not to touch it. Currents of air were avoided by stopping the ends of the tube with cotton. When the current passed through the test wire itself the temperature was determined by the expansion of the wire, and also by measuring its electrical resistance.

The wires tested were piano wires of two sizes, the larger 0.40 mm. in diameter, and the smaller 0.26 mm.

Preliminary Experiments.

The micrometer heads of the microscopes were first calibrated. It was found that for the left microscope 4000 divisions were equal to one mm., and for the right microscope 4013 divisions. In the calculations both have been taken as giving readings of $\frac{1}{4000}$ mm.

The temperature coefficient for electrical resistance was next determined for the larger wire. The wire was coiled on a piece of glass tubing covered with silk, and then immersed in a vessel of oil. The resistance was determined for temperatures ranging from near 0° to about 180°. Several determinations were made, and the curve showing the relation of resistance to temperature plotted; but the curves all showed quite a sharp bend at about

75°. This was found to be due to the fact that the viscosity of the oil at lower temperatures caused the temperature of different parts of the bath to be unlike. Greater care in keeping it thoroughly stirred secured a curve which was very nearly a straight line.

The following results were finally obtained: —

Resistance at 0° C. = 3.680.
Average change in R for 1° = 0.01428.
Coefficient of R compared with R_0 = 0.00388.
Resistance at 20° = 3.967.
Coefficient of R compared with R_{20} = 0.00360.

The observations were carried only a little above 100°, but as all the other results above that point had indicated a constant temperature coefficient, it was thought that no error would be introduced by not carrying the determinations further.

The diameter of the wire was measured with one of the microscopes, but it was found impossible to focus on the exact edge so as to make the determination certain. The diameter was therefore afterwards measured with a micrometer wire gauge and found to be 0.40 mm. The density, as computed from this diameter and the weight of a measured length, was found to be 7.785. The density was also determined by weighing a coil of the wire in water from which the air had been exhausted by an air pump, and also by weighing it in water in which the coil had been boiled so as to free it from air. The average density found from three determinations made in this way was 7.775. As this differed by only about one-tenth of one per cent from the result obtained from the micrometer measurement, that measurement was taken as the correct one.

Considerable difficulty was found in securing a uniform temperature in different parts of the glass tube which contained the wire. The helix was made at first of German silver wire, and at the lower temperatures seemed to give quite satisfactory results. But in some of the preliminary experiments a strong current was sent through the helix and it blackened unevenly, after which it was found that there were large variations in temperature between those parts of the tube where the wire was darkest and lightest. The helix was removed and a fresh one substituted, with the expectation that by using currents of only moderate strength it could be kept in good condition. It began to turn black in places, however, even when the temperature inside the tube was only about 150°. The attempt was made to oxidize it uniformly by sending a strong current through the parts that were least blackened, but this was found impossible. An iron coil was then tried and proved to be much more satisfactory.

The coefficient of linear expansion with rise of temperature was determined for each kind of wire.

The results obtained were as follows: —

LARGE WIRE.

	I.	II.
Length of wire	=891.181,	889.105 mm.
Average elongation for 1°=	43.57,	43.31 Mic. Div.
Coefficient of expansion =	0.0000122,	0.00001218.

Average coefficient of expansion = 0.00001219.

SMALL WIRE.

Length of wire = 893.0 mm.
Average elongation for 1° = 42.28 Mic. Div.
Coefficient of expansion = 0.00001183.

Arrangement of Apparatus.

After a number of preliminary experiments to determine the best arrangement of apparatus, that shown in Fig. 1 was finally adopted.

AB is a movable iron bed of the comparator, resting on the support *CD*. *GH* is the glass tube, resting on wooden blocks, and having openings at *E* and *F* through which the readings were taken by the microscopes *M* and *N*. The wire was fastened firmly to a wheel at *R*, passed over a grooved block of wood at *K*, through the glass tube *GH*, over the wheel *L*, to the weights suspended at *Q*. The wheel *L* was taken from an Atwood machine; the axle of the large wheel rests at each end between two smaller wheels. A pasteboard tube protected the portion of the wire between the glass tube and the wheel from currents of air. Only that portion of the wire between *E* and *F* was tested, but when the portion outside the glass tube was unprotected, drafts of air striking this outside portion had so much effect in changing the temperature of the part between *E* and *F* that no reliable results could be obtained. The wires for the current, and for measuring the resistance, were attached near *G* and *H*.

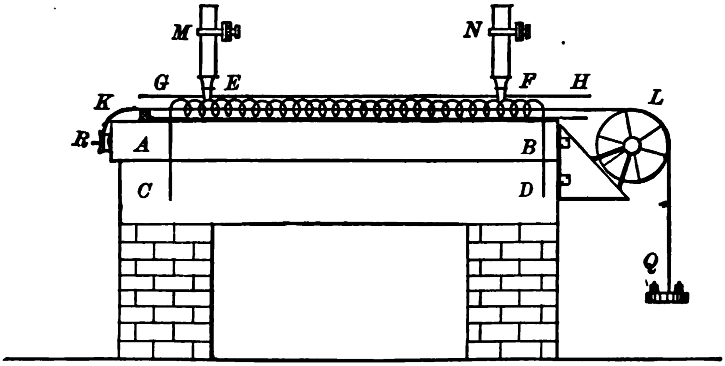

Fig. 1.

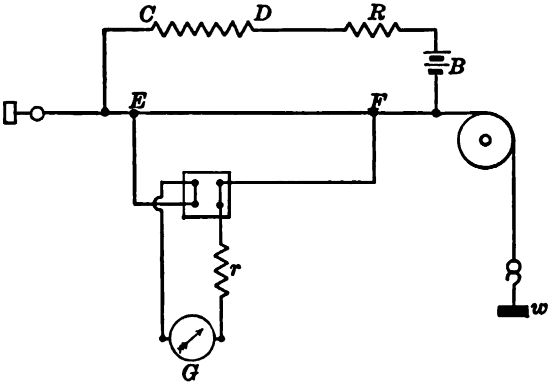

Fig. 2.

The plan of connections for making the resistance measurements is given in Fig. 2, the fall of potential method being used.

The portion *EF* of the wire to be tested, was in circuit with the battery *B*, the resistance *R* being adjusted so as to give the desired temperature. For some of the experiments in which a weaker current was desired, the circuit was closed through *R*, and *EF* connected in parallel with such a portion of *R* as would give the desired strength of current. *EF* was connected in parallel with the galvanometer *G*. The resistance box *r*, in the galvanometer circuit, was for the purpose of reducing the deflection, and also to make the resistance so large that the current in *EF* would not be appreciably diminished by closing the galvanometer circuit. *CD* is a compensated resistance in the circuit *EF*. By putting it in parallel with the galvanometer instead of *EF*, and comparing the deflections, the resistance of *EF* could be determined.

Considerable time was spent in finding what weights would give the most concordant results. Those used at first were too small to keep the wire horizontal; when weights were added to determine the elongation, the wire was drawn out of focus and the microscopes could not be accurately set. It was finally found that by using 2.6 kg. for the smallest weight, and increasing it twice by 0.4 kg. the wire could be kept in focus, and the elongations were such as could be read accurately and conveniently; a greater elongation would have required setting the microscopes nearer the edge of the field of view than could have been done with precision.

For most of the determinations two sets of readings were taken, each set consisting of the changes in length produced by adding 0.4 kg. twice, and removing the same weight twice, thus making the result the average of eight determinations. When the wire was heated, care was taken to see that it returned very closely to the original length when the weights were removed; if there was much variation it was concluded that the temperature had varied and the set was discarded. In many cases, however, it was found that the average results of these discarded sets would be almost identical with those obtained with constant temperature, since if the change was regular the effect upon the elongation with increasing weights would be balanced by the effect upon the contraction with decreasing weights. As it was impossible to tell when the change was regular and when it was irregular, the only safe way seemed to be to discard such sets of readings entirely. Since a change of one-tenth of a degree produced an elongation which was about the same as the difference in the elongation for 0.4 kg. made by a change of 40°, a very steady temperature was necessary to secure reliable results.

For each measurement the microscope was set twice and the average reading taken; if the readings differed from one another by as much as one or two divisions, more observations were made. The weights were generally placed in position a minute or two before the readings were taken. A comparison of readings made as quickly as possible after placing the weights, with those obtained after waiting a minute or two, showed more consistent results in the latter case. The aim was to make the conditions of determining the modulus as nearly identical in all cases as possible, so that the results at different temperatures might be strictly comparable.

In order to bring it to the "state of ease," each wire was stretched for 24 hours or longer by a weight somewhat greater than the maximum weight to be used in determining the modulus, before any observations were made.

The formula used in computing the modulus is

$$M = \frac{P \cdot L}{Q \cdot l},$$

where $P =$ weight in kg.,
$L =$ length of wire,
$Q =$ cross-section,
$l =$ elongation.

In computing M the length has been expressed in millimeters and the cross-section in square millimeters. Since the value of g in Ithaca is 980, the results given in the following tables must be multiplied by 980×10^6 in order to reduce to C.G.S. units.

Experiment 1.

DATA FOR CURVE A, FIG. 3.

Number.	T	Elongation for 0.4 kg.	No. of obs.	M
1	20.0	485.15	16	23388
2	42.5	490.35	14	23143
3	55.9	492.35	8	23046
4	65.9	495.50	4	22900
5	83.0	500.38	8	22676
6	103.5	513.15	10	22112
7	122.0	519.25	4	21852
8	147.0	539.50	9	21032
9	164.5	525.10	6	21609
10	200.0	550.75	6	20603

$L = 891.181$ mm. $Q = 0.12566$ sq. mm.

In the above table are given the elongations, expressed in divisions of the micrometer, for a weight of 0.4 kg. in the case of the first wire tested. The number of observations used in obtaining each result is also given, together with the corresponding values of the modulus. The corresponding curve is marked *A* in Fig. 3.

The higher temperatures were obtained, in this case, by means of a current through the helix surrounding the wire. Before any observations were made, the wire had been heated and cooled a great many times in making the preliminary experiments and in determining the coefficient of expansion. The annealing effect of this heating is probably the reason why the modulus is higher than was found afterwards for other pieces of the same kind of wire.

The curve shows plainly a decrease of elasticity with increasing temperature, and that the decrease is nearly proportional to the change in temperature.

Experiment 2.

Another piece of wire was taken from the same coil and heated as the first had been, by a current through the helix around the glass tube. The points marked with a cross near curve *B*, in Fig. 3, are those corresponding to the results of this experiment. The line marked *C* in the same figure is the same as *B*, only values for the modulus at nearly the same temperatures have been averaged instead of being plotted separately. The line is nearly straight, showing a decrease of elasticity which is proportional to the increase of temperature. The inclination of the line is very nearly the same as that of curve *A* up to about 70°; the slight change in the inclination of the first curve above that

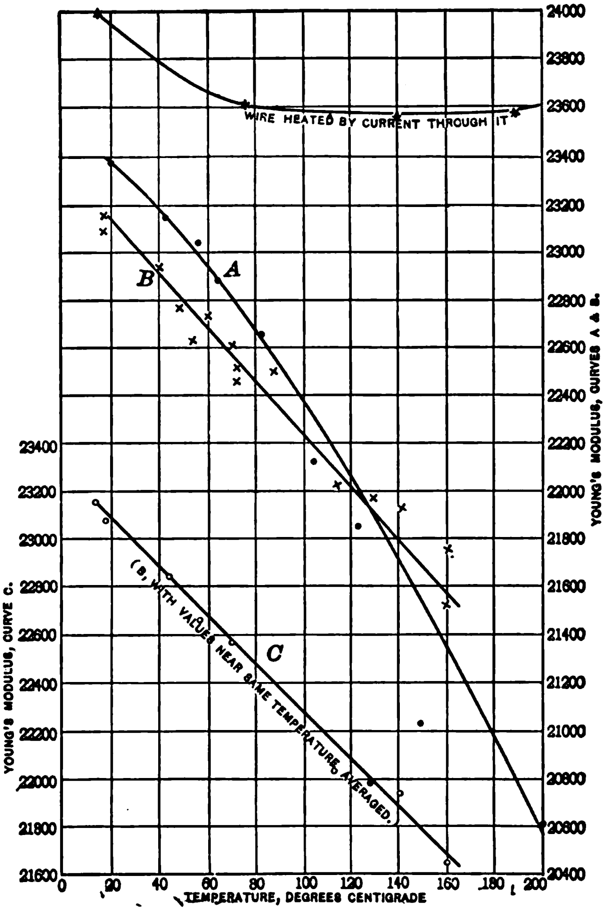

Fig. 3.

temperature is probably due to lack of accuracy in the determinations at the higher temperatures.

The first measurements in this experiment were made after the wire had been stretched so as to bring it to the "state of ease," but before it had been heated at all. The final result at about the same temperature showed an increase of about three-tenths of one per cent, doubtless due to the annealing effect of heating the wire.

EXPERIMENT 3.

The same wire was used as in the previous experiment, but this time it was heated by a current through the wire itself. The table gives the elongations and the corresponding moduli.

DATA FOR POINTS MARKED WITH A STAR IN FIG. 3.

Number.	T	Elongation for 0.4 kg.	No. of obs.	M
1	30.9	484.88	8	23344
2	16.5	471.9	10	23986
3	78.0	479.5	12	23606
4	140.0	480.38	8	23562
5	190.0	480.33	6	23565

$L = 889.105$. $Q = 0.12566$ sq. mm.

The first current, which heated the wire to about 31° as determined by the expansion, gave a higher modulus than had been obtained at the temperature of the room even after the annealing effect of a temperature of 160°. After the first determination, the current was increased so much that the wire broke. After it had again cooled, a still higher value of the modulus was found. It was then heated to different degrees by a current through it. The value of the modulus was somewhat less than when the second result in the table was found, but was still nearly two per cent greater than the greatest value that had been found before the current had been sent through it at all. The points marked with a star in Fig. 3 are those corresponding to this experiment.

The results seemed to indicate that the effect upon the elasticity due to heating the wire by a current around it and through it are very different, and that in the latter case the elasticity is considerably greater than it would be if the wire were heated to the same temperature by the former method.

Experiment 4.

The results obtained in the previous experiment were so unexpected that it seemed desirable to verify the phenomena. Another piece of the same kind of wire was accordingly adjusted. After the modulus had been determined at the temperature of the room, a weak current was sent through the wire; the modulus was found to be a little higher than before. The wire was then allowed to cool, and the modulus determined at the temperature of the room; it was found to be still higher. Currents of varying strength were then sent through the wire, and the modulus determined at the different temperatures. The values found were all very nearly alike.

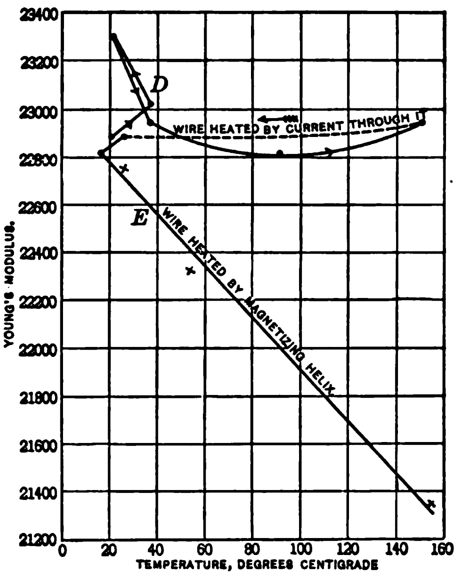

Fig. 4.

The curve marked *D* in Fig. 4 represents the results of this experiment. The temperatures are quite uncertain, as the adjustments for measuring the electrical resistance were incomplete, and it was found very difficult to determine the higher temperatures by the elongation. The change in length was rapid when the current was started, and was so large that the mark was carried a distance several times as great as the field of view.

There was no doubt, however, that there was a wide range in the temperature, and that the highest was 150° C. or above.

The results seem to indicate that the magnetizing effect of a current through the wire increases the modulus by an amount which very nearly balances the decrease that would be produced by the same increase of temperature secured by a current around the wire.

In order to see whether there had been such a permanent change in the molecular condition of the wire as would make the effect of heating it by a current around it different from what had been found in the other pieces, the modulus was determined at several temperatures secured by a current through the helix. The resulting curve, marked *E*, is shown in the same figure with the other results for the sake of comparison. The modulus is seen to decrease with increasing temperature, and the inclination of the line is very nearly the same that had been found in the previous experiments, as will be seen by comparing it with the lines given in Fig. 3.

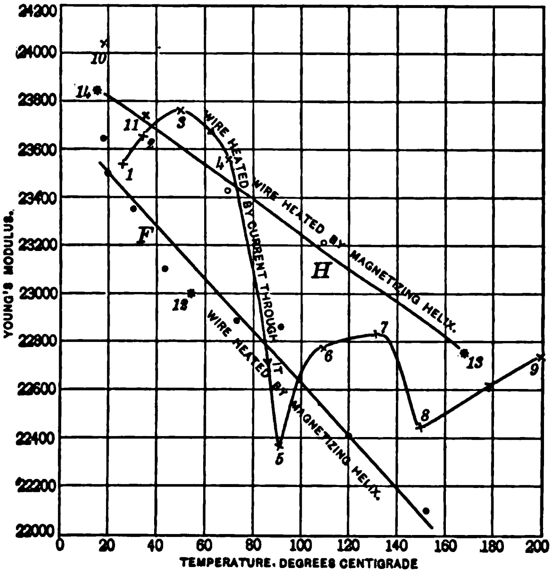

Fig. 5.

EXPERIMENT 5.

In this case a smaller wire was used. It was a piano wire having a diameter of 0.26 mm. After the modulus had been determined at the temperature of the room, the wire was heated to different temperatures by a current around it. The line marked *F* in Fig. 5 gives the curve for the relation of the modulus to the temperature. As in the case of the larger wire it is a straight line, and the inclination is almost identical with that found in a similar way in the preceding experiments. In this experiment, as in several others, the last value was found at the temperature of the room, after the other determinations had been made, in order to see how much effect the heating had produced upon the elasticity. It was found in every case to be a little increased; in this case the increase was about 0.6 per cent. With this wire the smallest weight was 1 kg., and 0.2 kg. was added twice and removed.

EXPERIMENT 6.

The wire used in the preceding experiment was heated by a current through it.

In Fig. 5 the points marked with a cross correspond to the results; they are numbered and a line drawn with arrow heads to show the order in which the observations were made. All except number 10 were obtained when a current was flowing through the wire. The temperatures are uncertain, and the results seem so irregular that there is hardly warrant for drawing conclusions from them. A few facts seem to be indicated, however. Comparing 1, 2, and 3 with the last result of the preceding experiment, an increase of elasticity is shown with a weak current, which corresponds with the result found in every case with the larger wire. Comparison of 1 and 10 shows a permanent increase of elasticity from the current through the wire which is about twice as great as the permanent change from heating by the current through the helix.

The wire was so small that the temperature changed easily, and great difficulty was found in keeping it constant; the results finally obtained are less certain than in the case of the larger wire. At 74° and 92° the results are particularly uncertain, as the wire outside of the glass tube was not sufficiently protected from currents of air. After number 11 the current was again sent through the helix and numbers 12 and 13 obtained, while 14 was without a current. In Fig. 5 the corresponding points are marked with stars. Without much doubt 12 is incorrect. If 13 and 14 are compared, it will be seen that the line *H* between them has an inclination not very different from that of the line *F*; a line from 10 to 13 would be still more nearly parallel to the line *F*.

A comparison of 10 and 14 would indicate that the longitudinal magnetization has decreased the permanent effect of the circular magnetization, though the modulus is still somewhat higher than before the current had been through the wire at all.

EXPERIMENT 7.

As the results for the modulus at the same temperatures obtained with currents around and through the wire were so different, it seemed certain that the magnetization had effect upon the elasticity in one of these cases, if not in both. In order to test the matter it seemed necessary to eliminate the magnetization. For this purpose a double coil was placed on the glass tube, and so arranged that the current would traverse one part of it in one direction and the other in the opposite direction, thus making it non-inductive.

The two points marked with circles near the line *H* in Fig. 5 were obtained with this coil, and the same piece of wire used in the preceding experiments. One determination made at 42° gave a modulus which was considerably higher than any other value with this piece of wire, but no inference can be drawn from a single result which is at variance with the other results. The final result without a current was also higher than had been found previously at the temperature of the room. The results at 72° and 109° compared with the average value found at the temperature of the room show about the same relation of temperature to Young's Modulus as was found with the magnetizing coil. Whether the deviation of the result at 42° is due to an error of observation or to some other cause I cannot tell. The fact that the other values, and also numbers 13 and 14 found with the magnetizing helix, are higher than those on the curve *F*, is accounted for by the permanent increase of elasticity produced by the current through the wire.

A piece of the larger wire was next adjusted and heated by the non-inductive coil. The line *I* in Fig. 6 corresponds to the results found. As had been found with the magnetizing helix, the elasticity decreases in proportion to the increase of temperature, and the inclination of the line shows that the rate of decrease is very nearly the same.

From these results it seems reasonable to conclude that if longitudinal magnetization has any effect upon Young's Modulus, that effect is very slight.

EXPERIMENT 8.

Most of those who have determined Young's Modulus have suspended the wires vertically instead of placing them horizontally as I have done. In order to see if the friction of the pulley introduced an error, pieces of each kind of wire were suspended vertically in a cupboard that had been arranged for a Kater's pendulum. A board was placed in front and openings made through which the same microscopes used in the other determinations could be inserted. Care was taken to have the microscopes horizontal and to so place the weights that the wire would be in the same position with the lighter and heavier weights; some measurements taken without special care with regard to these points were so discordant as to be entirely untrustworthy. Incandescent lamps were used to give illumination for the microscopes. A glass door was between them and the wire, and a screen of paper with openings large enough to permit the illumination of the microscopes. Even with these precautions it was found necessary to turn on the lights for some time before taking observations, in order not to have the variations of temperature introduce an error. The variations during the sets of observations retained were not more than one or two tenths of a degree.

The following results were obtained :—

	Elon.	No. of obs.	L	P	M	M_H	Diff. M_H and M_V.
Large Wire,	1113.95	20	989.08	0.8 Kg.	22631	22950	1.39%
Small Wire,	1316.50	18	990.0	0.4 Kg.	22663	23337	2.88%

M_H is the average result of the determinations made for each kind of wire placed horizontally before they had been heated.

The results obtained in the horizontal position are so concordant that it seems quite certain that the effect of the friction of the wheel is always about the same per cent of the result for the same kind of wire, so that the results are as strictly comparable with each other as if obtained in the usual way.

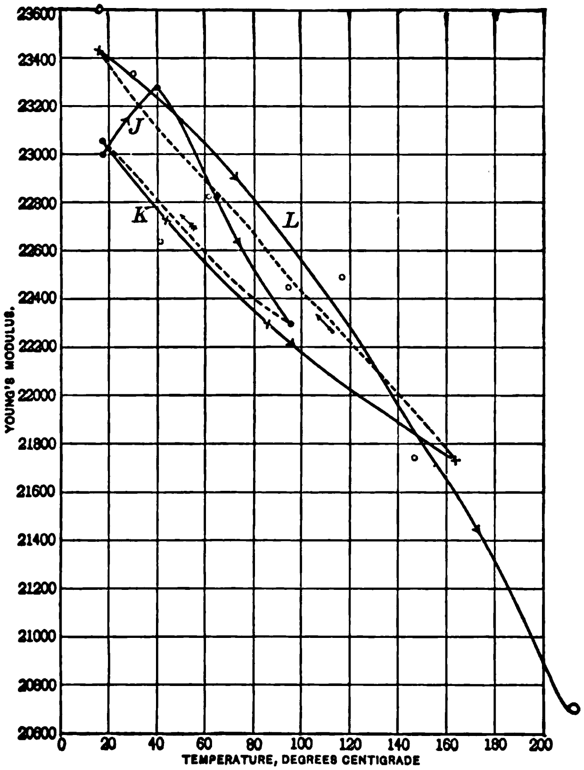

Fig. 6.

The fact that all of the results obtained are so high is probably because of the kind of wire; the best piano wires manufactured now have a modulus which is considerably higher than those usually given for iron or steel.

EXPERIMENT 9.

The same piece of large wire that had been used in the preceding experiment was placed in the tube and its modulus determined when heated by a current through it, the temperature being carefully determined. The following table gives the results obtained, and Fig. 6 shows the graphic representation of the determinations.

DATA FOR CURVES IN FIG. 6.

Number.	T	Elon. for 0.4 kg.	No. of obs.	M
1	20.0	491.02	8	22994
2	18.0	491.29	12	22981
3	18.8	491.19	8	22986
4	23.8	490.36	8	23025
5	40.2	485.06	8	23277
6	65.8	494.75	8	22821
7	95.1	506.69	8	22283
8	18.0	489.75	8	23053
9	43.1	495.75	8	22775
10	84.9	506.05	5	22311
11	145.0	515.36	7	21908
12	163.0	519.57	6	21735
13	17.3	482.25	4	23412
14	29.6	483.69	4	23342
15	40.8	498.50	6	22649
16	63.1	494.38	6	22838
17	95.8	502.70	10	22460
18	117.2	501.72	9	22503
19	152.3	519.94	8	21715
20	218.7	546.27	10	20668
21	17.7	478.41	8	23600

$L = 866.777$ mm. $Q = 0.12566$. $P = 0.4$ Kg.

The first determination was without any current; the second with a current so weak that it changed the temperature only a fraction of a degree; the third was again without a current. The three results agree within the limits of the errors of observation, showing neither temporary nor permanent effect from so weak a current. The next result, with a little stronger current, shows slight increase of elasticity; while the next, which raised the temperature to about 40°, shows a decided increase. The other two results of this series show a decrease quite similar to that obtained with a current around the wire, while number 8, which was without a current, shows only very slight permanent change from the heating. The next set, including 9 to 13, show regular decrease with increasing temperature, and a considerable permanent change. The last set, including the remaining observations, show a nearly regular decrease, with the exception of numbers 15 and 18; the former is too low and the latter too high to fall on the same line as the others. The permanent change as shown by the final result is considerably greater than before.

The results of this experiment seem to indicate that the effect upon the elasticity due to the *first* application of a current of given strength is different from that produced by a repetition of the current, especially with currents of such strength as to give a temperature in the neighborhood of 40°.

Experiment 10.

A piece of the smaller wire was used and heated by a current through it. The curve *M* in Fig. 7 represents the results graphically, the arrows indicating the order in which the results were obtained.

The increase of elasticity in the first set comes at a somewhat higher temperature than in the experiments with the larger wire, while a slight decrease is shown at 34°. The other two sets show a decrease of elasticity with increasing temperature, but not a regular decrease. There is a point of inflection in each line showing an increase in the modulus as compared with what would be expected at the temperature, and the point occurs at a higher temperature in the third than in the second set. The possibility of errors of observation is too great to be entirely sure that the positions of the points of inflection are correct; the departure from a straight line is greater, however, than was found in all but a very few of the experiments made when the wire was heated by a current around it; and the similarity of the two branches increases the probability that they are correct.

Experiment II.

Some of the results obtained for the modulus, when the wire was heated by a current through it, indicated the possibility that the first current sent through the wire had a different effect from subsequent currents. The object of this experiment was to test this subject, and also to see whether previous longitudinal magnetization would modify the effect of the current. Three sets of observations were made with the current always in the same direction, one with the current reversed, and one after the wire had been magnetized longitudinally. Of these five sets of observations the number of observations for the fourth and fifth sets was smaller than had been intended, since unfavorable circumstances made some of the results unreliable, and there was not time to repeat them.

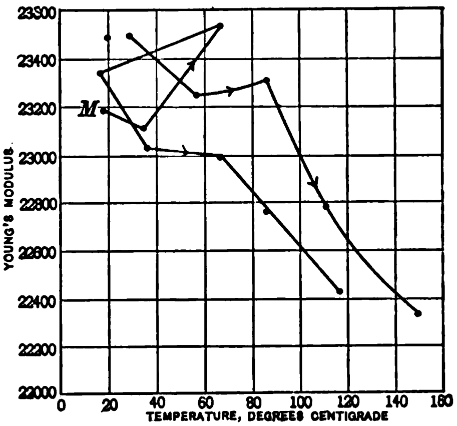

Fig. 7.

The lines in Fig. 8 represent the results. The first and second curves are somewhat similar, only the first does not descend as rapidly at first, and on the return curve shows a higher modulus with a weak current than after the current had ceased and the wire cooled. The first part of the third curve is like the corresponding part of the second; but with decreasing temperature some of the points are higher than at the same temperature on the first part of the curve, and as in the first curve, one is higher than after the wire had cooled.

The few results for the fourth line show no appreciable difference due to reversal of the current.

The fifth line descends less rapidly than the others, indicating that previous longitudinal magnetization may increase the effect that a current has upon elasticity. The results obtained in some of the earlier experiments with wires that had been repeatedly magnetized point toward the same conclusion.

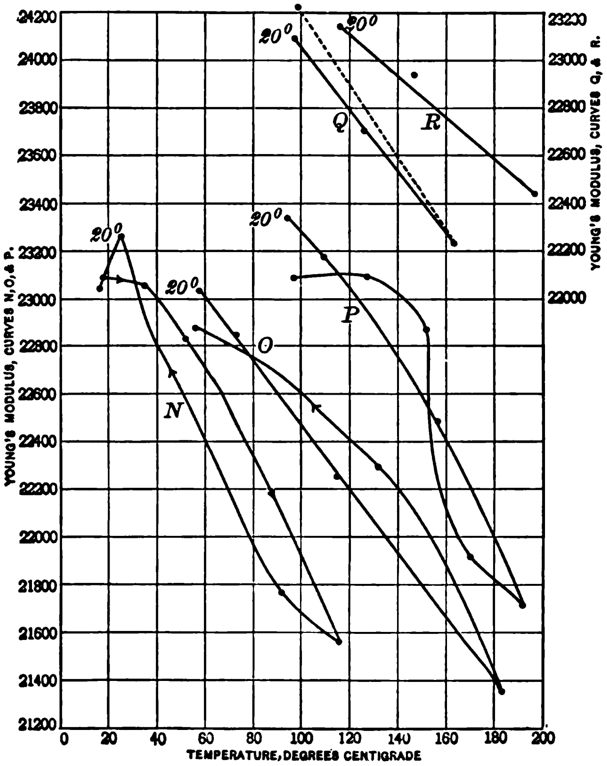

Fig. 8.

Conclusion.

The results of the experiments show plainly that the elasticity of a piano wire decreases as the temperature increases, with no indication whatever of the maximum that Wertheim supposed to exist between 100° and 200°.

As the decrease has been found to be proportional to the increase of temperature, the relation of Young's Modulus to temperature may be expressed by the formula, $M_t = M_0(1 - at)$.

To determine the value of a, a line was drawn which was the average of the results obtained when the wire was heated by a current through the helix; the modulus at zero degrees was obtained by producing the line. The decrease for 100° for the large wire was 5 per cent, or $a = 0.0005$. Hence the formula is $M_t = M_0(1 - 0.0005\,t)$. For the small wire, $a = 0.00046$.

This result is very nearly the same as that obtained by Kohlrausch and Loomis,[1] but is larger than the value given for a by some other experimenters.

Heating the wire up to about 200° increased the elasticity one or two per cent, the amount varying somewhat in the different experiments.

The various results obtained show that an electric current through a wire does perceptibly modify the elasticity, but that the effect varies with the conditions. The effect of the first current, which is strong enough to raise the temperature to 30° or 40°, was in almost all of the experiments to increase the modulus about one per cent. The heating effects of stronger currents generally decreased it again, but the irregularity of these results compared with the regularity of those obtained when the wire was heated the other way, show that an electric current must modify the elasticity. My results are not numerous or consistent enough to deduce the laws according to which the variations in the effect take place, but indicate that the effect depends to some extent on the previous treatment of the wire and on repetition of the current.

The results obtained with the non-inductive coil show that longitudinal magnetization has no appreciable effect upon elasticity.

This series of experiments was made in the Physical Laboratory of Cornell University, under the direction and with the assistance of Professor Edward L. Nichols.

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[1] Ann der Phys. u. Chem. 141, 480 (1870).

MINOR CONTRIBUTIONS.

ON MAGNETIC POTENTIAL.

BY FREDERICK BEDELL.

Introduction.

IN 1882, Professor Clausius[1] discussed at length the dimensional formulæ of the electrostatic and electromagnetic systems, and derived the dimensions for magnetic pole (m), from a consideration of the equivalence of a magnetic shell and a circuit carrying a current of electricity, as first pointed out by Ampère; thus,—

$$(1) \qquad \text{Area} \times \text{current} = \text{magnetic moment} = ml.$$

In the electromagnetic system, this gives a magnetic pole the dimensions $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$, which accords with Maxwell and subsequent writers. In the electrostatic system, however, this gives to magnetic pole the dimensions $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-2}$, for which Maxwell gives $M^{\frac{1}{2}}L^{\frac{1}{2}}$. The reasoning of Clausius was apparently correct, and he concluded Maxwell was in error. This conclusion was indorsed by Professor Everett.[2] The discussion[3] which followed "On the Dimensions of a Magnetic Pole in the Electrostatic System of Units" needs merely a mention here.

Professor J. J. Thomson pointed out that the equivalence of a circuit and a magnetic shell, as given by Ampère, is correct only in a medium with permeability equal to unity. The factor μ, denoting the permeability of the medium, should be introduced into the general statement, and (1) should be written—

$$(2) \qquad \mu \times \text{current} \times \text{area} = \text{magnetic moment}.$$

In the electrostatic system μ has the dimensions $M^0L^{-2}T^2$; hence m has

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[1] R. Clausius, "Ueber die verschiedenen Maassysteme zur Messung electrischer und magnetischer Grössen," Verhandl. des naturlust. Vereins des Preuss., Rheinlande und Westfalen, Vol. XXXIX., 1882; also, Philosophical Magazine, June, 1882, p. 381.

[2] J. D. Everett, Philosophical Magazine, May, 1882, p. 376.

[3] Philosophical Magazine, 1882,—

J. J. Thomson:	Vol. XIII., p. 427.	R. Clausius:	Vol. XIV., p. 124.
J. Larmor:	Vol. XIII., p. 429.	J. J. Thomson:	Vol. XIV., p. 225.
J. D. Everett:	Vol. XIII., p. 431.	O. J. Lodge:	Vol. XIV., p. 357.
C. K. Weak:	Vol. XIII., p. 530.	E. B. Sargent:	Vol. XIV., p. 395.

the dimensions $M^{\frac{1}{2}}L^{\frac{1}{2}}$, as given by Maxwell. In the electromagnetic system, permeability is a number without dimensions, and the presence of μ as a factor in (2) does not affect the dimensions of m. Professor Thomson derived these same dimensions from the expression for the work done in carrying a pole around a circuit. He also pointed out that the factor μ should enter into the expression for the attraction between magnetic poles; thus, —

$$f = \frac{mm'}{\mu r^2}. \tag{3}$$

Professor Clausius, however, still considered that he was correct, maintaining that, if a circuit and magnetic shell are equivalent, they must be mutually replaceable, and that a change in the medium would affect one as the other. Professor Everett subsequently concluded that Maxwell was correct, obtaining the dimensions of Maxwell from the galvanometer law and the law of Faraday, although he did not see wherein Clausius was wrong in obtaining his results from the equivalence of a circuit and a magnetic shell. Helmholtz,[1] too, showed lack of confidence in the equivalence of the magnetic shell, and stated that the Ampère-Weber hypothesis must be looked upon as neither verified or even completely worked out. He gave no consideration to the nature of the medium, which, as Professor Thomson pointed out, has a most important bearing on the question. It was left for Dr. Lodge to point out that a distinction should be made between the permeabilities of the medium of the space corresponding to the space of the equivalent shell and of the exterior medium. It was he who pointed out that the magnetic moment of a circuit is equal to the product of the current, the effective area, and the permeability of the *interior* space; the magnetic moment is entirely independent of the permeability of the surrounding space. The failure to recognize this fact was the cause of the erroneous views which had been set forth by such distinguished scientists.

It is the purpose of this paper to consider the effect of permeability upon the magnetic potential due to a magnet or a current of electricity, and to show more definitely the bearing of some of the above relations.

The Dimensions of Magnetic Potential.

The expressions for the magnetic potential at a point, as ordinarily given, are

$$\text{magnetic potential due to a magnet} = V = \Sigma\frac{m}{r}; \tag{4}$$

$$\text{magnetic potential due to a current} = V = I\Omega. \tag{5}$$

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[1] Von Helmholtz, Wiedemann's Annalen, 1882, No. 9, p. 42; also, Philosophical Magazine, Vol. XIV., p. 430.

In (5) I denotes the current, and Ω denotes the solid angle subtended by the circuit at the point, the potential of which is under consideration.

In the electromagnetic system these equations lead to the following dimensions for magnetic potentials: —

$$(6)\ [\text{from } (4)] \qquad V=\frac{m}{r}=\frac{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}{L}=M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1};$$

$$(7)\ [\text{from } (5)] \qquad V=I\Omega=M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}.$$

These equations (6 and 7) agree with each other and with the dimensions for magnetic potential in the electromagnetic system as usually given, and will be found correct.

In the electrostatic system, equations (4) and (5) give the dimensions:

$$(8)\ [\text{from } (4)] \qquad V=\frac{m}{r}=\frac{M^{\frac{1}{2}}L^{\frac{1}{2}}}{L}=M^{\frac{1}{2}}L^{-\frac{1}{2}};$$

$$(9)\ [\text{from } (5)] \qquad V=I\Omega=M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}.$$

These equations (8) and (9) are not concordant, although derived from (4) and (5), which gave correct results in the electromagnetic system. Equation (9) will be found to be correct; equations (8) and (4) are not.

If we denote the permeability of the space *exterior* to the magnet by μ_e, instead of (4), we should have

$$(10) \qquad V=\Sigma\frac{m}{\mu_e r},$$

which accords with (3), and is true in any system of units.

In the electromagnetic system the dimensions of μ are $M^0L^0T^0$; hence μ does not affect the dimensions in this system, and (6) and (7) are correct.

In the electrostatic system the dimensions of μ are $L^{-2}T^2$. Hence the dimensions of magnetic potential derived from (10) are: —

$$(11) \qquad V=\frac{m}{\mu_e r}=\frac{M^{\frac{1}{2}}L^{\frac{1}{2}}}{L^{-2}T^2L}=M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2},$$

which agrees with (9), and is correct.

In a Uniform Medium the Magnetic Potential due to a Current is Independent of Permeability.

The introduction of μ in the expression for the magnetic potential due to a magnet (4) gives the same dimensions to magnetic potential (see 11), as derived from (5), the expression for the magnetic potential due to a current.

That the permeability of the medium should be taken into consideration, as above, in the expression for the magnetic potential due to a magnet is clear; but should we not expect the permeability to enter in a similar manner in the corresponding expression for the magnetic potential due to a current (see 5)? The equivalence of a current and a magnetic shell as stated by Ampère would lead to this conclusion. Such, however, is not the case; equation (5) holds true as it is written. The magnetic potential at a point due to a current is independent of the permeability, — a fact the explanation of which merits attention.

In *air*, the magnetic potential at a point due to a current may be obtained from the principle of the magnetic shell as follows. Consider the circuit replaced by a shell consisting of elementary magnets, with axes in the direction of the thickness of shell. If r and r' are distances of the point P from the two poles of one of these elementary magnets, of length l and pole strength dm, the magnetic potential at the point P, due to this magnet, is

$$dV=\frac{dm}{r}-\frac{dm}{r'}. \tag{12}$$

Inasmuch as the length of the magnet is small compared with its distance from the point P, for r' we may write $r+l\cos\alpha$, where α is the angle between the axis of the magnet and a line connecting it with the point P. Hence

$$dV=dm\left(\frac{1}{r}-\frac{1}{r+l\cos\alpha}\right)=\frac{l\,dm\cos\alpha}{r^2}, \tag{13}$$

the last term being obtained by approximation. But from the equivalence of a magnetic shell in air (see 1) we have

$$l\,dm=I\,dA; \tag{14}$$

where dA represent the polar area of an elementary magnet. Making this substitution in (13), and remembering that $dA\cos\alpha\div r^2=d\Omega$, the solid angle at P subtended by the polar surface of the elementary magnet, we have

$$dV=I\,d\Omega;\text{ and }V=I\Omega. \tag{15}$$

This reasoning holds true for air; but (12), (13), (14) are true only for air, or a medium whose permeability is unity.

For *media other than air* equation (15) still holds. If μ_e is the permeability of the space *exterior* to the space occupied by the magnetic shell to which the current is considered equivalent, instead of (12) and (13), we have [see equation (10)]: —

$$dV=\frac{dm}{\mu_e r}-\frac{dm}{\mu_e r'}=\frac{1}{\mu_e}\frac{l\,dm\cos\alpha}{r^2}. \tag{16}$$

If μ_i is the permeability of the *interior* space, or that occupied by the equivalent magnetic shell, instead of (14), we have

$$ldm = \mu_i I dA. \tag{17}$$

Substituting (17) in (16), we have

$$dV = \frac{\mu_i}{\mu_e} \frac{I dA \cos\alpha}{r^2} = \frac{\mu_i}{\mu_e} I d\Omega. \tag{18}$$

$$V = \frac{\mu_i}{\mu_e} I\Omega. \tag{19}$$

This is the expression for the magnetic potential at a point P, due to a current I, in a circuit subtending the angle Ω at the point P, when μ_i is the permeability of the space occupied by an hypothetical equivalent shell, and μ_e is the permeability of the surrounding medium.

In all cases usually considered $\mu_i = \mu_e$; that is, in any uniform medium, the magnetic potential due to a current is

$$V = I\Omega, \tag{20}$$

and is independent of the permeability of the medium.

A METHOD FOR THE STUDY OF TRANSMISSION SPECTRA IN THE ULTRA-VIOLET.

BY ERNEST NICHOLS.

THE method to be described is a very simple one, and has long been in use in other branches of spectrum work. It is to photograph, side by side, two spectra arising from different portions of the same slit; one a spectrum of sunlight, the other a spectrum of sunlight changed by passing through a layer of the substance to be examined. The apparatus used in the present study consisted of a Rowland concave grating with a 2 m. radius of curvature, and 568 rulings to the millimeter; a narrow slit 4 cm. long; a heliostat with a silver mirror, and a dark box or camera. The grating was placed in the path of a beam of sunlight sent through the slit in the window shutter from the heliostat; and so adjusted that the image of the slit, due to the grating mirror, was reflected back upon the slit, and was of the same size as the slit itself. According to the approximate law for the grating, the spectrum should come to focus on the circumference described on the line connecting the center of the grating and the middle of the slit as a diameter. The film holder in the camera was warped to form an arc of this circle. The camera was further provided with a sliding front. After the camera had been adjusted so that the whole visible spectrum was in focus on a part of the film holder, a flexible celluloid film, 20 × 5 cm. was slid into the holder where the extreme violet and ultra-violet rays would fall upon it during the exposure. To give a negative of anything like equal intensity throughout its length a differential exposure was required. The film was exposed by sliding back the front of the camera, exposing first the extreme ultra-violet and in succession the longer wave-lengths, until a region near the G lines was reached; the motion was then reversed and the opening gradually closed. An exposure of from 30 to 40 seconds for the shortest wave-lengths, and from 2 to 4 seconds for the region of the G lines, was found, under the conditions of the present study, to give the best results. But little store can be set by such values, for the requisite exposure varies inversely as the width of the slit, the reflecting power of the heliostat mirror and grating, and the rapidity of the films; and varies directly as the dispersion. The ratio of the exposure required by the two extremes of this region under ordinary conditions should, however, be about as 16 to 1. Plates of the transparent solids to be examined were brought up against the back of the slit in such a way as to cover the lower half of it. In this way two spectra were brought together, side by side, in the camera. The spectrum from the upper half of the slit was the record of unchanged sunlight, and that from the lower half, of sunlight which had passed through the substance under examination. Solutions were examined in a glass cell equal in depth to the length of the slit, and filled half full of the solution to be studied. The only difference between the two spectra thus obtained in the camera, must be due to the characteristic absorption of the solution in question. The estimation of wave-length in the ultra-violet spectrum, which might otherwise require more or less complicated machinery, is very easily and accurately made by reference to the Fraunhofer lines. In the present study, wave-lengths were written down in accordance with the photographs of the solar spectrum made by Professor Rowland in 1888. The method gives quantitative results of only the roughest sort, for quantitative measurements must depend upon the estimation of depth of shade, which it is always difficult to determine with exactness; and behind this lies the uncertain relation between the change produced in silver salts and the amount of light received by them. Fortunately, in work of this sort qualitative results are the more important, and quantitative results, on account of their dependence on the thickness of the layer, or upon the degree of concentration of the solution under examination, are only secondary. The present paper contains the results from the examination of thirty-eight different substances, twelve of which showed more or less selective absorption in the region studied. The results appear in Tables I. and II. Among the substances showing selective absorption were several kinds of glass, a sample of white mica, and solutions of uranium nitrate and uranium acetate, and a fluorescent solution obtained in the charring of sucrose with strong sulphuric acid.

Reproductions of the photographs of a number of these spectra are given in the accompanying plate.

TABLE I.

SOLUTION.

(Thickness of layer, 1 cm.)

Substance.	Concentration.	Region studied.		Transmission.
		λ=(in μ)	to λ=	
Water (distilled)	—	0.41	0.31	Absorption very slight and not selective.
Alcohol (absolute)	—	—	—	Same as Water.
Lithium chloride	2 grams in 20 cc. water	—	—	Same as Water.
Potassium chloride	2 grams in 20 cc. water	—	—	Same as Water.
Sodium chloride	2 grams in 20 cc. water	—	—	Same as Water.
Strontium chloride	2 grams in 20 cc. water	—	—	Same as Water.
Thallium chloride	1 gram in 20 cc. water	—	—	Same as Water.
Indium chloride	¼ gram in 20 cc. water	—	—	Same as Water.
$(NH_4)_2Al_2(SO_4)_4$	Saturated at 0°	—	—	Same as Water.
$K_2Al_2(SO_4)_4$	Saturated at 0°	—	—	Same as Water.
$(NH_4)_2Fe_2(SO_4)_4$	Saturated at 0°	—	—	See Fig. 1.
Iodine in CS_2	½ gram in 20 cc. CS_2	0.43	0.325	Total from .43 to .386; decreases to .377; None from .377 to .325.
Chlorophyll in alcohol	—	0.4	0.325	None.
Fluorescent substance[1]	—	0.45	0.30	Absorption slight up to λ = 0.40. Then gradually increases as λ becomes less.
Uranine	Weak solution in water	0.4	0.325	Nearly total, absorption uniform.
Uranine	Stronger solution in water	0.4	0.325	Absorption considerable, but uniform.
Fluorescein	—	0.4	0.325	Practically same as Uranine.
Stramonium seeds	Alcoholic solution	0.4	0.325	None.
Uranium nitrate	4 grams in 20 cc. water (saturated)	—	—	See Fig. 2.
Uranium acetate	20 cc. water 20 drops HCl	—	—	See Fig. 3.

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[1] Obtained by charring sucrose with strong sulphuric acid.

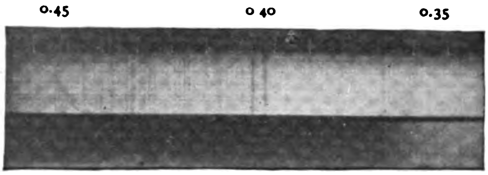

Fig. 1.

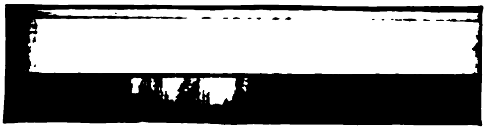

Fig. 2.

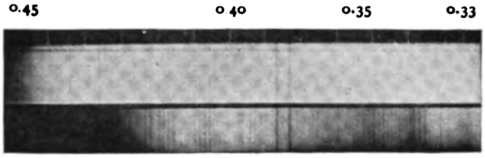

Fig. 3.

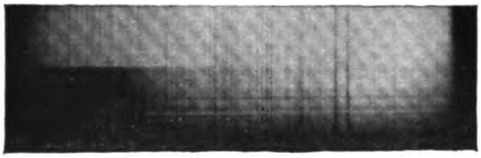

Fig. 4.

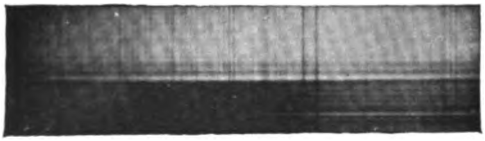

Fig. 5.

ERNEST NICHOLS: ULTRA VIOLET ABSORPTION.

Table II.

SOLIDS.

Substance.	Thickness (in mms).	Region studied.		Transmission.
		$\lambda=\mu$ t	o $\lambda=\mu$.	
Rock salt	6.0	0.4	0.31	Total.
Glass (common window)	2.4	—	—	Absorption complete from $\lambda=0.325$ on.
Mica (white)	0.07	—	—	Absorption begins at $\lambda=0.33$ and becomes complete at $\lambda=0.30$.
Quartz	3.2	0.4	0.30	Total.
Gypsum	3.6	0.4	0.30	Total.
Alum (solid)	3.2	0.4	0.30	Total.
Fluorspar	3.5	0.4	0.30	Total.
Plate glass	8.0	0.4	0.32	Like common glass above.
Flint glass	3.5	0.4	0.31	Like common glass above.
Black glass	2.0	0.4	0.30	None.
Red glass	2.0	0.4	0.30	None.
Brown agate	3.1	0.4	0.30	Uniform, nearly opaque.
Uranium glass	31.0	0.4	0.30	None.
Tourmaline (blue)	5.9	0.4	0.31	None.
Tourmaline (green)	5.0	0.4	0.31	None.
Cobalt glass	2.0	—	—	See Fig. 4.
Dydinium glass	7.5	—	—	See Fig. 5.
Calcite[1]	10.0	0.4	0.31	Total.

[1] Plate cut perpendicular to optic axis.

Colgate University, April and May, 1894.

The Photography of Manometric Flames.

By William Hallock.

WHILE occupied with a research upon the use of the nasal cavity in singing, which had been suggested by Dr. Floyd S. Muckey, and was largely carried out by him, it became desirable to record our results by photographing the manometric flames which we used.

For the attempted analysis of the voice, with and without the use of the nasal cavity, we arranged a set of König resonators, each having a manometric capsule in the back of it, connected by a rubber tube to the series of small gas burners, situated one above the other. The resonators were tuned to Ut$_2$ and its first seven over-tones, Ut$_3$, Sol$_3$, Ut$_4$, Mi$_4$, Sol$_4$, "number 7," and Ut$_5$. The whole arrangement was identical with the familiar apparatus of König.

The flames being viewed in a rotating mirror appeared as a bright line when that particular resonator was not responding, and as a wavy line when the air in the resonator vibrated and caused the membrane of the manometric capsule to vibrate also, and the flame to jump up and down.

It is impossible to photograph what one sees in the rotating mirror on account of the irregularities of the mirror and its rotation, hence some other method of recording the vibration of the flame must be devised. Professor Merritt[1] has accomplished this by causing the sensitive plate to travel across the back of the camera, so that at each vibration the image of the flame would fall upon a new part of the plate, and thus be continuously recorded. It might be possible to cause the lens to move across the front of the camera, and thus accomplish the same result.

In our case a camera was mounted upon a piece of board, which was pivoted approximately under the optical center of the lens, thus enabling us to rotate the camera about the center of the lens. When the camera is so rotated, the image of any object in front of it will traverse the plate, and be photographed as a line, or otherwise, as the case may be. A large black screen was so arranged on the stationary part of the apparatus that the flames, and the flames only, were visible to the camera through a slit about an inch wide in the screen, directly in front of the camera in its central position.

The method of operating was as follows. The camera was first focused upon the flames. The cap may be left off all the time, and no stop is needed. The plate-holder was then placed in the camera, which was directed toward the black screen on one side of the slit. When all was ready, and the flames were behaving properly (as seen in the rotating mirror), the slide was withdrawn from the plate-holder and the camera quickly rotated through about 90 degrees, until it pointed toward the screen on the opposite side of the slit; the slide was then replaced in the holder, and the plate was ready for development. The eight flames of our apparatus recorded themselves either as straight lines or as wavy lines, according as their resonators responded to overtones or not. The record was made while the camera was pointed toward the flames through the slit in the screen. Springs were arranged to rotate the camera at definite and constant velocities, but it was found that it could be rotated with the hand quite well enough. When a poor lens (not rectilinear) is used, or great accuracy is necessary in the comparison of the number of vibrations in any two notes, two momentary exposures of the plate to the flames, with the camera at rest, and so stationed that the images fall once on one edge of the plate and again on the opposite edge, will give the fiducial points from which to count, eliminating any distortion due to the lens. By this means very common lenses may be used, with very large apertures.

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[1] Ernest Merritt, Physical Review, Vol. I., p. 166, 1893.

Heretofore it has been considered necessary to increase the actinic power of the little flames by surrounding them with oxygen, or other means. We found this unnecessary if we took the precaution to turn up the flames until a characteristic bright point appeared just at the top of the blue. We used an old 4 × 5 camera with a fair 5 × 7 lens, and Cramer's "isochromatic C." dry plates with ordinary developer. A good negative was obtained in this way with a lens costing only fifty cents. This lens was plano-convex, with a diameter of three inches and an eight-inch focus, and was used with full aperture.

PHYSICAL LABORATORY, COLUMBIA COLLEGE,
NEW YORK, August, 1894.

NEW BOOKS.

A Treatise on Astronomical Spectroscopy: being a translation of Die Spectralanalyse der Gestirne. By Professor Dr. J. Scheiner. Translated, revised, and enlarged by Edwin Brant Frost. 8vo, pp. 482. Boston, Ginn & Co., 1894.

When the German original of this work appeared in 1890, the thought must have occurred to many that rarely had any book come so perfectly in season. A new science had recently arisen by differentiation from astronomy and physics. An immense quantity of observations, of all degrees of value, had been made, and then recorded in journals of every character. Theories, often of most contradictory natures, had been published. What was needed was a man competent and willing to reduce the observations and to discuss the theories. Scheiner tried to do this and more; and his book has been of untold usefulness to the scientific world. In the past four years, however, material has been accumulating at an enormous rate, especially in this country; and the time has come for a revision of Scheiner's treatise. It is most gratifying to American scholarship to realize that this work was undertaken by a professor of Dartmouth College, Mr. Edwin B. Frost, and that in his revision the amount of new matter introduced entitles the book to be called an enlargement as well.

Professor Frost's work is, in the highest degree, satisfactory. His arrangement of the subjects is the same as Scheiner's. The book is divided into four parts. Part I. treats of spectroscopic apparatus. Part II. gives a discussion of Kirchhoff's Law and Doppler's Principle, as these subjects were understood in 1890. Part III. deals with the results of spectroscopic observations of the sun, the planets, comets, nebulæ, the stars, and the aurora. There is also a chapter on the displacements of spectral lines. Part IV. contains various spectroscopic tables, notably Rowland's new table of standard wave-lengths and Kayser and Runge's lists of iron lines. The bibliography, one of the most important features of Scheiner's original work, is extended so as to include all publications through the year 1893. The changes and additions in the revised work are in every case distinct improvements.

By far the most important of the changes is the substitution of Rowland's scale of wave-lengths for the Potsdam scale, which is based upon Müller and Kempf's work. Apart from the question of relative accuracy of these two scales (and a good argument might be based on this), the great advantage arising from the use of Rowland's scale is the securing of uniformity over almost the entire scientific world. With Young, Keeler, Pickering, and Campbell in America, and Kayser and Runge in Germany using Rowland's scale, it would be folly for any observer to adhere to a different system. Uniformity is of the utmost importance; and Professor Frost has rendered science great service in reducing to Rowland's scale the lists of measurements, which for years to come will be standards for reference.

The new observations and theories which have been published in the past four years receive due attention. The book is wonderfully well up to date. It is true that since its publication fresh light has been thrown on several disputed points, especially on the spectrum of the Great Nebula in Orion and the theory of Nova Aurigæ; but in stellar spectroscopy only a weekly or monthly magazine can hope to lay any claim to having stated all the facts known. Professor Frost has profited, too, by the kindly assistance of other American investigators. Campbell contributes a section on the reduction of spectroscopic observations of motions in the line of sight. Young publishes a partial revision of the chromospheric lines. Hale's spectroheliograph is fully described.

Altogether, Professor Frost's book is a great step in advance, as was made necessary by the unexampled progress of astrophysics itself. But, in spite of this, there are certain points, common to the original and the newly published books, which cannot be regarded but as defects; and again there are many statements which should be received with caution. It is in the first part of both books, that devoted to spectroscopic apparatus, that they are least satisfactory. It is true that the descriptions of the apparatus in use in the great observatories is most complete; many interesting theories and discussions are given; and experiments such as those on the effect of changes in temperature upon the index of refraction are reported in full. In each one of these particulars, too, the American book is superior to the German. The point where they both fail is just here: the ordinary observer who has a telescope or spectroscope or both, and who wishes to do what he can under existing conditions, gets very little help or suggestion. From a German standpoint this is not very important; but here in America, where there are innumerable colleges and also amateur observers, each with instruments, often extremely good, and each anxious to take part in this new and fascinating work in astrophysics, it is the most serious matter possible. No one can tell how much work, and good work too, might be done if only proper guidance could be given. It would have been much better if, instead of describing so many finished forms of apparatus, more space had been given to the provisional forms, or to the home-made pieces that every observatory has to use. In any case, attention should surely have been called to Keeler's paper in the *Sidereal Messenger*, X., 1891, "On Elementary Principles governing the Efficiency of Spectroscopes for Astronomical Purposes." Few articles so helpful as this have ever appeared. This same section of the book could have been greatly improved, too, if the instrumental adjustments and corrections had been described more in detail. The author and the translator, as Professor Frost modestly calls himself, may have left these out purposely; but surely the book would have had a much wider field of usefulness if they had been included.

There are other points of weakness, too. One is, of course, the system of classification of the stellar spectra; and to the defects of this Professor Frost himself calls attention, saying that many spectroscopists regard any classification as provisional in the light of our present ignorance. This last fact is especially true for two reasons, one being that in Scheiner's classification undue importance is attached to the visual spectra; the other being that the interpretation of the observed spectra is so exceedingly uncertain. In our complete ignorance as to the cause of so many stellar radiations, and in the almost complete absence of experimental data as to changes produced in spectra by alterations in temperature or pressure, or by the presence of other substances, it certainly seems unwise to make use of theoretical explanations. The discontinuous spectra emitted by the stars may be due to temperature effects, they may be caused by electrical discharges, or at least accompanied by chemical changes; and, as we know so little about it, the application of theoretical deductions from Kirchhoff's law is decidedly unjustifiable. The question, too, of identification of the stellar lines with those due to substances known to us is by no means so easy or so certain as Scheiner would have us think. Identification of lines is one of the most arduous tasks of the spectroscopist; and it is by no means a matter of accuracy of measurement simply. It is difficult enough in many cases in the solar spectrum; but it is many times more so with stellar spectra, owing to physical changes in the spectra, such as the disappearance of lines and groups, changes in relative intensities, new shadings on different sides. Of course it is very easy to account for these changes on theoretical grounds unsupported by experiment, especially if the theory is assumed; but the reader of the book now under discussion receives, I think, an entirely wrong impression of our actual *knowledge* of these subjects.

The book is, however, a perfect storehouse of facts; and through it the work of every investigator will be made easier. One wishes that more attention had been called to what we do not know, so that the future work of new observers could be directed into useful courses. But the book, as it is, is one that will be instantly read by all workers with telescope and spectroscope, and will be kept at hand for daily reference. The appearance of the book is a credit not alone to the author, but also to the publishers, who surely show their deep interest in scientific education in its production. The paper, type, diagrams, and illustrations are all that could be desired.

JOSEPH S. AMES.

Magnetische Kreise, deren Theorie und Anwendung. By Dr. H. DU BOIS. 8vo, pp. xiv+382. Berlin, Julius Springer, 1894.

To one who has read the titles of the numerous books on magnetism and its applications which have appeared during the last few years, it might seem that to produce anything new in this field was an impossibility. The volume before us demonstrates, however, the falsity of such a conclusion. Although Dr. du Bois's book contains little matter which is new in the sense of appearing for the first time in print, yet the choice and arrangement of the material, and especially the manner in which it is presented, render the book quite different from works on magnetism which have heretofore appeared. It cannot be likened to Ewing's treatise, for the effects of magnetism upon the physical properties of matter, and the differences exhibited by various bodies in their behavior when subjected to magnetizing forces, are not discussed; while methods of magnetic measurement, although mentioned, are treated only briefly. Still less is it to be compared with the works of S. P. Thompson, for the methods of presentation are entirely different. And finally, although the more important practical applications are briefly described, yet the book has little in common with most technical works on magnetism.

Speaking generally, it may be said that the aim of the book is to present together, in a connected and consistent form, *all* the more important methods of treating magnetic problems. The chapters dealing with technical applications serve to illustrate the methods and principles discussed. In consequence of the magnitude of the task which the author has undertaken, the treatment is in many cases of necessity brief. Yet the reader will be able to obtain here a broad view of the subject, such as could otherwise be reached only by a long and tedious search through the original literature.

The book is divided, naturally, into two parts, the first of which concerns itself wholly with magnetic theory, while the second deals with practical applications, and methods of magnetic measurement. A general knowledge of the subject on the part of the reader is assumed; yet the first two chapters are devoted to a review in elementary form of fundamental phenomena and notions, and will be found interesting reading on account of several novel features in the mode of treatment. Evidently impressed by the ridicule that has recently been thrown on magnetic poles and the law of inverse squares by a well-known English writer, the author has avoided the use of the term "pole" altogether. In the first chapter the innovation is very successful. Strength of field is defined, for instance, by reference to the current induced in a small test coil, when the latter is rotated through 180°. In the second and following chapters, however, it becomes difficult to dispense with the conception of pole-faces, and the terms "end" and "end element" are introduced. The new terms are used in the same sense in which the expressions "pole" and "pole-face" are rightfully employed, and the advantage gained is not obvious. Yet it must be admitted that the new expressions soon lose their strangeness, and are by no means inappropriate. The distinction made in the first chapter between paramagnetic and ferromagnetic bodies — viz. that the latter exhibit hysteresis, while the former do not — seems uncalled for.

After two chapters devoted to the essential features of the ordinary mathematical theory of permanent magnets, and of induction in isotropic media, we come, in Chapter V., to the problem of the induction in a toroid, or anchor ring, containing a radial slit. The author may be said to base his whole treatment of discontinuous magnetic circuits upon this typical case. The discussion in Chapter V. is in accordance with the theory developed in the preceding chapters, and is quite different from that with which, I think, most American readers are familiar. It will perhaps prove of especial interest for this very reason. Successive approximate expressions are developed for the demagnetizing factor and the leakage coefficient, and are shown to agree with the results of experiment within reasonable limits.

Part II., which comprises about two-thirds of the book, begins with a discussion of circuits in which the magnetizing coils are unsymmetrically distributed. The tendency of the total flux of induction to remain the same in all parts of such circuits, in spite of the lack of symmetry in the original magnetizing field, is explained, not by reference to the analogy of electric currents, but as a result of the reaction of the "end elements" at points of the circuit where leakage occurs. Strangely enough the explanation of this fundamental property of magnetic circuits is rarely given, except perhaps by a fleeting reference to the electric analogy, which must often appear to the student as not wholly satisfactory. The two clearly written paragraphs which Dr. du Bois devotes to the "compensating action of leakage" are therefore doubly welcome. The discussion of Hopkinson's synthetic method, which follows, does not do justice to this most important method of treating magnetic problems. Here, as in many other parts of the book, English and American readers will regret the absence of concrete *numerical* illustrations.

Chapter VII. is devoted to the analogies between magnetic induction and various forms of current flow. A brief history of the development of such analogies, and a table in which they are compared, add to the interest of the chapter. The greater portion of Chapter VII. naturally deals with the electric current analogy, and Dr. du Bois points out quite clearly many of the dangers which must be avoided in its application. That such dangers exist is only too true. I have known a serious attempt to be made, for example, to compute hysteresis loss by reference to a formula analogous to the well-known C^2Rt of Joule. But any analogy may lead to equally ridiculous results when carelessly applied. A perfect analogy would be an identity. The statement, or rather the implication, made in Chapter VII. that the analogue of Ohm's law may lead to false results, even when applied with due caution, does not, however, appear to be justified by the example cited. The author lays great stress on results obtained with a uniformly wound anchor ring containing a narrow slit. Experiment shows that the leakage coefficient, defined as the ratio of the total flux to that portion which goes directly across the slit from pole to pole, grows less as the iron becomes more saturated, and apparently approaches unity as a limit. A superficial consideration of the problem from the standpoint of the electric analogy would indeed lead one to expect an opposite result. But if we imagine the iron ring replaced by a conductor immersed in a poorly conducting liquid, provide a *distributed* E. M. F. to correspond with the uniform winding of the magnetizing coil, and imagine the specific resistance of the conductor to approach that of the liquid, the conclusions would correspond exactly to the results obtained by the experiments referred to.

In Chapters VIII. and IX. are discussed the various forms of magnetic circuit as they occur in the dynamo, electro-magnet, and the transformer. The limited space at the author's disposal has prevented a detailed treatment of any of the cases considered, so that the designer will find little here which is new. But for the physicist who wishes to obtain a glimpse of the applications of his science, these chapters will prove most interesting reading. Of especial interest is the description of a large electro-magnet, constructed by the author for developing strong fields. The essential features to be considered in the design of such magnets are discussed in some detail, and the result obtained, viz. a field of $H = 40{,}000$, shows that the principles explained have been successfully applied.

The book closes with a brief but comprehensive résumé of the various methods of measuring H and B. The discussion of the magneto-optical method for the measurements of strong fields is deserving of special mention.

ERNEST MERRITT.

"Light": An Elementary Text-Book Theoretical and Practical for Colleges and Schools. By R. T. GLAZEBROOK, F.R.S., Assistant Director of the Cavendish Laboratory. Cambridge Natural Science Manuals. 8vo, pp. x + 213. University Press, 1894.

This little volume of 200 pages is an elementary text-book, and represents a course for medical students given at the Cavendish Laboratory. It is designed to be used by beginners who are accompanying their class work with experiments. In connection with the text about thirty simple experiments are described as practical exercises for the pupil, illustrating the principles and laws studied. The first six chapters deal with the laws of light intensity, the measurement of its velocity, giving brief outlines of Fizeau's and Foucault's, as well as of the Astronomical methods, reflection at plane and curved surfaces, and the properties of lenses. In the three concluding chapters are discussed the eye in its relation to vision, the more important optical instruments, the leading facts relating to color and color blindness, the formation of the spectrum, and the difference in character between the spectra of solids or liquids and gases.

Many questions and examples are given at the end of each chapter.

That Light is a wave motion in the ether is mentioned, but the whole treatment is by the method of geometrical optics. It is not even attempted to deduce the law of refraction from the wave theory. This entire exclusion of physical optics from an elementary text-book seems to us a mistake. Would it not be more profitable for the student to spend less time over calculations relating to lenses and curved mirrors, and learn something of interference, diffraction, and the colors of thin films, and so in a way be able to appreciate the grounds of the wave theory, and to think of light as a wave motion ? Probably the pupil would sacrifice some facility in solving lens problems, but he would have a far deeper insight into the science of optics.

The diagrams are excellent, and we are glad to note the tracing of *pencils* of light instead of *rays* in optical instruments, as this important point is too often neglected.

A. L. KIMBALL.

The Steam Engine and Other Heat Engines. By J. A. Ewing, F.R.S., Professor of Mechanism and Applied Mechanics in the University of Cambridge. 8vo, pp. xiv + 400. Cambridge Press, 1894.

This is an outgrowth of the article prepared by Professor Ewing for the Encyclopedia Britannica, and is expanded into a treatise intended to serve as a university text-book. It covers, therefore, a field a little different from that of Cotterill and Peabody on the thermodynamics of the steam engine, and different from that of Thurston and Whitham, in the field of constructive steam engineering. It partakes a little of both, and in addition devotes some most interesting space to air, gas, and oil motors.

From his nearness to original sources, the author has been able to present in his first chapter some most excellent illustrations of the early steps in the utilization of heat for steam making, and then devotes 130 pages to a discussion of the theory of the steam engine as a heat motor. This discussion seems to be most satisfactory and useful; without taking the position of a full presentation of the theorems of thermodynamics, it is in no way open to the criticism of being superficial. The discussion of the Carnot cycle and its realization in the actual engine is especially clear. At the close of this discussion some space is devoted to the reversal of the heat cycle, in refrigerating machinery, and likewise to the use of engines with more than one working substance. The chapters on the actual behavior of steam in the cylinder and the effects of superheating and jacketing, high speed, etc., are particularly full.

The second part of the work covers the development of the compound and multiple-expansion engines, the testing of engines, the kinematics of valve gear and governing, and the forms of boiler and engine. The references under these latter heads to a number of modern forms, and the results of tests of steam turbines, give special interest to this part of the discussion. The concluding chapter on air, gas, and oil engines, is of particular interest in America as the development of the direct combustion engine has been much wider in England and on the Continent than here. Within the limits which the author set himself the discussion is full and interesting, but of course goes no further than a presentation of typical arrangements. The book concludes with a table of the properties of saturated steam from a temperature of 32° to 401° Fahrenheit, the superior limit corresponding to a pressure of 250 pounds to the square inch.

The book is of a type excellently well adapted to serve as a text-book for students in engineering where the design, construction, and operation of the steam engine is not to be made an exclusive specialty. It will serve also most admirably as a text-book and work of reference for this latter class where lectures and other exercises are to supplement the course laid down in the text-book which is used. It will form a valuable addition to the library of every engineer.

F. R. Hutton.

Elementary Hydrostatics. By John Greaves, M.A., Fellow of Christ's College. 8vo, pp. xi.+204. Cambridge, University Press, 1894.

This unassuming title carries with it a wholesome lesson to contributors to the scientific literature of our day. Science can scarcely gain an advance through the multiplication of text-books of a general nature, merely stating the old principles in new garb and covering fields already fully harvested. Because there is an occasional John Tyndal who may translate the deeper truths of science into terms of the popular tongue, the needs of modern science are none the less sharply confined, in general, to progress along specific lines. Every new book which does not aim to fill a distinct want in the literature of the subject treated, lays an unmerited burden upon the workers in that field, who must individually satisfy themselves of its purport.

The volume under discussion is designed for students preparing for the First Part of the Mathematical Tripos, and in the fulfillment of this purpose has its birthright. This may serve as an opportune occasion for the writer to register, in no feeble terms, his entire disbelief in the scientific value of books written for such a purpose, and his lack of sympathy with the English idea of making every permanent feature give way to temporary triumph in these examinations. The treatment of Hydrostatics is essentially from the abstract side, and, as such, could hardly be expected to be the most useful to the student who wishes to grasp the subject in its concrete bearings. There are evidences of good proof-reading and of excellent printer's work. The lavish use of broad-faced type, to make prominent the salient points, is, æsthetically, a mistake, and its object might have been accomplished more tastily.

The definitions, usually, are good. However, in the attempt to exclude everything but the idea defined, the author has laid himself open to the criticism that some of his definitions are not broad and full, as vehicles of the general idea, but are, possibly, too special and technical. In Chapter I. the relations between stress and strain and the definitions of the properties of matter are clear, concise, and well illustrated by examples. On page 11, a possible weakness in treatment is shown in the dependence of the proof upon § 16 which comes later on in the book.

In general, the whole discussion might be made clearer by the judicious use of more diagrams and by the graphic representation of the unit areas and volumes with the forces acting upon them. The attempt is made to show everything on one plane where isometric projection would add largely to clearness. A potent lesson in this regard may be drawn from the treatment, by Professor Church, of the subject of Hydrostatics in his *Mechanics of Fluids*, pp. 515–603. As a text-book for preparation for the Mathematical Tripos, the volume seems quite satisfactory. As a broad treatment for general purposes, it is subject to some criticism.

ELON HUNTINGTON HOOKER.

Recent Text-Books in Laboratory Physics.

The little manual[1] by W. G. Woollcombe is evidently intended to precede the author's *Practical Work in Heat* published last year, and is presumably to be followed by others covering the remaining divisions of physics. It is an excellent introduction to laboratory work, covering fifty experiments. Of these, nineteen are devoted to measurements of length, including areas and volumes. The next twenty cover measurements of mass, and the remaining eleven are divided between barometers, the simple pendulum, and capillarity.

The book is thoroughly practical, the experiments well selected and arranged, and the instructions and explanations sufficiently full in detail to render an ordinarily intelligent boy of fourteen independent of the teacher.

All the experiments are quantitative, and as many of them involve the use of liquids, the student receives a good drill in manipulations. It would seem desirable to conduct a course of this sort in a room fitted up for chemical experiments, with convenient sinks, troughs for mercury, and possibly aprons for the students.

The author's rule (on p. 13) to express all lengths in centimeters, instead of the customary use of millimeters for small dimensions, if universally adopted, would do much to diminish mistakes and confusion.

In the laboratory manual[2] by H. N. Chute, the subject is covered in some 128 experiments, of which general measurements and mechanics take forty-three, heat seventeen, magnetism and electricity twenty-five, sound eleven, and light thirty-two. Half a day each week is supposed to be given to the laboratory, and in addition home work is required in the preparation of reports on the results.

The book will commend itself to those teachers who wish to utilize their lecture apparatus in the laboratory, as a large number of the experiments are qualitative, demonstrating natural laws and repeating those of the lecturer. Some of the practical hints are excellent, and the shelf of reference books in the laboratory, if the students can be encouraged to use them in the way proposed, should be an important addition.

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[1] *Practical Work in General Physics.* By W. G. WOOLLCOMBE. pp. xii + 83. The Clarendon Press, 1894.

[2] *A Physical Laboratory Manual for Use in Schools and Colleges.* By H. N. CHUTE. 8vo, pp. xiv + 213. D. C. Heath & Co., 1894.

The volume is well printed and illustrated, and the binding of a nature to stand the rough usage of a laboratory.

In *Practical Lessons in Physical Measurement,*[1] by Alfred Earl, the hope is expressed by the author that the treatise may serve as a Practical Arithmetic and Practical Grammar, as well as an Introduction to Practical Physics. He might have added Practical Geometry and Practical English Composition, on the strength of Chapters V. and VIII.

The aim of the book is to take young boys and give them a thorough foundation for future experimental work. The course begins in the very rudiments of measurement, and the selection and construction of standards is thoroughly discussed. The whole 270 experiments are devoted to measurements of length, mass, and time, and every detail tending to accuracy is carefully considered.

Great stress is laid by the author on accuracy of expression and correctness of language in the preparation of the reports on the results. The use of the pen in the description of the experiment is considered of equal importance to the use of the instruments themselves.

The book would be much improved by a chapter of instructions addressed to the teacher, explaining which portions are to be read over by the student at home, how the laboratory is to be arranged, and how the work is to be distributed.

A new English edition[2] of Kohlrausch's *Physical Measurements* is a welcome addition to our laboratory literature. The translation from the second German edition which has been doing duty up to the present time as the American Kohlrausch was exceedingly incomplete, being only half the size of the present volume. In addition it was marred by numerous mistakes, and poorly printed. The arrangement of the four-place logarithms, printed on both sides of the same leaf, and so far from the back cover as to make them inconvenient to find, was sufficient to disgust the average physicist. Those who were not conversant with the German language preferred the English edition of 1883, even at the increased price, on account of the superior value. The want of a new edition has been felt.

In this new edition, the translation closely follows the German, and is well done, the lack of sufficient cuts and of detail in the directions and descriptions being the fault of the original.

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[1] *Practical Lessons in Physical Measurement.* By ALFRED EARL. pp. xv + 350. Macmillan & Co., 1894.

[2] *An Introduction to Physical Measurements.* By F. KOHLRAUSCH. Third English, translated from the seventh German edition, by T. H. WALLER and H. R. PROCTOR. pp. 476. D. Appleton & Co.

The book is replete with suggestions, covering all the best methods in use at the present time, and is particularly good in the enumeration of the sources of error attending the various operations. But it can hardly be properly called an "Introduction," presupposing, as it does, a good acquaintance with the subject. In such branches as the proper use of the balance, the calibration of thermometers, and parts of the optical work and magnetic determinations, it is unexcelled.

We have sought in vain for the "notes, appendices, and tables signed 'TR.,' added by the translators" mentioned in the preface.

HOLBROOK CUSHMAN.

A Treatise on the Measurement of Electrical Resistance. By WILLIAM A. PRICE, M.A. 8vo, pp. xvi+199. Oxford, The Clarendon Press, 1894.

The author of this book writes from knowledge acquired by long practical experience in the construction of resistance coils, and the work contains a large amount of useful information, and precise directions relating to the winding and mounting of resistance coils, including standards. While it cannot be said that a large part of this information is not to be found elsewhere, yet it is here assembled in a very satisfactory and accessible form.

In the chapter on materials for resistance coils the author gives a succinct account of the general principles, derived from Matthiessen's investigations, which should govern the choice of an alloy for purposes of resistance. The wide divergence of different samples of German silver wire is recognized, and attention is drawn to the brittleness or rottenness which it sometimes exhibits, particularly when exposed to corroding gases, or when frequently heated by an electric current. This undesirable property is probably due to the zinc constituent of the alloy. A single section only is devoted to the alloy platinoid, and no new light is thrown on the behavior of either this alloy or of the newer one known as manganin. A *resumé* is given of Dr. Lindeck's paper on that subject before the British Association in 1892. It is well known that the experts of the Berlin Reichsanstalt direct that manganin must be "artificially aged," by subjecting it to a temperature of 140° C. for five hours after it has been wound on its bobbin, and heavily coated with shellac varnish. It is altogether likely that other alloys intended for resistance coils would be improved by similar treatment. The resistance of a platinum-silver standard coil is lowered by having its temperature only moderately raised for an hour or so.

The Wheatstone's bridge is very satisfactorily treated in a separate chapter which leaves little to be desired. Much useful information on the slide wire bridge will be found in the succeeding chapter. The only contact device described is defective in the very important point that all the pressure which the operator applies to the knob is transmitted to the bridge wire, which may in this way be indented and injured. A device is easily made in which the contact pressure is determined entirely by the tension of a small spiral spring.

Among the methods of comparing resistances, one is surprised not to find that best one of all — the Carey Foster method. This omission is the more remarkable because the method is of English origin and is used at the Cavendish laboratory for the comparison of the B. A. standards.

Three methods of measuring the resistances of batteries and electrolytes are quite fully described. Two of these depend upon the electromotive force of the battery under test. Since perceptible polarization takes place with an external resistance as high as 10,000 ohms, and with low resistance in a small fraction of a second, any method which is vitiated by polarization is inapplicable to cells which are incompetent to maintain a uniform continuous current. The condenser method is applicable to all kinds of batteries and is readily applied, but no mention is made of it. The theory of Mance's and Beetz' method "rests on the assumption that the values of the electromotive force, and the resistance of a battery, are independent of the current passing through it." But the internal resistance of a battery has a definite relation to the current. This fact is exemplified in a striking manner by the resistance of the best "dry cells." In a dry cell recently tested by the condenser method with some special appliances, with an external resistance of 80 ohms and a current of 0.0177 ampere, the internal resistance was 3.32 ohms; while with an external resistance of one ohm and a current of 0.921 ampere the internal resistance was only 0.61 ohm. The intermediate values lie on a smooth curve, apparently an hyperbola, when currents and internal resistances are plotted as coördinates. All that can be affirmed of the internal resistance of a battery is that it is the ratio between the potential difference between the terminals and the particular current flowing, or $rI = e$. Both r and e are functions of the current.

It is unfortunate that the author should not have made himself familiar with the practical details of determining the unit of resistance in absolute measure, in order that he might have made his book still more valuable by adding a plain account of one or two of the best methods. It would have been much more to the point than the mathematical appendices on the Wheatstone's bridge and the theory of Mance's method, which have little more to commend them than an "academical interest."

HENRY S. CARHART.

Volume II. *March–April, 1895.* *Number 5.*

THE
PHYSICAL REVIEW.

To face the plate at p. 305. PHYSICAL REVIEW No. 10.

ERRATA.

The numbers indicating wave-lengths in Figs. 1 and 3 are incorrect. They should increase from left to right, instead of from right to left, the division at the left end of each figure being 0.33 μ. The value of each division of the scale is 0.01 μ. As a further check it may be remembered that the prominent double line (H) in each figure should lie just to the left of the division for 0.40 μ.

pose the crystalline structure is due to a symmetrical arrangement of similar particles, and that either (1) these are closer together (or farther apart) in a plane perpendicular to the optic axis than in the direction of this axis; or (2) that they are longer (or shorter) in the direction of the axis than in the plane perpendicular to it, where all their dimensions are the same. Let us cut out a spherical piece of the mass. Imagine a cork-borer whose cross-section is of the dimensions of that of a particle. If we bore with this along a diameter parallel to the optic axis, we shall inclose in it a different number of particles from that which we should get if we bored with it in any direction through the center of the sphere perpendicular to the axis. We might then say that the densities in these two directions were measured by the number of particles included.

In order to find out how such a crystal would attract a particle at an outside point, I have made calculations based on several conjectures as to the structure of the mass. Of these the following will serve as an example. I have supposed the sphere made up of separate equidistant layers of homogeneous matter parallel to a plane perpendicular to the optic axis: this contains both the suggestions above mentioned. In framing such an hypothesis as to the arrangement of the particles, it has to be remembered that the optic axis is only a direction in the body, and that all lines parallel to it have the same properties; also that all the layers must be of the same nature, and differ only as to size. Taking the formula for the potential of a disk, in terms of spherical harmonics at an external point, whose angular distance from the optic axis is θ, and summing for a number n of disks which will just make up the sphere, terms in $\cos\theta$ enter; these terms have coefficients of the dimensions $\frac{1}{n}$, $\frac{1}{n^2}$, etc.[1] Now n has to be taken very large (of the order of magnitude of the number of particles in a unit of length), and so these terms are negligible. Other suggestions were followed out to the same conclusion, as might be expected, that no arrangement of the constituent particles could produce a measurable difference in the magnitude of the force of attraction of such a crystal at an outside point, however its optic axis might be oriented with respect to that point — provided the law of attraction were that of Newton. Of course such proof is entirely negative, — the guesses as to formation may be far from the truth, and Newton's law may require modification, — and so it was thought that it might be well to try experiment. When two spherical masses, at a distance apart comparable with their radii, attract each other, we know from the experiments of Cavendish,[2] Reich,[3] Baily,[4] and

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[1] See Appendix.

[2] Phil. Trans. 1798.

[3] Freiberg, 1838, and C.R. 1837.

[4] Phil. Mag. 1842.

Cornu and Baille,[1] that the attraction obeys the Newtonian law within the limits of the agreement of their results, and that "distance between them" means the distance between their centers, which also admits of mathematical proof; but it should be remembered that because this law is true for heavenly bodies it does not follow that it should hold good for the case investigated by Cavendish. It is of interest to find that it is upheld as this experiment is performed with apparatus of smaller and smaller dimensions. Professor Boys' investigation, promised in his paper in the *Proceedings Royal Society*, Vol. 46, ought to be of special interest.[2] At the end of this paper will be given the results of a series of experiments made to test this law directly. They are necessarily rough, as the apparatus was designed for qualitative measurements. In the case of crystals, it is still less allowable to assume that Newton's law is obeyed, and to say that a spherical mass of crystalline matter will give the potential at an external point the same as if it were an equal mass concentrated at its center of figure. It was thought worth while to test the case experimentally, and the following experiments undertaken at the suggestion of Professor Rowland were begun with that end in view. The form of the apparatus described is similar to that employed by Professor Boys in his paper referred to. In the following pages will be given a description of the apparatus and methods employed, and the results found. It will be seen that there is no deviation from the law of Newton, within the limits of observation, either in the case of attracting crystalline matter with reference to its optic axis, or in the case of isotropic matter at small distances.

Description of Apparatus.

The suspended system consisted of a light beam *AB* (Figs. 1 and 2) joined to the hook of the suspending fiber by a rod *CD*, which also carried the mirror *m*; a brace *ACB*, being a continuation of the beam, was used to give sufficient rigidity. The whole was made of very fine aluminium wire. The mirror was of plain glass silvered on the back, and was 1 cm. in diameter. At the points *A* and *B* the small masses *a* and *b* were suspended at unequal distances below the beam by means of silk fibers. This system was inclosed in a rectangular brass box (Fig. 3) whose internal dimensions are: height 11.7 cm., width 3.1 cm., and depth 1.55 cm. This box stood upon a heavy base with tripod and screws for leveling purposes, and above carried a projection upon which fitted, with carefully made cone bearings, a brass cap carrying the glass tube containing the suspending fiber. This fiber was of quartz, about 0.0003 inches in diameter. By means of the cone bearings the whole suspension could be rotated very easily, and the small masses set symmetrically within the box. The large attracting spheres were supported upon the same base that carried the box. Two pillars, *g* and *h*, screwed on to the base, carried a rectangle *cdfe*, pivoted at the middle of its two sides, *df* and *ce*, upon the pillars; upon the other two sides, *cd* and *ef*, were put the large masses, *M* and *M'*. Thus, by simply rotating the rectangle, the balls could be made to take up their position of maximum attraction, or a neutral position; they were so placed on one side and the other of the middle of the wires *ef* and *cd* that when in their positions for maximum attraction they were exactly opposite the corresponding small masses; and, accordingly, they were always rotated in one direction only from their neutral position.

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[1] C. R., Vols. 70, 76, and 86.

[2] Since this was written Professor Boys has published a paper on this subject. See "Nature," Aug. 2, 1894.

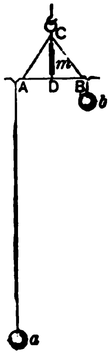


Fig. 1.

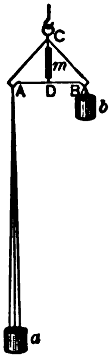


Fig. 2.

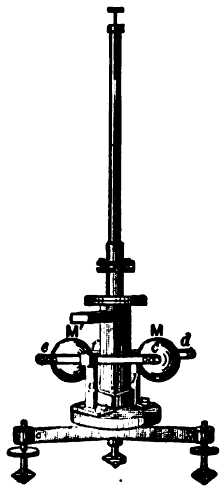
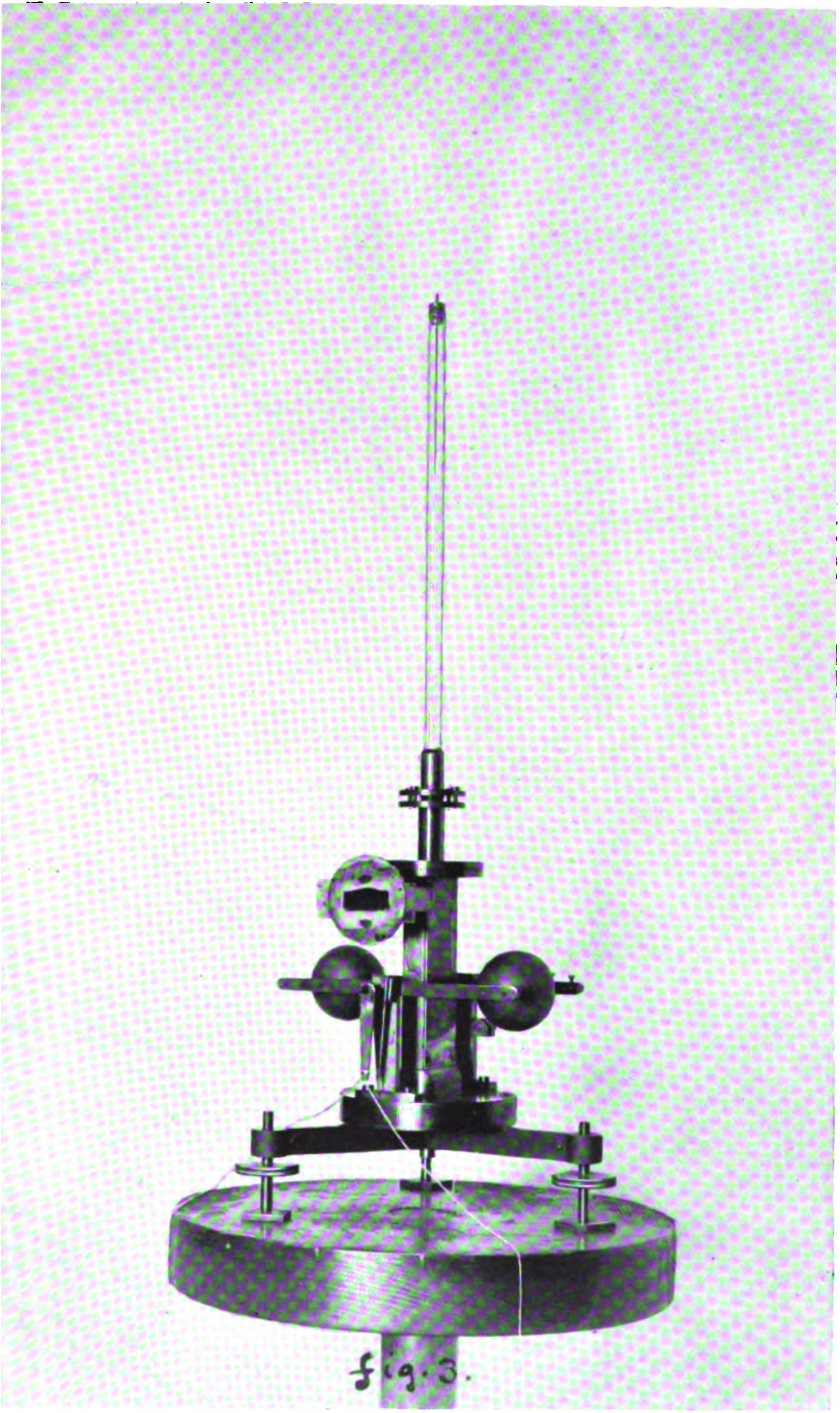


Fig. 3.

At the upper part of the box, opposite the mirror, was a hole about which fitted a funnel-shaped projection of rectangular cross-section, the end of which was covered with plane parallel glass. Through this passed the beam of light to



A. STANLEY MACKENZIE: ATTRACTION BETWEEN MASSES.

the mirror. The motion of the suspended system was observed by means of a telescope and straight scale.

As a large deflection was sought in order to show any differences that might exist, the fiber was made as small as possible and the period large. Twice during the experiments the fiber broke in the middle, and all the previous results had to be discarded and a fresh start made. In each case the fiber had been up and under strain for weeks, had received no jar, and broke in the night when not in use, and not while it was in motion. This may have been due to bad fibers, although all fibers were carefully examined before use under the microscope in order to detect unevennesses. It was more likely due to the fact that the fiber was too heavily loaded. On account of the rapid dying out of the swings, due to the peculiar form of suspension, the period was never accurately found; but the mirror soon took up its position of rest, and that only was observed.

The box was made air-tight with "universal wax," and to still further protect the apparatus from the effects of air-currents, the whole instrument was covered by another case with a glass window opposite the window in the box. In the later experiment still further precautions were taken in order to get rid of the effects of temperature, and they will be described at a later period.

The instrument was set up in a room at about 10 feet below the surface of the ground, on a heavy table whose feet stood in boxes of coarse sawdust. In this way the surface earth tremors and the effects of sudden changes in outside temperature were entirely gotten rid of. A set of readings of the temperature of the room was taken in order to see how much it varied, due to the presence of the observer and the burning gas-jet. It was found to rise from $\frac{1}{2}$ to $\frac{3}{4}$ of a degree in two or three hours. The extremes of temperature during the whole period covered in the experiments were 14° and 21° C., and no variation of the deflections was observed due to this cause.

FIRST SERIES OF EXPERIMENTS.

Attraction of large masses of calc-spar for small lead masses.

In this series of measurements the small balls *a* and *b* (Fig. 1) were of lead, and weighed each 0.5310 grams; and the total weight suspended was 1.1653 grams. In the early part of the experiments it was observed that there were variations in the zero-point due to the presence of iron in the lead. The balls finally used were made of electrolytically deposited lead, and showed no such effects. The vertical distance between the balls was 8.80 cm., and the horizontal distance 1.50 cm.

The attracting masses M and M' were two pieces of optically perfect calc-spar cut into the form of spheres. The cutting was a difficult operation, and was done by Mr. Bohrer, lapidary, of Philadelphia. They were of different sizes, due to the grinding away of a flaw that developed in one, and had radii of 2.65 and 2.32 cm., and weighed 212.860 and 142.143 grams respectively. The supports *ef* and *cd* of the balls were pointed rods fitting into small conical holes drilled in the spar at opposite ends of a diameter. The balls could thus be turned about this diameter into any required position. A set of readings was made thus: the balls were placed horizontally in their neutral position, and the reading of the zero-point of the scale taken. They were then placed in their position of maximum attraction, and after three or four semi-vibrations the mirror was at rest, and the reading of the scale again taken. These readings were repeated several times, and the mean of the deflections taken. The crystals were then rotated into another position, and the operation of taking a set of readings repeated. The actual taking of a set of readings was very laborious and required a great deal of time, for it was generally found that the zero kept shifting more or less erratically for the first hour or two during which observations were made; it was found that this was the case even under the most favorable circumstances, and the explanation does not seem very patent. It was some time before this fact was noted, and the readings given are those taken after the zero became nearly constant. Accordingly, observations were generally continued for three or five hours at a time, depending upon the time taken by the zero-point to become constant. The best readings were taken after ten o'clock at night, when everything was quiet and still, and trains had ceased running in the neighborhood. During one sitting readings were taking for eight continuous hours with almost exactly agreeing results during the last five. As a rule, the zero-point was never absolutely constant, but had a slowly shifting motion. This was much more apparent for the first week or so after setting up the suspended system; but by taking the average of a series of deflections no appreciable error was caused by it. Various methods of attaching the quartz fibers were tried in order to see if that would obviate this shifting of the zero-point, but loaded so near to the breaking-point as mine were, no method I tried turned out to be entirely effectual. Fibers drawn on a revolving wheel were tried, but not found so trustworthy as those made by myself after the original plan of Professor Boys by means of a bow and arrow.

The direction of the optic axis was found by means of the polariscope. The ends of the diameter parallel to this direction were marked *A* and *B* (Fig. 4). The axis upon which the crystal was rotated, was through *O* perpendicular to the section in this figure, being the line of intersection of the gliding plane through the center and the equatorial plane of *AB*. *AB* is the direction of the optic axis, and six other points are marked on the ball at 45° apart. In the following table of readings "*A* and *A*" means that when the attracting masses are rotated into their position for giving maximum deflection, the point on each ball marked *A* is nearest the corresponding small ball. The numbers given are the deflections on the scale in centimeters found as described above. The numbers in a column between two horizontal lines were taken at the same sitting.

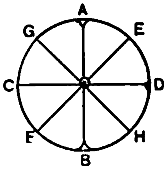


Fig. 4.

A and A.	B and B.	C and C.	D and D.	E and E.	F and F.	G and G.	H and H.
6.1	6.0	6.0	5.9	5.9	5.8	5.9	5.8
5.9	6.1	6.1	5.95	6.1	6.2	6.1	6.1
6.1	6.1	6.1	5.95	5.9	6.0	6.0	5.9
5.9	6.0	6.0	6.0	5.9	5.9	6.2	6.1
5.9	6.0	6.0	5.9	5.95	6.1	5.8	6.0
6.1	6.0	6.0	6.0	6.15	5.9	6.0	6.0
6.0	5.9		6.05	6.0	6.0	6.1	6.0
6.0	5.9		5.95	5.9	5.9		6.1
60.	5.9		5.9	6.0			6.1
6.05	6.0		6.0	5.95			6.0
6.05	5.95		6.1				6.0
6.1	6.05		6.0				6.0
6.0			6.0				6.1
6.1			6.0				6.0
6.0			6.0				6.0
5.9			5.9				6.0
			6.1				
			6.05				
			6.05				
			6.1				
Mean 6.01	5.99	6.03	5.995	5.975	5.957	6.03	6.01

Some other positions of the balls were then tried with similar results.

A and B.	A and C.	C and D.	E and F.	G and H.
6.15	5.9	6.1	6.0	5.9
5.95	6.0	6.0	6.2	6.1
5.9	5.9	6.0	5.8	5.9
5·9	6.0	5.9	6.0	
6.0	6.0	6.1	6.0	
	6.1	5.9	6.1	
	5.95		5.9	
	6.05		6.0	
Mean 5.98	5.99	6.00	6.00	5.97

These results agree with the others, and also show that there is nothing of the nature of polarity exhibited by crystals in their attractions.

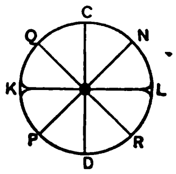


Fig. 5.

The axis of rotation was then changed to AB: Fig. 5 is a section through the center of the crystal perpendicular to this diameter: KL (Fig. 5) was the axis in the previous measurements, and C and D are the same points as before.

K and K.	L and L.	C and C.	D and D.	C and D.	N and R.
5.9	5.85	6.2	6.2	6.0	6.0
6.1	6.05	6.0	5.9	6.0	6.1
6.0	5.85	5.9	6.1	5.9	6.1
6.2	6.05	6.1	5.9	5.9	6.1
6.0	5.9	6.0		6.1	5.9
6.0	6.0	5.9		5.9	6.0
5.9	6.1	6.0		6.0	5.8
6.1	6.1	5.9		5.9	6.1
6.0	6.1	6.1		6.0	5.9
	6.1	6.0			6.0
	5.9	6.0			5.9
	5.9	5.9			6.1
	6.0	6.1			6.0
	6.1	6.0			5.9
	6.1				6.1
	6.0				5.95
	6.1				6.15
	5.9				5.9
					6.0
Mean 6.02	5.995	5.99	6.025	5.97	6.00

It will be seen from these numbers that there is no difference between these deflections. The scale was only read to ½ mm., and these results vary among themselves by less than that amount. The average of all the mean readings given is 6 cm., and the greatest divergence of the means from it is 0.03 cm., or $\frac{1}{200}$ of the whole deflection, and most of the results are much closer, especially where many readings of the same quantity are taken. To within this amount then we find no variation of the force of attraction of a crystal at a point at a given distance from the center of the crystal, as this point is taken in different directions with respect to the optic axis.

It might be noted that in the columns marked "*K* and *K*" and "*L* and *L*" the line joining the crystal and small ball lies in a cleavage plane of the crystal, and that the attraction in this position is the same as in every other.

The conclusion must be drawn, therefore, that in so far as these measurements go, a crystal acts exactly like an isotropic body in its gravitational properties; that is, a spherical mass of it obeys Newton's law, and acts as the same mass concentrated at its center.

A series of measurements with opaque crystal balls had been made the previous year and gave similar results, but the total deflection was only 3 cm., and it was thought advisable to increase the deflection and at the same time use transparent crystals.

SECOND SERIES OF EXPERIMENTS.

Attraction of large masses of calc-spar for small masses of the same material.

In order to make the investigation more complete, it was thought interesting to substitute for the small lead balls two small masses of calc-spar, and so get the attraction of one crystal for another. If any difference did exist, this method would double the variation in the deflection: at the same time the total deflection was increased by lengthening the beam of the balance.

In this series of experiments the whole suspended system (Fig. 2) was new. The small masses were cylinders of calc-spar with the optic axis the axis of figure, and vertical. This shape was chosen on account of the difficulty of cutting such small masses into perfect spheres. The dimensions of the cylinders were 6 mm. in length and 6.99 mm. in diameter, and weighed 0.6096 and 0.6066 grams respectively.

The fiber was of quartz 0.0003 inches in diameter and 10⅛ inches long. The weight of beam and mirror was 0.1248 grams, and the total weight supported by the fiber 1.35 grams. The cylinders were suspended by three cocoon fibers each, so as to reduce the rotation, and their centers were 2.25 cm. distant horizontally, and

8.70 cm. vertically. The attracting masses were the same as before, but shifted 0.375 cm. to the right and left respectively, so as to be opposite the small cylinders. Sets of readings were taken in the same way as before described. The particular arrangement of masses and suspending fibers gave the beam an aperiodic motion, which was unexpected. The time required for the beam to come to rest was about 15 minutes. The distance from the mirror to the scale was 340.8 cm.

The following are the readings taken: not so many positions of the large masses were examined as in the earlier series, but only a few typical ones, and more readings of each taken in order to eliminate the errors of observation. In the first four columns the axis of rotation of the crystal was *KL* (Fig. 5); in the fifth column it was *AB* (Fig. 4).

A and A.	C and C.	F and F.	C and D.	C and C.
8.9	8.9	8.75	9.05	9.0
8.9	8.9	9.0	9.0	9.0
8.9	8.9	8.75	8.95	8.9
8.7	9.1	9.05	9.1	9.1
9.0	8.9	9.05	9.0	9.0
8.8	8.9	8.75	8.9	9.0
9.1	9.0	9.15	8.95	9.0
8.9	9.1	9.0	9.05	8.9
8.9	8.9	9.0		8.9
9.0	8.9	9.1		9.1
9.0	8.9	8.8		9.0
8.9	9.1	9.0		9.1
9.1	8.9	8.8		
9.0	9.0	9.0		
9.0	9.0	9.0		
9.0	8.9			
9.1	8.8			
8.75	8.9			
8.8	8.9			
8.9	9.1			
9.0	8.8			
9.0	8.9			
9.1	9.1			
8.9	9.0			
9.0	9.0			

A and A.	C and C.	F and F.	C and D.	C and C.
9.0	9.1			
8.8	9.0			
8.9	9.0			
9.0	9.0			
9.0	8.8			
9.0	9.0			
8.85	8.8			
9.0	8.9			
8.9	9.1			
9.05	9.0			
9.0	8.9			
8.9	9.1			
8.95	8.9			
8.9	8.9			
9.0	9.1			
9.0	8.9			
9.05	9.1			
9.0	8.8			
9.0	9.0			
	9.0			
	9.0			
	8.9			
	9.1			
	8.8			
	8.9			
	8.95			
	8.95			
	8.95			
	8.95			
	8.95			
	8.95			
Mean 8.95	8.96	8.95	9.00	9.00

From this second series of experiments the same conclusion must be drawn as before from the first series; that there is no difference in the deflections, and hence none in the attractions, when the crystal sphere is turned in any way about its center, which remains fixed. The total error in observation is the same as before, but the total deflection is larger, and the means closer together. Where a large number of observations was made, the means differ only by 1 part in 900; and even in those cases where only a few readings were taken, the greatest difference in the means is 1 in 200.

THIRD SERIES OF EXPERIMENTS.

Verification of the law of masses.

As a further proof that these crystal spheres attract according to the law of Newton, with a force proportional to the product of the attracting masses, the large spheres of calc-spar were taken off and replaced by accurately turned spheres of lead of diameter 5.42 cm. each, and weighing 927.767 and 928.047 grams respectively. They were held by light steel wires, *cd* and *ef* (Fig. 3), passing through their centers. They were placed so that when in the position for maximum attraction, and when the little masses had come to their new position of rest, the distances apart of the centers of the attracting masses should be the same as that in the last series of experiments. By calculation and measurement this distance was 3.591 cm. in this series and 3.590 cm. in the last; that is, they were the same within the limit of error in the measurements.

The irregular motion of the beam was more pronounced in these readings than in the smaller deflections of the two preceding series, as will be seen from the readings which follow: —

46.6	46.6	46.8
47.0	46.4	46.8
46.6	47.0	46.4
46.7	46.7	46.9
46.6	46.7	46.6
		46.7
		46.8
		46.6
Mean of all 46.7.		

In order to compare this deflection 46.7 with that in the case of the crystals 8.95, at the same distance, we must pass from the scale readings to the actual angles of twist of the fiber. Calling α and α' the angles of torsion, we have

$$\tan 2\alpha = \frac{8.95}{340.8}, \text{ and } 2\alpha = 1^\circ\ 30'\ 15''.6;$$

$$\tan 2\alpha' = \frac{46.7}{340.8}, \text{ and } 2\alpha' = 7^\circ\ 48'\ 10''.$$

Therefore $$\frac{\alpha}{\alpha} = 5.187.$$

If we calculate the ratio of the forces of attraction in the two cases from the law of Newton, making corrections for the fact that the little masses are drawn from their zero positions, and that the big balls exercise a slight attraction on the small masses which are farther off as well as upon those that are near, we find these attractions should be as 5.214 : 1; or, in other words, the reading 46.7 should have been 47.0, in order that the experimental and theoretical numbers might coincide. The variation from Newton's law is only, therefore, about one-half of one per cent. This agreement is satisfactorily close under the circumstances, but the method is capable of better results, for, as will be seen in the next series of experiments, the instrument was not yet sufficiently guarded against slight draughts or changes of temperature, and their effects became more noticeable, the larger the deflection.

FOURTH SERIES OF EXPERIMENTS.

Direct experimental test of Newton's law of the distance.

It was thought that with the apparatus described it would be interesting to test the law of the inverse square of the distance, when the distance between the centers of the attracting bodies was small. In the Cavendish experiment, the distance was about 93 cm.; in Cornu's, about one-quarter of this; in Reich's, 16.8 to 19 cm.; and in the present investigation, 3.7 to 7.4 cm. The apparatus described was not at all designed for quantitative measurements, and accordingly was not well suited for this series of experiments; but after some preliminary observations of what I could do with it, I thought I would go on and take a series of readings, and at the expense of time and care I was able to get much better results than might have been expected. The chief difficulty to be overcome was, of course, the measurement of the distances. The placing of the outer or larger balls symmetrically with respect to the instrument was only a matter of care in the manufacture of the instrument, and was done satisfactorily; the greatest difficulty was met in trying to place the inner beam with its suspended small masses midway between the larger masses. This difficulty arose from the fact that the inside masses could not be seen through the observing window, and so their positions could not be directly observed. A diagram will make the matter clearer. Let the figure (Fig. 6) represent the projection of the essential parts of the apparatus on a horizontal plane: *cd* and *ef* are the wires carrying the attracting masses.

The distance *MM'* or *df* can be accurately measured by the dividing engine. Also by means of the cathetometer the inner mass *b* can be set as much above the horizontal plane containing *M* and *M'* in their positions of zero attraction as *a* is below it. The remaining essential point is to have *O*, the middle point of the beam, exactly midway between *M* and *M'*, and the beam itself parallel to *cd* or *ef*. The method adopted was the following, which proved successful although tedious. Before closing the case, the beam was set as near its correct position as was possible; then the case was closed, and the covers placed over it, and a series of deflections observed to the right and left, with the large balls in the positions *M* and *M'* (Fig. 6). If the deflection on one side differed from that on the other, the position of the beam was slightly changed in the way suggested by the results, and its correct position thus determined by trial and error.

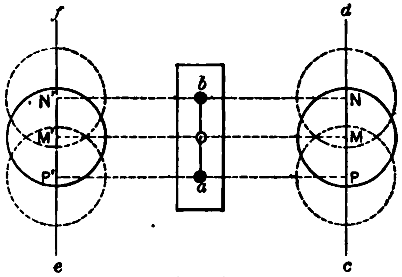


Fig. 6.

Some further precautions in addition to those already described were taken in order to overcome the slight variations of the motion of the beam that have always existed. Fitting over the whole instrument, already made air-tight, was a copper box, and over all a great pasteboard box lined with cotton wool. Each of these coverings was made practically draught-tight where it touched the table. The large balls were moved by strings passing through these covers. It was also found effective to have a paper bottom to the table upon which the instrument stood, in order to ward off the radiations from the floor.

The frame supporting the large balls played between stops which allowed the balls to be in the position of zero attraction, or to be exactly opposite the inner balls in the position for maximum attraction. As an extra precaution, the frame carried a pointer moving over a graduated circle which could be observed from the outside through the glass windows. The setting of the beam exactly was of so much importance that these trial measurements were made with the large balls at *M* and *M'*, at *N* and *N'*, and at *P* and *P'*. From two to three weeks' constant work was required before this central and symmetrical position for the beam was exactly found and the apparatus ready for measurements to be made.

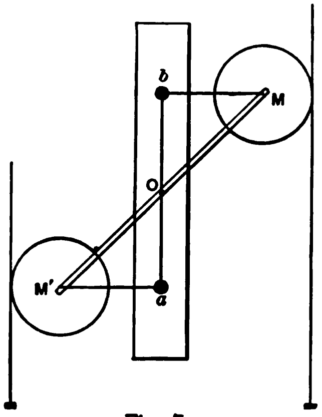


Fig. 7.

Next we have to find the horizontal distance between *M* and *M'* when in their positions for attracting, as in Fig. 7. This was done in three ways in order to be more free from mistake, as any error made is doubled. The first method was to use plumb-lines of quartz fibers set on the edges of the spheres of lead by means of a magnifying glass, and to measure their distance apart by means of a microscope mounted on the platform of a dividing engine. The second method was to measure *OM* and *Ob* accurately and calculate *bM*. These two ways were found equally good, and gave results agreeing to $\frac{1}{20}$ mm. The third way was to note the angle *OMb* which the pointer moved through, and knowing *OM* to calculate *bM*; but as the angle could not be observed as accurately as the distances, this third method was not used.

Three different distances *bM* were used with the same pair of attracting balls. They were 7.421, 5.565, and 3.714 cm.; they are nearly in the ratio of 4 : 3 : 2. The actual deflections of the little masses have to be subtracted; namely, 0.015, 0.025, and 0.069 cm., respectively, making the actual distances 7.406, 5.540, and 3.645 cm. If we now take the sum of the attractions of *M* for *b*, and *M'* for *a*, and subtract the sum of the attractions of *M'* for *b*, and *M* for *a*, we find that the resultant attractions should be as 1 : 2.05 : 5.25. The small masses were those described in the second series of experiments, and the large balls those described in the third. If we take as our error in the distance 2 *Mb* a distance of $\frac{1}{10}$ mm., the error in *Mb* is one-half of this, and the error in the attraction will again double it, which gives us at most 1 in 400 for the shortest distance, and 1 in 800 for the longest.

The deflections were observed as before by means of a telescope and scale at a distance of 323 cm. from the mirror. In spite of all the precautions taken to prevent changes of temperature, the zero-point never remained absolutely steady, but shifted slowly. However, by taking a series of readings and using the average, this was obviated to a large extent; and the averages have a very close agreement, as will be seen. The following are the observed deflections for each of the distances: —

LEAST DISTANCE.

41.0	41.1	40.9	40.95	41.0
41.0	41.1	41.1	41.1	41.1
41.0	41.0	41.1	41.05	41.0
41.1	40.9	40.9	41.0	41.0
41.0	41.1	40.9	41.0	41.0
	41.1	41.1	41.0	40.9
	40.9	41.0	41.1	41.0
	41.1	41.0	41.1	41.1
	41.1	41.0	41.1	41.0
	41.0		40.9	40.95
Mean 41.02	41.04	41.00	41.03	41.00

Mean of all 41.02.

MEAN DISTANCE.

16.1	16.1	16.2
15.9	15.9	15.9
15.95	16.1	16.0
15.9	15.9	15.95
16.05	16.0	16.0
15.95	16.0	15.95
16.0	15.9	16.0
16.05	15.9	16.0
16.0	16.0	
Mean 15.99	15.98	16.00
Mean of all 15.99.		

GREATEST DISTANCE.

7.8	7.8	7.9	7.7	7.9	7.7
7.7	7.6	7.6	7.9	7.65	7.9
7.8	8.0	7.9	7.8	7.85	7.8
7.75	7.7	7.7	7.9	7.8	7.7
7.8	7.8	7.8	7.8	7.8	7.8
7.8	8.0	7.8	7.7	7.8	7.75
7.8	7.7	7.7	7.8	7.8	7.8
7.7	7.9	7.8	7.85		7.75
7.9	7.7	7.8	7.8		7.7
7.65	7.8	7.8	7.7		
7.8	7.8	7.7	7.85		
7.8		7.8			
7.9		7.8			
7.7		7.8			
7.8		7.7			
7.8		7.8			
Mean 7.78	7.80	7.78	7.80	7.80	7.77
Mean of all 7.79					

The number of readings at the mean distance was the last taken, and is not as great as for the other two on account of the instrument having been slightly disturbed by a blow; but the readings are so consistent that they are sufficient. An attempt was now made to find the deflection due to the wires *cd* and *ef* alone, but it could not be done at all well. It seemed to be about 1 mm. for the nearest distance. This is about 1 part in 400, and as that is about the amount of error in the general method, the effect of the wires was neglected.

The tangents of twice the angles of deflections are then $\frac{7.79}{323}$, $\frac{15.99}{323}$, and $\frac{41.02}{323}$, and hence the angles of deflection are 82.84, 170.05, and 434.36, minutes respectively, which are in the ratio of 1 : 2.04 : 5.24. If we compare this with the calculated ratio 1 : 2.05 : 5.25, we find the agreement remarkably good, being out less than 1 part in 500 for the case of the greatest and least distances.

In conclusion, then, the law of Newton has been found to be true for the attractions of non-isotropic masses and for isotropic masses at distances apart as small as 3 or 4 cm.

APPENDIX.

To find the potential at a point outside a sphere made up of equidistant homogeneous disks, parallel to the plane of XY.

Let AB be a disk of radius b, z its vertical distance from the plane XY, $OP=r$, $COP=\theta$, $R=\sqrt{b^2+z^2}$; to find the potential at P, of a number of such disks which just make up the sphere.

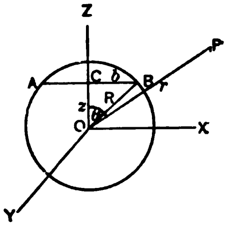


Fig. 8.

We have for the potential of a disk the equation

$$V=\frac{\pi\lambda}{r}\Bigg[(R^2-z^2)+(R^2-z^2)z\frac{P_1(\theta)}{r}+\frac{1}{2}\left\{3\frac{R^2-z^2}{2}z^2-1\frac{R^4-z^4}{4}\right\}\frac{P_2(\theta)}{r^2}$$

$$+\left\{5\frac{R^2-z^2}{2}z^3-3\frac{R^4-z^4}{4}z\right\}\frac{P_3(\theta)}{r^3}+\text{etc.}$$

$$+A\left\{a_n\frac{R^2-z^2}{2}z^{2n}+a_{n-1}\frac{R^4-z^4}{4}z^{2n-2}+\text{etc.}+a_0\frac{R^{2n+2}-z^{2n+2}}{2n+2}\right\}\frac{P_{2n}(\theta)}{r^{2n}}+\text{etc.}\Bigg],$$

where A is a constant, and a_n, a_{n-1}, $\cdots$ a_1, a_0, are the coefficients of $P_{2n}(\theta)$ in the formula

$$P_{2n}(\theta) = \frac{1}{2^{n+1}}\left(a_n\mu^{2n} + a_{n-1}\mu^{2n-2} + \cdots + a_0\right)$$

The reason for using even harmonics will be seen presently.

Simplifying this expression, we have

$$V = \frac{\pi\lambda}{r}\Bigg[\{R^2 - z^2\}\frac{P_0(\theta)}{r^0} + \{R^2z - z^3\}\frac{P_1(\theta)}{r} - \tfrac{1}{8}\{R^4 - 6R^2z^2 + 5z^4\}\frac{P_2(\theta)}{r^2}$$

$$+ \text{etc.} + \frac{A}{2}\left\{\frac{a_n}{1}z^{2n}R^2 + \frac{a_{n-1}}{2}z^{2n-2}R^4 + \text{etc.} + \frac{a_0}{n+1}R^{2n+2}\right.$$

$$\left. - \left(\frac{a_n}{1} + \frac{a_{n-1}}{2} + \text{etc.} + \frac{a_0}{n+1}\right)z^{2n+2}\right\}\frac{P_{2n}(\theta)}{r^{2n}} + \text{etc.}\Bigg],$$

where λ is the surface density.

Now we wish to sum up the separate potentials of a pile of such disks which make up a sphere.

Call the thickness of each disk $\equiv t$, and distance between any two consecutive ones $\equiv d$, and let there be m such disks on each side of the central one. In our problem m is very large, of the order of magnitude of the number of molecules in a unit of length. Using ρ for the volume density, we have $\lambda = \rho t$. Calling the vertical distances of the centers of the disks from the center of the sphere

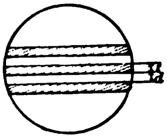


Fig. 9.

$$z_0,\ z_1,\ z_2,\ \cdots\ z_m,\ z_{-1},\ z_{-2},\ \cdots\ z_{-m},$$

we have

$$z_0 = 0,\quad z_1 = t + d,\quad z_2 = 2(t+d),\quad z_m = m(t+d),$$

$$z_{-1} = -(t+d),\quad z_{-2} = -2(t+d),\quad z_{-m} = -m(t+d),\ \text{etc.}$$

Substitute in V these values of z, and we get

$$z = 0 : dV = \frac{\pi\rho t}{r}\Bigg[R^2P_0(\theta) - \tfrac{1}{8}R^4\frac{P_2(\theta)}{r^2} + \text{etc.}$$

$$+ \frac{A}{2}\cdot\frac{a_0}{n+1}R^{2n+2}\frac{P_{2n}(\theta)}{r^{2n}} + \text{etc.}\Bigg] \qquad (1)$$

$$\begin{aligned}
z=t+d:dV=\frac{\pi\rho t}{r}\Big[&\{R^2-(t+d)^2\}\frac{P_0(\theta)}{r^0}+\{R^2(t+d)-(t+d)^3\}\frac{P_1(\theta)}{r}\\
&+\text{etc.}+\frac{A}{2}\Big\{\frac{a_n}{1}R^2(t+d)^{2n}+\frac{a_{n-1}}{2}R^4(t+d)^{2n-2}+\text{etc.}\\
&+\frac{a_0}{n+1}R^{2n+2}-\left(\frac{a_n}{1}+\frac{a_{n-1}}{2}+\text{etc.}+\frac{a_0}{n+1}\right)(t+d)^{2n+2}\Big\}\frac{P_{2n}(\theta)}{r^{2n}}+\text{etc.}\Big].
\end{aligned}\tag{2}$$

$$\begin{aligned}
z=-(t+d):dV&\\
=\frac{\pi\rho t}{r}\Big[&\{R^2-(t+d)^2\}P_0(\theta)-\{R^2(t+d)-(t+d)^3\}\frac{P_1(\theta)}{r}+\text{etc.}\\
&+\frac{A}{2}\Big\{\frac{a_n}{1}R^2(t+d)^{2n}+\frac{a_{n-1}}{2}R^4(t+d)^{2n-2}+\text{etc.}+\frac{a_0}{n+1}R^{2n+2}\\
&-\left(\frac{a_n}{1}+\frac{a_{n-1}}{2}+\text{etc.}+\frac{a_0}{n+1}\right)(t+d)^{2n+2}\Big\}\frac{P_{2n}(\theta)}{r^{2n}}+\text{etc.}\Big].
\end{aligned}\tag{3}$$

$$\begin{aligned}
z=m(t+d):dV=\frac{\pi\rho t}{r}\Big[&\{R^2-m^2(t+d)^2\}P_0(\theta)\\
&+\{R^2m(t+d)-m^3(t+d)^3\}\frac{P_1(\theta)}{r}+\text{etc.}+\frac{A}{2}\Big\{\frac{a_n}{1}R^2m^{2n}(t+d)^{2n}\\
&+\frac{a_{n-1}}{2}R^4m^{2n-2}(t+d)^{2n-2}+\text{etc.}+\frac{a_0}{n+1}R^{2n+2}\\
&-\left(\frac{a_n}{1}+\frac{a_{n-1}}{2}+\text{etc.}+\frac{a_0}{n+1}\right)m^{2n+2}(t+d)^{2n+2}\Big\}\frac{P_{2n}(\theta)}{r^{2n}}+\text{etc.}\Big].
\end{aligned}\tag{4}$$

$$\begin{aligned}
z=-m(t+d):dV&\\
=\frac{\pi\rho t}{r}\Big[&\{R^2-m^2(t+d)^2\}P_0(\theta)-\{R^2m(t+d)-m^3(t+d)^3\}\frac{P_1(\theta)}{r}+\text{etc.}\\
&+\frac{A}{2}\Big\{\frac{a_n}{1}R^2m^{2n}(t+d)^{2n}+\frac{a_{n-1}}{2}R^4m^{2n-2}(t+d)^{2n-2}+\text{etc.}+\frac{a_0}{n+1}R^{2n+2}\\
&-\left(\frac{a_n}{1}+\frac{a_{n-1}}{2}+\text{etc.}+\frac{a_0}{n+1}\right)m^{2n+2}(t+d)^{2n+2}\Big\}\frac{P_{2n}(\theta)}{r^{2n}}+\text{etc.}\Big].
\end{aligned}\tag{5}$$

Now when we go to sum, it is evident that the terms with odd harmonics disappear two and two, because they contain z to odd powers only; thus (2) and (3) cancel, (4) and (5), and so on. Using the symbol

$${}_mS_r\equiv 1^r+2^r+3^r+\text{etc.}\cdots+m^r,$$

we have

$$\Sigma dV = V = \frac{\pi\rho t}{r}\Big[\{(2m+1)R^2 - 2\,{}_mS_2(t+d)^2\}P_0(\theta) + \text{etc.}$$

$$+A\Big\{-\Big(\frac{a_n}{1}+\frac{a_{n-1}}{2}+\text{etc.}+\frac{a_0}{n+1}\Big){}_mS_{2n+2}(t+d)^{2n+2}+\frac{a_n}{1}{}_mS_{2n}R^2(t+d)^{2n}+\text{etc.}$$

$$+\frac{a_0}{n+1}R^{2n+2}\Big\}\frac{P_{2n}(\theta)}{r^{2n}}+\text{etc.}\Big].$$

Now $t+d=\frac{1}{m}R$, and if we call $t=\frac{1}{k}(t+d)$, then $t=\frac{1}{km}R$. Substituting these values, we get

$$V=\frac{\pi\rho}{kr}\Big[\Big\{\frac{2m+1}{m}-2\frac{{}_mS_2}{m^3}\Big\}R^3P_0(\theta)+\text{etc.}$$

$$+A\Big\{-\Big(\frac{a_n}{1}+\frac{a_{n-1}}{2}+\text{etc.}+\frac{a_0}{n+1}\Big)\frac{{}_mS_{2n+2}}{m^{2n+3}}+\frac{a_n}{1}\,\frac{{}_mS_{2n}}{m^{2n+1}}+\text{etc.}+\frac{a_0}{n+1}\,\frac{1}{m}\Big\}$$

$$\frac{P_{2n}(\theta)}{r^{2n}}R^{2m+3}+\text{etc.}\Big].$$

Now (from Todhunter's Algebra, p. 415) the term of ${}_mS_r$ with the highest power of m is $\frac{1}{r+1}m^{r+1}$, but all the coefficients of the general term of the last equation are of the form

$$\frac{1}{m^{r+1}}\,{}_mS_r=\frac{1}{r+1}+A\frac{1}{m}+B\frac{1}{m^2}+\text{etc.}$$

Hence the only part of that term which is of the first order of magnitude is

$$A\left\{-\frac{\Big(\frac{a_n}{1}+\frac{a_{n-1}}{2}+\text{etc.}+\frac{a_0}{n+1}\Big)}{2n+3}+\frac{\frac{a_n}{1}}{2n+1}+\frac{\frac{a_{n-1}}{2}}{2n}+\text{etc.}+\frac{\frac{a_0}{n+1}}{1}\right\},$$

which can be written

$$A\Big\{\frac{a_n}{2n+1}+\frac{a_{n-1}}{2n-1}+\text{etc.}+\frac{a_1}{3}+\frac{a_0}{1}\Big\}\frac{2}{2n+3}.$$

From Todhunter's "Functions of Laplace, Lamé, and Bessel," p. 17, we have

$$\frac{1}{2^{n+1}}\left(\frac{a_n}{k+2n+1}+\frac{a_{n-1}}{k+2n-1}+\text{etc.}+\cdots\right)=\frac{k(k-2)(k-4)\cdots(k-2n+2)}{(k+2n+1)(k+2n-1)\cdots(k+1)},$$

where k has any positive value. Let $k=0$; then

$$\frac{a_n}{2n+1}+\frac{a_{n-1}}{2n-1}+\text{etc.}+\frac{a_1}{3}+\frac{a_0}{1}=0,$$

or the part of the coefficients of the harmonics that is of the first order is zero in every case, and V can only have terms in the spherical harmonics of the form $\left(\frac{A}{m}+\frac{B}{m^2}+\text{etc.}\right)P^2_n(\theta)$. Hence, neglecting terms which are not of the first order of magnitude, our equation for V reduces to

$$V=\frac{\pi\rho}{kr}\left[\frac{2m+1}{m}-2\frac{{}_mS_2}{m^3}\right]R^3=\frac{\pi\rho}{kr}\left[2+\frac{1}{m}-\frac{1}{3}\frac{m(m+1)(2m+1)}{m^3}\right]R^3.$$

Or again, neglecting $\frac{1}{m}$, $\frac{1}{m^2}$, etc.,

$$V=\frac{\pi\rho}{kr}\frac{4}{3}R^3=\frac{1}{k}\cdot\frac{4}{3}\frac{\pi R^3\rho}{r},$$

which is the potential of such a sphere as ordinarily given, assuming the mass $\frac{1}{k}$ of what it would be were it solid.

Hence no terms in θ enter of the first order of magnitude.

Bryn Mawr College, Bryn Mawr, Pa.,
June, 1894.

THE INFLUENCE OF TEMPERATURE UPON THE TRANSPARENCY OF SOLUTIONS.

BY EDWARD L. NICHOLS AND MARY C. SPENCER.

THE experiments to be described in this paper deal with the transparency of certain aqueous solutions, for various wave-lengths of the visible spectrum and at temperatures ranging between 20° and 80° C. For the purposes of these measurements the horizontal slit photometer was used. The instrument was mounted upon a track, at the ends of which two similar incandescent lamps were placed. These lamps were so adjusted as to give as nearly as possible the same quality of light, and in front of one of them, at the observer's right hand, was placed a cell containing the solution to be tested. In Fig. 1, which shows the arrangement of the apparatus, L and L_1 are the lamps, C is the cell containing the solution, and HP is the spectrophotometer.[1]

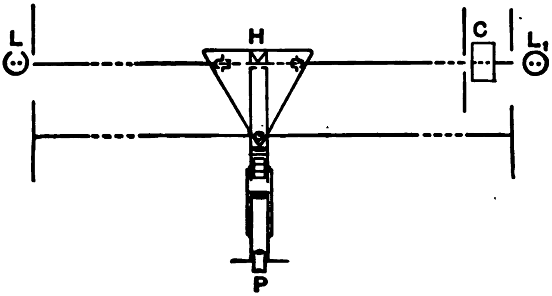


Fig. 1.

Figure 2 gives a cross-section of the cell used for heating the solutions, the transmitting power of which was to be tested. It consisted of a cylinder of brass about 15 cm. in diameter. This cylinder was provided with broad flanges on either end, against which sheets of thin plate glass were securely clamped. A neck in the top of the cylinder allowed of the introduction of the solution and also of the insertion of a thermometer. The distance between the outer faces of the cell was 8.9 cm., and the thickness of the contained layer of liquid 8.5 cm.

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[1] For a description of the horizontal-slit photometer see Transactions of the American Institute of Electrical Engineers, 1890; also PHYSICAL REVIEW, Vol. 2, p. 138.

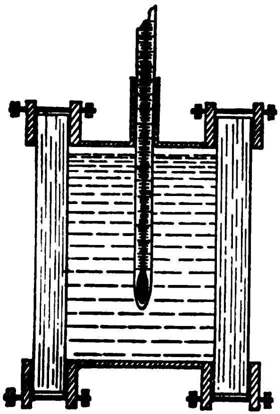

Fig. 2.

The cell was heated by means of an electric current which traversed a coil of wire surrounding the brass cylinder. This coil is not shown in the diagram.

The method of procedure was as follows:—

The observing telescope of the photometer was set so as to bring a given portion of the visible spectrum into the field of view, and readings were made of the positions upon the bar at which the region under inspection was equally bright in the spectra of the two lamps. The cell containing the solution was then interposed and the light ratio was redetermined. A series of readings was taken in this way for the wave-length in question, the temperature of the solution being varied from time to time. The results of these observations were then indicated by means of a curve in which ordinates represented transmitting powers and abscissas temperatures, and the process was repeated until a series of such curves, for different regions of the spectrum, had been obtained. From these it was possible to derive the data for the construction of other curves showing the relative distribution of energy in the absorption spectrum of the solution at any desired temperature within the range covered by the experiment.

It is evident that the method just outlined will give much more definite and precise information concerning the influence of temperature upon the transparency of a solution than it is possible to obtain by means of the methods pursued by previous observers. Although a considerable amount of information upon this subject had already been massed by Gladstone,[1] Melde,[2] Hartley,[3] Conroy,[4] and others, all of whom agree in the general statement that temperature lessens the power of transmitting light, no one, so far as the present writers are aware, had undertaken to determine quantitatively, wave-length by wave-length, degree by degree, the exact amount of this decrease. This, by means of the apparatus just described, the writers have been able to do in the case of a considerable number of solutions.

The solutions tested were for the most part colored salts dissolved in distilled water, the concentration being such as to allow an easily measurable quantity of light to be transmitted, but at the same time to show distinctly the character of the selective absorption, where such existed. An equally important field, that of the study of the influence of temperature upon colorless solutions possessing definite absorption bands of great density (as, for example, the solutions of the salts of the rare earths), was not included in our investigation.

Several modes of expressing the results obtained by the method just indicated suggest themselves. One may, for example, give the transmitting power of the cell containing each of the solutions studied, in terms of that of the same cell when filled with distilled water and subjected to the same temperature changes; or, having determined the loss by reflection and by absorption within the glass walls of the cell, one may reduce the transparency in each case to that of a layer of unit thickness. For the purposes of the present investigation it was deemed sufficient to give the transmission of the filled cell in each case, since it was the comparison of the transmitting power of the solution when cold with that which it possessed at higher temperatures which was desired.

The substances tested were distilled water and aqueous solutions, chiefly of various salts. These, grouped according to their behavior when heated, are given in the following list: —

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[1] Gladstone, Philosophical Magazine (4), Vol. 14, p. 423.

[2] Melde, Annalen der Physik und Chemie, Vol. 126, p. 284.

[3] Hartley, Proceedings of the Royal Society, Vol. 22, p. 241.

[4] Conroy, Philosophical Magazine (5), Vol. 31, p. 371.

(A)
- Cobalt-ammonium sulphate,
- Potassium bichromate.

(B)
- Potassium chromate,
- Sodium bichromate,
- Copper sulphate,
- Chromic sulphate,
- Potassium ferric cyanide.

(C)
- Distilled water,
- Cochineal,
- Sodium nitrate,
- Nickel sulphate.

(D)
- Copper acetate,
- Ferric chloride,
- Potassium permanganate,
- Copper chloride,
- Nickel chloride.

Measurements at a single temperature were also made with the empty cell interposed between the spectrophotometer and the source of light, with results which are given in Table I.

TABLE I.

TRANSPARENCY OF THE EMPTY CELL.

Wave-length	0.6867 μ	0.5896	0.4700 μ	0.4080 μ
Transmission	0.709	0.763	0.783	0.785

Experiments upon the substances of Group C having shown the influence of heat upon the transmitting power of the filled cell to be inappreciable, it was deemed unnecessary to make measurements with the empty cell at high temperatures.

The quantity denoted by the term "transmission" in the above, and in all subsequent tables, is the ratio of the light reaching the photometer through the cell to that which reaches the instrument directly when the cell is withdrawn.

The results of our measurements upon the solutions of the preceding list are contained in the following tables, and they are indicated graphically in some cases by means of the two species of diagram already referred to.

The first of these is an isochromatic curve showing the change in transmitting power of a single wave-length with varying temperature; the other is an isothermal curve by means of which the distribution of transparency in the visible spectrum is given for a single temperature.

It will be seen from the following tables that the solutions tested fall naturally into the four groups already given: —

(1) Solutions suffering *temporary* change of transmitting power throughout the spectrum when heated (Group A).

(2) Solutions suffering *temporary* decrease of transparency in certain regions, but not throughout the entire spectrum (Group B).

(3) Liquids unaffected as regards transparency by temperature changes up to 80° (Group C).

(4) Solutions suffering *permanent* change of color as the result of heating (Group D). Such solutions do not return to their original condition when cooled.

Measurements upon Group D were made only for a single region of the spectrum. It will be seen that as a rule they suffer diminution of transmitting power with rise of temperature. This sometimes occurs continuously, but with increasing rapidity, as in the cases of ferric chloride and copper chloride, while sometimes a sudden and very marked falling off of transparency occurs at a definite temperature, as in the case of copper acetate and nickel chloride. One of the solutions of this group, potassium permanganate, gains materially in transparency for the wave-length tested, a result due to a gradual decomposition brought about by heating.

GROUP A.

Solutions showing temporary loss throughout the spectrum.

TABLE II.

COBALT AMMONIUM SULPHATE (8 GRMS. IN 300 CC. H_2O).

	Wave-length 0.6867 μ.						
Temperature	18° 22°	28° 31°	38° 42°	48° 56°	59° 64°	69° 74°	78° 82°
Transmission	0.76	0.76	0.69	0.68	0.65	0.64	0.61
	Wave-length 0.5896 μ.						
Temperature	19° 23°	26° 33°	38° 42°	48° 52°	58° 61°	68° 72°	78° 81°
Transmission	0.58	0.57	0.56	0.56	0.52	0.51	0.50

Table II. (*continued*).

Wave-length 0.5500 μ.							
Temperature	20°	30°	39° 38°	49° 50°	60°	71°	80°
Transmission	0.21	0.20	0.19	0.19	0.17	0.15	0.15

Wave-length 0.5200 μ.							
Temperature	20°	31°	40°	50°	60°	69°	80°
Transmission	0.07	0.06	0.06	0.05	0.05	0.05	0.04

Wave-length 0.4900 μ.							
Temperature	21°	30°	40°	49°	61°	69°	80°
Transmission	0.07	0.067	0.05	0.06	0.06	0.05	0.05

Wave-length 0.4700 μ.							
Temperature	20°	31°	40°	49°	60°	70°	80°
Transmission	0.14	0.12	0.11	0.10	0.10	0.10	0.10

Wave-length 0.4550 μ.							
Temperature	22°	30°	39°	50°	60°	69°	80°
Transmission	0.08	0.07	0.07	0.07	0.06	0.05	0.05

The above results for cobalt ammonium sulphate, which may be regarded as typical, are indicated graphically by means of both forms of diagram in Fig. 3.

In this and in all solutions tested, excepting those which suffered permanent change as the result of heating, the isochromatic diagrams (within the limits of accuracy) *were straight lines.*

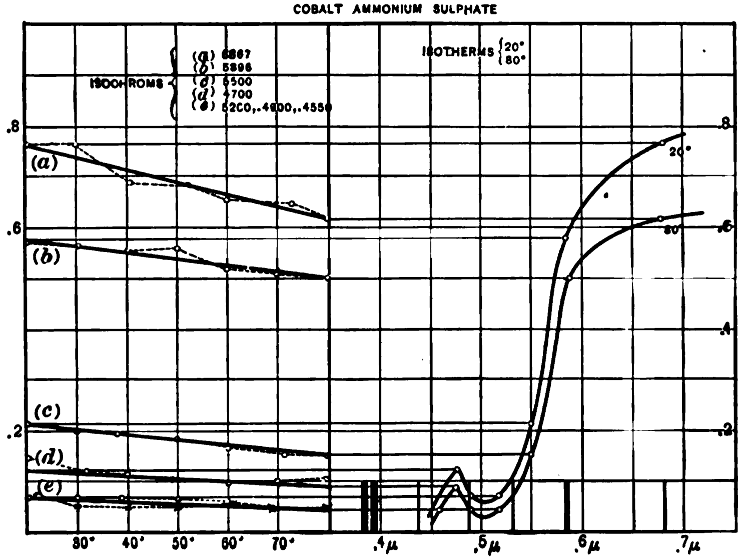

Fig. 3.

Table III.

POTASSIUM BICHROMATE (4 grms. in 300 cc. H_2O).

Wave-length 0.6867 μ.

Temperature	20°	29°	40°	50°	60°	69°	79°
Transmission	0.76	0.75	0.74	0.73	0.72	0.71	0.69

Wave-length 0.6200 μ.

Temperature	21°	31°	41°	51°	59°	69°	80°
Transmission	0.89	0.85	0.85	0.89	0.84	0.85	0.85

Wave-length 0.5650 μ.

Temperature	20°	26°	31°	38°	50°	60°	70°	79°
Transmission	0.66	0.64	0.64	0.63	0.58	0.54	0.53	0.52

Wave-length 0.5600 μ.

Temperature	20°	30°	40°	50°	60°	69°	79°
Transmission	0.44	0.44	0.40	0.38	0.35	0.32	0.28

Wave-length 0.5500 μ.

Temperature	21°	30°	39°	50°	58°	70°	79°
Transmission	0.18	0.17	0.15	0.12	0.10	0.09	0.09

Wave-length 0.5375 μ.

Temperature	20°	29°	43°	50°	60°	70°	80°
Transmission	0.11	0.10	0.09	0.08	0.07	0.06	0.00

Isotherms of potassium bichromate, for 20° and 80° respectively, are given in Fig. 4.

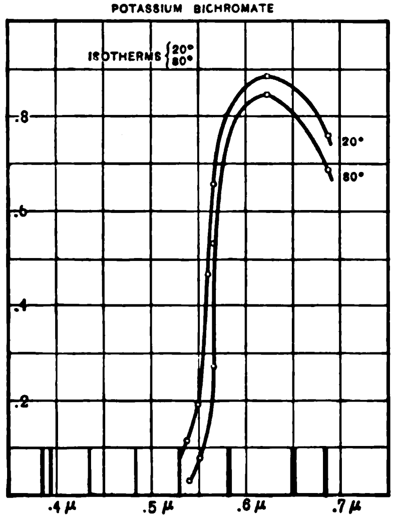

Fig. 4.

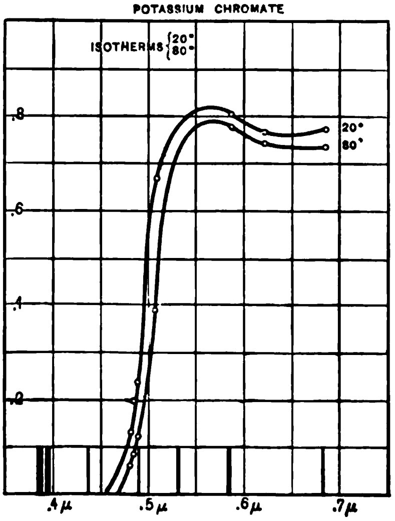

Fig. 5.

TABLE IV.

POTASSIUM CHROMATE[1] (4 GRMS. IN 300 CC. H_2O).

Wave-length 0.6867 μ.

Temperature	21°	28°	40°	49°	60°	69°	80°
Transmission	0.78	0.78	0.77	0.75	0.76	0.76	0.72

Wave-length 0.6200 μ.

Temperature	21°	31°	42°	50°	60°	70°	80°
Transmission	0.76	0.78	0.77	0.77	0.75	0.76	0.76

Wave-length 0.5896 μ.

Temperature	27°	39°	47°	54°	60°	65°	73°	82°
Transmission	0.80	0.80	0.79	0.79	0.77	0.79	0.79	0.77

Wave-length 0.5050 μ.

Temperature	20°	33°	42°	53°	59°	70°	78°	84°
Transmission	0.63	0.63	0.61	0.50	0.50	0.43	0.39	0.36

Wave-length 0.4900 μ.

Temperature	20°	28°	40°	46°	58°	65°	76°
Transmission	0.23	0.21	0.21	0.19	0.19	0.16	0.13

Wave-length 0.4860 μ.

Temperature	22°	29°	48°	62°	73°	82°
Transmission	0.20	0.18	0.13	0.11	0.09	0.08

Wave-length 0.4800 μ.

Temperature	24°	46°	59°	70°	75°
Transmission	0.12	0.10	0.09	0.07	0.00

Isotherms of potassium chromate, for 20° and 80° respectively, are given in Fig. 5.

---

[1] Potassium chromate forms an intermediate member of Groups A and B. In one of the regions explored (0.6200 μ – 0.5900 μ), the influence of temperature was almost imperceptible.

GROUP B.

Solutions showing temporary loss throughout a portion of the spectrum.

Table V.

SODIUM BICHROMATE (4 grms. in 450 cc. H_2O).

Wave-length 0.6867 μ.

Temperature	22°	30°	40°	50°	60°	69°	80°
Transmission	0.74	0.78	0.80	0.78	0.72	0.73	0.74

Wave-length 0.6200 μ.

Temperature	22°	29°	39°	50°	59°	69°	80°
Transmission	0.82	0.83	0.83	0.83	0.83	0.84	0.83

Wave-length 0.5896 μ.

Temperature	20°	30°	40°	50°	59°	70°	80°
Transmission	0.71	0.71	0.73	0.71	0.72	0.71	0.71

Wave-length 0.5500 μ.

Temperature	21°	30°	40°	50°	60°	70°	79°
Transmission	0.51	0.48	0.47	0.44	0.41	0.38	0.37

Wave-length 0.5350 μ.

Temperature	22°	30°	39°	49°	59°	71°	80°
Transmission	0.13	0.11	0.10	0.09	0.08	0.09	0.07

Wave-length 0.5230 μ.

Temperature	21°	29°	39°	49°	61°	70°	79°
Transmission	0.10	0.09	0.08	0.08	0.07	0.06	0.05

Isotherms of sodium bichromate, for 20° and 80° respectively, are given in Fig. 6.

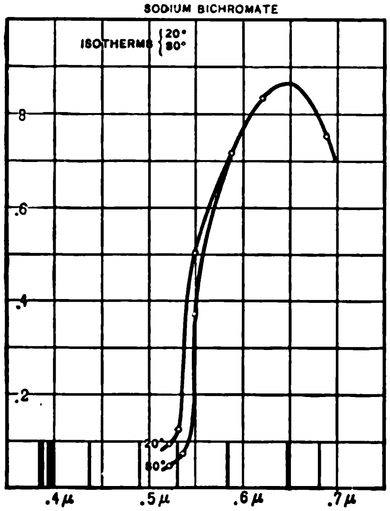

Fig. 6.

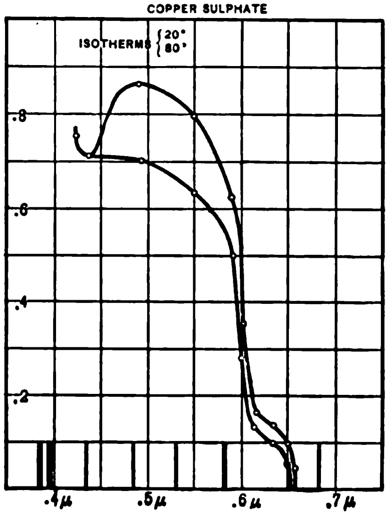

Fig. 7.

TABLE VI.

COPPER SULPHATE (8 GRMS. IN 300 CC. H_2O).

Wave-length 0.6550 μ.

Temperature	20°	30°	47°
Transmission	0.06	0.01	0.008

Wave-length 0.6500 μ.

Temperature	20°	30°	40°	52°	60°	70°	78°
Transmission	0.10	0.09	0.08	0.08	0.07	0.07	0.06

Wave-length 0.6300 μ.

Temperature	21°	29°	40°	49°	60°	69°	77°	81°
Transmission	0.16	0.15	0.15	0.13	0.12	0.11	0.11	0.11

Wave-length 0.6200 μ.

Temperature	21°	30°	41°	51°	59°	69°	81°
Transmission	0.19	0.17	0.16	0.15	0.15	0.14	0.13

Wave-length 0.6000 μ.

Temperature	20°	30°	40°	50°	60°	70°	80°
Transmission	0.36	0.34	0.34	0.32	0.32	0.30	0.30

Wave-length 0.5896 μ.

Temperature	21°	31°	40°	51°	59°	71°	80°
Transmission	0.63	0.60	0.56	0.53	0.52	0.52	0.51

Wave-length 0.5500 μ.

Temperature	21°	30°	39°	52°	60°	69°	81°
Transmission	0.80	0.80	0.72	0.72	0.69	0.67	0.65

Wave-length 0.4900 μ.

Temperature	21°	30°	40°	49°	59°	70°	80°
Transmission	0.85	0.81	0.82	0.80	0.78	0.76	0.76

Wave-length 0.4350 μ.

Temperature	20°	32°	40°	50°	60°	70°	80°
Transmission	0.71	0.71	0.71	0.70	0.73	0.73	0.73

Wave-length 0.4250 μ.

Temperature	22°	30°	40°	50°	59°	70°	79°
Transmission	0.76	0.74	0.75	0.75	0.76	0.77	0.75

Isotherms of copper sulphate, for 20° and 80° respectively, are given in Fig. 7.

TABLE VII.

CHROMIC SULPHATE (7 GRMS. IN 800 CC. H_2O).

Wave-length 0.6867 µ.

Temperature	22°	30°	40°	49°	61°	69°	79°
Transmission	0.52	0.50	0.49	0.49	0.48	0.44	0.43

Wave-length 0.6200 µ.

Temperature	20°	29°	39°	49°	59°	71°	80°
Transmission	0.25	0.24	0.24	0.23	0.22	0.22	0.21

Wave-length 0.5896 µ.

Temperature	21°	29°	30°	39°	60°	70°	79°
Transmission	0.13	0.12	0.12	0.11	0.11	0.10	0.10

Wave-length 0.5500 µ.

Temperature	20°	30°	39°	50°	60°	69°	80°
Transmission	0.14	0.14	0.13	0.13	0.12	0.12	0.12

Wave-length 0.5200 µ.

Temperature	22°	30°	41°	51°	60°	70°	80°
Transmission	0.37	0.37	0.37	0.37	0.37	0.37	0.37

Wave-length 0.4700 µ.

Temperature	20°	31°	40°	50°	60°	70°	80°
Transmission	0.56	0.56	0.56	0.54	0.50	0.47	0.44

Wave-length 0.4350.

Temperature	22°	30°	40°	50°	59°	71°	80°
Transmission	0.45	0.43	0.43	0.42	0.42	0.39	0.38

Isotherms of chromic sulphate, for 20° and 80° respectively, are given in Fig. 8.

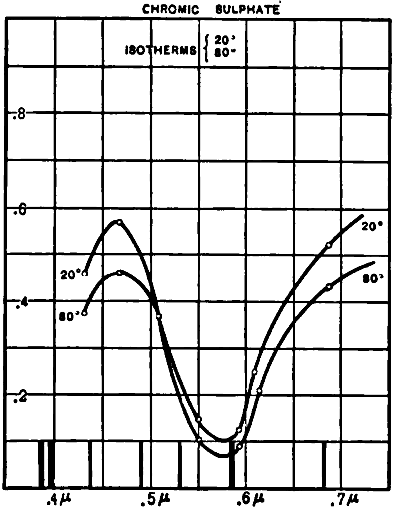

Fig. 8.

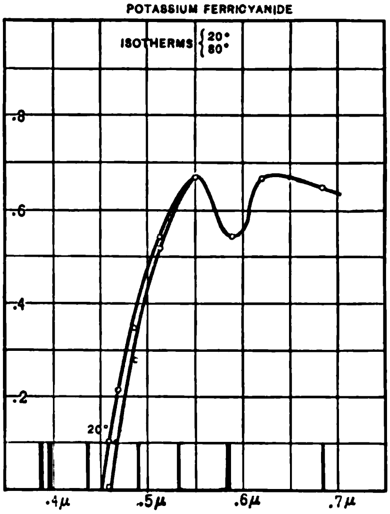

Fig. 9.

TABLE VIII.

POTASSIUM FERRICYANIDE (4 GRMS. IN 150 CC. H_2O).

Wave-length 0.6876 μ.							
Temperature	24°	30°	44°	58°	69°	80°	
Transmission	0.61	0.64	0.64	0.64	0.64	0.63	
Wave-length 0.6200 μ.							
Temperature	20°	32°	39°	49°	60°	70°	80°
Transmission	0.69	0.62	0.67	0.66	0.67	0.69	0.67
Wave-length 0.5896 μ.							
Temperature	22°	35°	54°	62°	69°	78°	
Transmission	0.54	0.55	0.54	0.53	0.54	0.54	
Wave-length 0.5500 μ.							
Temperature	20°	32°	40°	49°	59°	69°	81°
Transmission	0.67	0.68	0.67	0.67	0.68	0.67	0.67
Wave-length 0.5200 μ.							
Temperature	22°	38°	45°	55°	64°	74°	
Transmission	0.59	0.56	0.56	0.56	0.56	0.53	
Wave-length 0.4900 μ.							
Temperature	20°	31°	41°	51°	60°	69°	80°
Transmission	0.36	0.33	0.32	0.31	0.30	0.29	0.29
Wave-length 0.4700 μ.							
Temperature	21°	41°	53°	66°	75°	82°	
Transmission	0.19	0.17	0.16	0.13	0.11	0.10	
Wave-length 0.4625 μ.							
Temperature	20	40°	52°	63°	70°		
Transmission	0.12	0.07	0.07	0.06	0.00		
Isotherms of potassium ferricyanide, for 20° and 80° respectively, are given in Fig. 9.							

GROUP C.

Liquids unaffected by change of temperature.

TABLE IX.

DISTILLED WATER.

Wave-length 0.6867 μ.

Temperature	22°	30°	40°	50°	60°	70°	80°
Transmission	0.82	0.82	0.83	0.83	0.83	0.83	0.83

Wave-length 0.5200 μ.

Temperature	22°	29°	40°	49°	59°	69°	80°
Transmission	0.72	0.73	0.72	0.72	0.71	0.71	0.71

Wave-length 0.4250 μ.

Temperature	20°	30°	40°	50°	60°	70°	80°
Transmission	0.81	0.82	0.81	0.82	0.82	0.83	0.82

TABLE X.

COCHINEAL (3 GRMS. IN 600 CC. H_2O).

Wave-length 0.6867 μ.

Temperature	23°	31°	41°	49°	59°	70°	80°
Transmission	0.76	0.76	0.76	0.75	0.75	0.75	0.76

Wave-length 0.6200 μ.

Temperature	20°	30°	40°	50°	60°	70°	80°
Transmission	0.33	0.35	0.34	0.34	0.35	0.34	0.34

Wave-length 0.5896 μ.

Temperature	22°	30°	40°	50°	60°	70°	80°
Transmission	0.18	0.17	0.17	0.16	0.16	0.17	0.17

Wave-length 0.5350 μ.

Temperature	20°	30°	40°	50°	59°	70°	80°
Transmission	0.40	0.41	0.40	0.39	0.39	0.40	0.40

TABLE XI.

SODIUM NITRATE (4 GRMS. IN 300 CC. H_2O).

Wave-length 0.6867 μ.							
Temperature	20°	30°	40°	50°	61°	70°	79°
Transmission	0.78	0.79	0.79	0.80	0.80	0.80	0.80
Wave-length 0.5500 μ.							
Temperature	20°	30°	40°	50°	60°	70°	80°
Transmission	0.88	0.88	0.86	0.86	0.84	0.84	0.86
Wave-length 0.4350 μ.							
Temperature	20°	30°	40°	50°	60°	70°	80°
Transmission	0.84	0.85	0.85	0.82	0.83	0.82	0.82

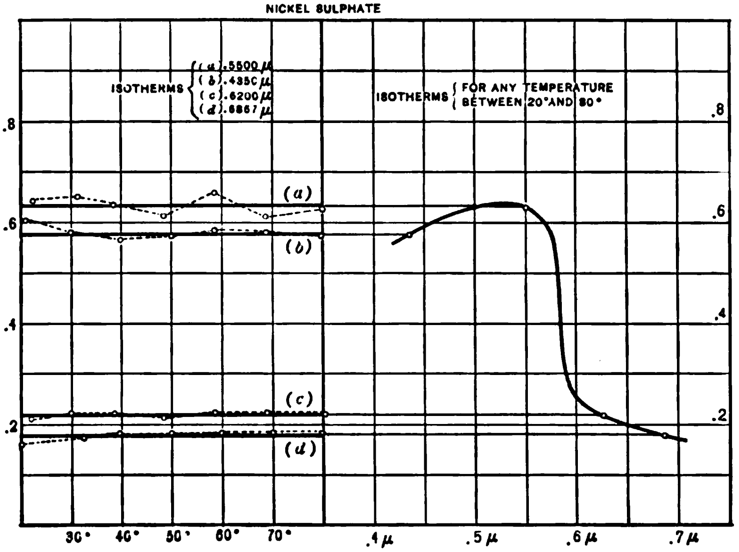

Fig. 10.

TABLE XII.

NICKEL SULPHATE (4 GRMS. IN 300 CC. H_2O).

Wave-length 0.6867 μ.							
Temperature	20°	32°	40°	50°	60°	70°	80°
Transmission	0.17	0.18	0.19	0.19	0.19	0.19	0.19

Table XII. (*continued*).

Wave-length 0.6200 μ.

Temperature	23°	30°	39°	49°	59°	69°	81°
Transmission	0.21	0.22	0.22	0.21	0.22	0.22	0.22

Wave-length 0.5500 μ.

Temperature	22°	31°	39°	49°	59°	69°	80°
Transmission	0.64	0.65	0.63	0.61	0.66	0.61	0.62

Wave-length 0.4350 μ.

Temperature	21°	30°	40°	50°	59°	69°	80°
Transmission	0.61	0.58	0.57	0.58	0.59	0.59	0.58

A typical example of Group C, in which the isochroms are all horizontal straight lines is nickel sulphate, the characteristics of which are indicated by both forms of diagram in Fig. 10. The observational errors, always large in colorimetric work, are more than usually prominent in this case, particularly in the region 0.55 μ, but it is evident from the diagram that in none of the regions studied is the transparency of nickel sulphate modified to a determinable degree by change of temperature.

GROUP D.

Solutions suffering permanent change of color as the result of heating.

Table XIII.

COPPER ACETATE (4 grms. in 300 cc. H_2O).

Temperature	22°	30°	39°	50°	60°	70°	80°	λ = 0.550 μ.
Transmission	0.52	0.51	0.48	0.46	0.42	0.40	0.00	

Table XIV.

FERRIC CHLORIDE (4 grms. in 150 cc. H_2O).

Temperature	24°	38°	41°	50°	60°	λ = 0.505 μ.
Transmission	0.15	0.14	0.13	0.07	0.00	

Table XV.

POTASSIUM PERMANGANATE (1 GRM. IN 1000 CC. H_2O).

Temperature	20°	30°	40°	50°	59°	69°	80°	$\lambda = 0.620\ \mu$.
Transmission	0.27	0.35	0.37	0.39	0.41	0.42	0.49	

Table XVI.

COPPER CHLORIDE (4 GRMS. IN 300 CC. H_2O).

Temperature	21°	29°	40°	50°	60°	$\lambda = 0.470\ \mu$.
Transmission	0.61	0.58	0.54	0.43	0.00	

Table XVII.

NICKEL CHLORIDE (4 GRMS. IN 300 CC. H_2O).

Temperature	20°	30°	40°	50°	60°	$\lambda = 0.550\ \mu$.
Transmission	0.12	0.12	0.11	0.09	0.00	

The results of the above measurements upon Group D are embodied in the diagrams of Fig. 11.

With reference to the results given in numerical and graphical form in this paper, it is probably not without significance to note that in general the influence of a rise of temperature upon the color of pigments and of solutions is the same. The tendency in both cases is towards increased absorption, the loss of transmitting power being selective rather than of uniform distribution throughout the visible spectrum. It is of interest, moreover, to note from Table IX. that water itself suffers no loss of transparency and no appreciable change of color when heated.

From a more detailed study of various solutions, especially of the group considered last, much that is interesting and important might doubtless be derived. The importance, likewise, of making measurements of this character upon solutions possessing definite absorption bands, to which reference has been made in a previous paragraph, is obvious. The investigation might be extended with great interest also to the infra-red and to the ultra-violet, in which important regions of the spectrum the transmitting power of very many substances, so far as quantitive measurements are concerned, to say nothing of the influence of temperature upon that property, remains as yet unknown. If the incomplete experiments described here serve to indicate, even dimly, the direction to be given to further research in this field, the publication of them will have fulfilled its purpose.

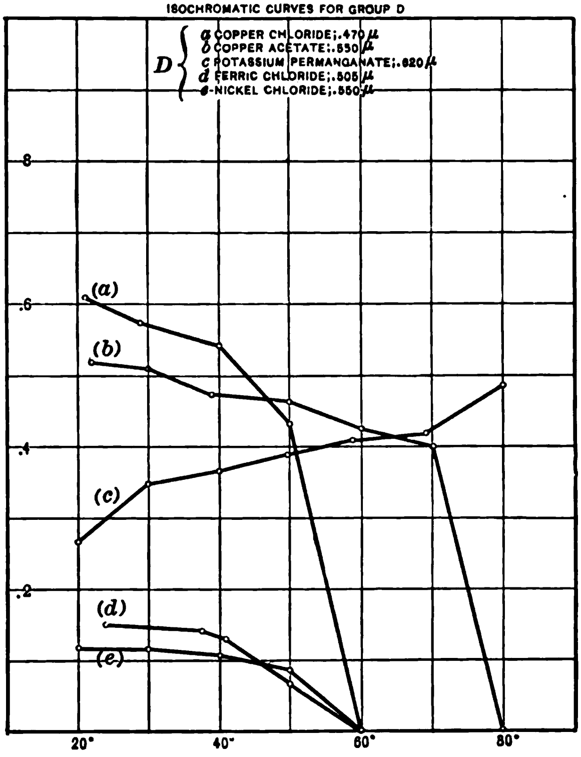

Fig. 11.

AUGUST, 1894.

DETERMINATION OF THE ELECTRIC CONDUCTIVITY OF CERTAIN SALT SOLUTIONS.

By Albert C. MacGregory.

Introduction.

DURING the last few years a great deal of work has been done in the line of determining the electric conductivity of certain salts in their water solutions. Two objects are accomplished by means of such observations; namely, the determination of the velocities of the so-called ions, and the gaining of some insight into the condition of the salts when in solution. It is true that the same objects may be accomplished by observations of the depression of the freezing-point, but owing to the ease and accuracy of making the observations, the following method seems to be preferable.

The following determinations fill in a few vacancies in the list of results heretofore obtained for salts of the alkaline earths and acetates.

Apparatus.

The resistance bottles used were the same as those employed by Professor Kohlrausch in his work.

As the solutions varied in strength from $m = 0.0001$ to $m = 1.00$ (m denotes the gram equivalent in the liter of solution), it was necessary to employ two resistance bottles, one for the solutions from $m = 0.0001$ to $m = 0.01$, and the other for the remaining ones.

In measuring the most dilute solutions I employed a Becher glass of 80 mm. diameter and 130 mm. high: the edge was ground off even and fitted with a hard rubber cover 10 mm. thick and 110 mm. across. By means of three small pieces of rubber on the under side all lateral movement was avoided. Three holes were bored through the cover: two of them were situated on either side of the center at a distance of 25 mm. from the same, and through them passed heavy copper wires. At the upper end of these wires binding posts were placed, while at the lower end were fastened large platinum wires ending in the electrodes of heavy platinum foil, 40 mm. square. The wire and foil were fastened together by welding and soldering, and were afterwards platinized in the usual manner. The third hole contained the thermometer, which entered the solution midway between the electrodes on a line connecting the outer edge.

With a normal filling of 500 cc. the electrodes dipped about 20 mm. under the surface of the liquid. The glass was entirely surrounded by felt.

The thermometer used was graduated to 0.°2, thus giving readings to 0.02°. The thermometer was tested by comparison with one from the Reichsanstalt. The bottle used for the other solutions consisted of two small glass bottles joined by a tube about 6 mm. in diameter. The bottles were provided with glass stoppers. After being filled, the bottle was placed in a water bath and the thermometer hung next to the connecting tube. Owing to the heat produced by the passage of the current the bridge readings were made as rapidly as possible. It was always necessary to wait until the temperature of the solution was the same as that of the bath before the readings were taken. The Kohlrausch roller bridge which was used was supplied with resistances of 1, 10, 100, and 1000 ohms, but for the most part only the 100 and the 1000 were used, their actual value being carefully determined by comparison with a normal instrument. By the use of a rheostat having resistances up to 4000 ohms the bridge wire was calibrated for every $\frac{1}{100}$ of its length. Whenever it was possible the resistances were determined by the use of both the 100 and 1000 ohms, and the middle taken as the true value. If the two values did not agree within 0.2 per cent, the bridge was again calibrated. Twelve calibrations in all were made during the course of the work. At one end of the wire the corrections remained quite constant, while at the other end there was a variation between the beginning to the completion of my work equivalent to about 0.0005 of the entire length of the wire.

Resistance Capacity of the Bottles.

The resistance capacities of the two bottles were determined by the use of normal and part normal solutions of KCl and NaCl. Before the solutions were used they were carefully tested by comparing their conductivity and specific gravity with existing data.

The capacity of the larger bottle was found by the use of solutions having a strength $m = 0.05$, $m = 0.02$, $m = 0.01$, the conductivity of these solutions at 18° being, according to Kohlrausch, 5415, 2246, and 1147 respectively for KCl, and 4485, 1876, and 962 for NaCl (Hg. = 1). These values are to be multiplied by 10^{-10}. For the solutions used in calibrating the smaller bottle the values of m were 1.00, 0.5, and 0.1.

The mercury capacities thus found for the larger bottle were as follows:—

Salt	$m =$ 0.05	0.02	0.01
NaCl	1073.9	1072.5	1073.2
KCl.	1071.0	1075.0	1073.0

Middle $1073 \cdot 10^{-8}$ ohms.

For the smaller bottle: —

Salt	$m =$ 1.00	0.5	0.1
KCl.	1652.4	1653.0	1650.1
NaCl	1650.7	1651.3	1653.0

Middle $1651.7 \cdot 10^{-6}$ ohms.

Making the Solutions.

Whenever it was possible the solutions were made by weighing out the required amount of the pure salt, dissolving it in water, and then diluting to a liter of solution. When this was not possible, a concentrated solution was made and then brought to the normal strength by the specific gravity method. To accomplish this it was necessary to use tables giving the relation between the specific gravity and the per cent of salt. The following tables were used: from Kohlrausch for $SrCl_2$ and $Ca(NO_3)_2$, from Franz for $Ba(C_2H_3O_2)_2$, and from Hager for $Ca(C_2H_3O_2)_2$.

The normal solutions thus obtained were tested by comparing with tables of the specific gravity from other authors, and, when possible, with tables of the electric conductivity.

The values thus found for the specific gravity of the normal solutions were as follows: —

Salt.	Sp. Grav. $\frac{18}{4}$.	Salt.	Sp. Grav. $\frac{18}{4}$.
$\frac{1}{2}$ $CaCl_2$	1.0435	$\frac{1}{2}$ $SrCl_2$	1.06765
$\frac{1}{2}$ $Ca(NO_3)_2$	1.0611	$\frac{1}{2}$ $Sr(NO_3)_2$	1.0816
$\frac{1}{2}$ $Ca\ (C_2H_3O_2)_2$	1.0386	$\frac{1}{2}$ $Sr(C_2H_3O_2)_2$	1.0669
$\frac{1}{2}$ $Ba(C_2H_3O_2)_2$	1.0843	$\frac{1}{2}$ $K_2C_2O_4$	1.0575

The solutions of $m=0.5$ to $m=0.05$ were made by diluting the normal solutions in quantities of 100 cc. (The smaller bottle held about 40 cc.) For the more dilute solutions I used a method suggested by Professor Kohlrausch,[1] which is as follows: The Becher glass was first filled with 500 cc. of water and its conductivity determined. Then I drew out 9.5 cc. of water, and using very exact pipettes added successively 0.5 cc., 0.5 cc., 2.0 cc., and 2.0 cc., of the solution for which $m=0.1$, determining the conductivity after each addition. Then 0.5 cc., 2.0 cc., and 2.0 cc. of the solution for which $m=1.0$ were added in the same way. After each addition the pipettes were rinsed out three or four times.

In this manner I obtained solutions having the following strengths: —

After the addition of the solution $m=0.1$: —

0.5 cc.	$m = 0.0001018$
0.5	0.0002034
2.0	0.0006079
2.0	0.0010090

From solution $m=1.0$: —

0.5 cc.	$m = 0.002016$
2.0	0.006024
2.0	0.010000

---

[1] Wied. Ann. XXVI., 175, 1885.

The conductivity was determined for eleven (11) degrees of concentration between the values $m = 1.0$ and $m = 0.0001$.

In order to compute the molecular conductivity from the conductivity of the solution, the latter was diminished by the conductivity of the water, and this difference was divided by the values for m as given above.

As the resistance in the Becher glass depended upon the amount of solution, it was necessary to know the variation in the capacity for different degrees of filling. Instead of making new determinations I employed the values obtained by Professor Kohlrausch for the same glass and the same degrees of filling.

Filling	460 cc.	470	480	490	500	510	520
Capacity	1.0685	1.0489	1.0314	1.0155	1.000	0.9855	0.9715

The value for 500 cc. (Hg. at 0°) was determined as given above.

Temperature and Temperature Coefficient.

With very little difficulty the observations were all made at 18°. The room in which I worked had the sun only for a short time in the morning.

If the solution in the bottle became too warm, it was cooled from the outside by means of a piece of ice, or by moistening the outside and thus cooling by evaporation. If the solution became too cool, it was warmed by the hand until the correct temperature was obtained. I am sure that the variation in the temperature was never greater than 0.02° from 18°, as the thermometer could be read to this degree of exactness. In two or three cases I began with a temperature of 18°, and after forty minutes there was no perceptible change.

To avoid parallax I used a vision line drawn on the window, its position being determined in the usual manner. The temperature coefficients were determined from the normal solutions.

Salts Investigated.

The following salts were investigated: Calcium sulphate, calcium chloride, calcium nitrate, calcium acetate, strontium chloride, strontium nitrate, strontium acetate, barium acetate, silver acetate, potassium oxalate.

The following methods were employed for making the individual solutions: —

$CaSO_4$: The water of crystallization was first driven off and then the required amount of salt weighed out. Owing to the insolubility of the salt, only solutions having strengths of $m = 0.01$ and under were investigated. 0.3400 g. was weighed out and then diluted to ½ liter.

$CaCl_2$: The salt was first heated for two hours above 200° in order to drive off the water of crystallization, then cooled in a dessicator. 27.713 g. were weighed out and diluted to ½ liter.

$Ca(NO_3)_2$: Made according to specific gravity. Spec. Grav. of the original solution 1.1141. According to Kohlrausch this denotes 13.822 g. of salt in 100 g. of solution. Since a normal solution requires 82.04 g. salt in a liter, 593.02 g. of the solution were weighed out and diluted to a liter.

$Ca(C_2H_3O_2)_2$: Made according to specific gravity. Spec. Grav. of the original solution 1.1443. According to Hager this denotes 27.234 g. of salt in 100 g. of solution. Normal requires 79.01 g. of salt in a liter of solution. So 289.86 g. were weighed out and diluted to a liter.

$SrCl_2$: Made according to specific gravity. Spec. Grav. of original solution 1.1343. According to Kohlrausch this denotes 13.922 g. salt in 100 g. of solution. Normal requires 79.20 g. in a liter. So 568.38 g. were weighed out and diluted to a liter.

$Sr(NO_3)_2$: The salt was first recrystallized, dried at 75–80°, and then 105.79 g. were weighed out.

$Sr(C_2H_3O_2)_2$: The amount of water of crystallization was estimated and found to be 4.1 per cent, thus denoting the presence of ½ molecule of water. Then 107.2595 g. were weighed out.

$Ba(C_2H_3O_2)_2$: Made according to specific gravity. Spec. Grav. of original solution 1.1724. According to Franz this denotes 22.349 g. of salt in 100 g. of solution. Normal solution requires 127.509 g. of salt in a liter of water. 570.03 g. of the solution were weighed out and diluted to a liter.

$K_2C_2O_4$: The salt was first well dried over H_2SO_4, and then 92.113 g. were weighed out. (Reckoning 1 mol. of water.)

$AgC_2H_3O_2$: The salt was weighed out directly. Owing to its slight solubility, solutions from $m=0.01$ and under were the only ones investigated. (The salt is soluble 1 part in 98 of water.) An attempt was made to make a solution $m=0.05$, but after this had stood in a dark room for three days a precipitate began to form, and so the attempt was abandoned. All the investigations with this salt were made in a room illuminated by gaslight only. For the solution $m=0.01$, 0.8347 g. of the salt was weighed out and diluted to ½ liter. The specific gravities of the normal solutions are given above.

The Water Used.

The water for the very dilute solutions was obtained by triple distillation, the last two distillations being made in a tinned copper boiler having a tin cooler. The boiler was heated by a large Bunsen flame, and the products of combustion were led off into the chimney. The required purity was attained in about an hour and a half after the water began to boil; then for about three hours water was obtained having a conductivity between $0.75 \cdot 10^{-10}$ and $1.5 \cdot 10^{-10}$. In a few cases the conductivity was only $0.68 \cdot 10^{-10}$, so that its average conductivity was about $1.00 \cdot 10^{-10}$.

The water was kept in glass stoppered bottles covered by a bell jar.

It has been noticed by others, as well as by myself, that after the water has stood for a few days the conductivity has decreased from what it was directly after distilling. For example, water distilled May 5 had a conductivity of $1.095 \cdot 10^{-10}$. On the 12th it was $1.019 \cdot 10^{-10}$, and on the 18th $0.997 \cdot 10^{-10}$.

Results.

Professor Kohlrausch having investigated the solutions of $CaCl_2$, $Ca(NO_3)_2$, and $SrCl_2$, for values of $m = 1.00$ and above, I used only those having a less value. His work was based upon per cent of salt, and not upon the gram equivalent, and so his values for the normal solutions and their multiples are interpolated. In the case of $Ca(NO_3)_2$ a slight difference occurred, in that he found $K = 541.0 \cdot 10^{-8}$ (K=molecular conductivity), while I found $536.4 \cdot 10^{-8}$.

The first columns in the following table contain the values for *m*; the second the observed conductivity diminished by the conductivity of the water. All the observations were made at 18° unless otherwise indicated.

TABLE I.

½ $CaSO_4$.

m	$k_{18} \cdot 10^{10}$.
water	(1.465)
0.0001	10.789
0.0005	51.234
0.0010	97.714
0.0050	402.30
0.0100	719.81
0.0100	(815.42 $t = 23°.77$)

$AgC_2H_3O_2$.

m	$k_{18} \cdot 10^{10}$.
water	(1.206)
0.0001053	8.395
0.0002083	16.687
0.0006000	48.108
0.0009615	76.844
0.0050	388.68
0.0100	750.40

½ $CaCl_2$.

m	$k_{18} \cdot 10^{10}$.
water	(1.603)
0.0001018	10.983
0.0002034	21.686
0.0006079	64.084
0.001009	105.17
0.002016	208.69
0.006024	594.35
0.01	964.67
0.05	4341.6
0.10	8218.0
0.50	35008.0
1.0	63310.0
2.0	108300.0
3.0	138900.0
4.0	158300.0
5.0	166600.0
6.0	164400.0

½ $Ca(NO_3)_2$.

m	$k_{18} \cdot 10^{10}$.
water	(1.663)
0.0001018	11.138
0.0002034	22.058
0.0006079	64.499
0.001009	105.99
0.002016	208.11
0.006024	597.59
0.01	963.3
0.05	4266.0
0.10	7948.0
0.50	31711.0
1.0	53640.0
2.0	81800.0
3.0	94600.0
4.0	98300.0
5.0	94600.0
6.0	84500.0

½ $Ca(C_2H_3O_2)_2$.

m	$k_{18} \cdot 10^{10}$.
water	(1.345)
0.0001018	7.387
0.0002034	14.417
0.0006079	42.154
0.001009	69.084
0.002016	135.44
0.006024	383.61
0.01	616.34
0.05	2596.9
0.10	4663.6
0.50	15972.0
1.00	23708.0
1.00	(28120.0 $t = 26°.00$)
2.00	29752.0
3.00	27108.0

TABLE I. (*continued*).

$\frac{1}{2}$ $SrCl_2$.		$\frac{1}{2}$ $Sr(NO_3)_2$.	
m	$k_{18} \cdot 10^{10}$.	m	$k_{18} \cdot 10^{10}$.
water	(1.024)	water	(1.095)
0.0001018	11.449	0.0001018	10.623
0.0002034	22.564	0.0002034	21.056
0.0006079	65.769	0.0006079	61.652
0.001009	107.90	0.001009	101.37
0.002016	212.05	0.002016	198.74
0.006024	608.34	0.006024	568.26
0.01	986.62	0.01	919.98
0.05	4404.7	0.05	4009.0
0.10	8492.0	0.10	7462.8
0.50	35396.0	0.50	28858.0
1.0	63890.0	1.0	47837.0
2.0	108200.0	1.0	(56820.0
3.0	138700.0		$t = 26.^\circ 00$)
3.5	149900.0	3.00	80530.0

$\frac{1}{2}$ $Sr(C_2H_3O_2)_2$.		$\frac{1}{2}$ $Ba(C_2H_3O_2)_2$.	
water	(1.222)	water	(1.019)
0.0001018	7.837	0.00009990	7.699
0.0002034	15.626	0.0001996	15.373
0.0006079	46.367	0.0005964	45.260
0.001009	76.356	0.0009901	74.266
0.002016	149.02	0.001978	145.84
0.006024	426.97	0.005911	415.84
0.01	687.53	0.009814	670.09
0.05	2936.6	0.05	2939.2
0.10	5328.7	0.10	5390.6
0.50	18920.0	0.50	19754.0
1.0	29126.0	1.0	(31169.0
2.0	36577.0	1.0	$t = 26.^\circ 00$)
3.0	34976.0	2.0	42203.

$\frac{1}{2}$ $K_2C_2O_4$.	
m	$k_{18} \cdot 10^{10}$.
water	(0.997)
0.0001018	11.926
0.0002034	23.368
0.0006079	69.648
0.001009	114.75
0.002016	226.57
0.006024	650.73
0.01	1054.67
0.05	4693.4
0.10	8839.1
0.50	37570.0
1.0	68783.0
1.5	95900.0

In Table II. the gram equivalents m and the molecular conductivity $\frac{k}{m}$ are given.

TABLE II.

½ $CaSO_4$.		½ $CaCl_2$.		½ $Ca(NO_3)_2$.	½ $Ca(C_2H_3O_2)_2$.
m	$\frac{k}{m} \cdot 10^8$.	m	$\frac{k}{m} \cdot 10^8$.	$\frac{k}{m} \cdot 10^8$.	$\frac{k}{m} \cdot 10^8$.
0.0001	1079.0	0.0001	1078.9	1094.1	725.6
0.0005	1025.0	0.0002	1066.2	1084.5	708.8
0.001	977.1	0.0006	1054.2	1061.0	693.4
0.005	804.6	0.001	1042.3	1050.5	684.7
0.01	719.8	0.002	1035.2	1032.3	671.8
		0.006	986.7	992.0	636.8
$AgC_2H_3O_2$.		0.01	964.7	963.3	616.3
		0.05	868.3	853.2	519.4
0.0001	797.2	0.1	821.8	794.8	466.4
0.0002	801.1	0.5	700.2	634.2	319.4
0.0006	801.8	1.0	633.0	536.4	237.1
0.001	799.2	2.0	541.0	409.0	148.8
0.005	777.4	3.0	463.0	315.0	90.4
0.01	750.4	4.0	396.0	246.0	
		5.0	333.0	189.0	
		6.0	274.0	141.0	

½ $SrCl_2$.		½ $Sr(NO_3)_2$.	½ $Sr(C_2H_3O_2)_2$.	½ $Ba(C_2H_3O_2)_2$.	½ $K_2C_2O_4$.
m	$\frac{k}{m} \cdot 10^8$.	$\frac{k}{m} \cdot 10^8$.	$\frac{k}{m} \cdot 10^8$.	$\frac{k}{m} \cdot 10^8$.	$\frac{k}{m} \cdot 10^8$
0.0001	1124.7	1043.5	769.8	770.7	1171.5
0.0002	1109.4	1035.2	768.3	770.2	1148.9
0.0006	1081.9	1014.2	762.7	758.9	1145.7
0.001	1069.4	1004.7	756.8	750.1	1137.4
0.002	1051.8	985.8	737.4	737.3	1123.9
0.006	1009.9	943.3	708.8	703.5	1080.2
0.01	986.6	920.0	687.5	682.8	1054.7
0.05	880.9	801.8	587.3	587.8	938.7
0.1	849.2	746.3	532.9	539.1	883.9
0.5	707.9	577.2	378.4	395.1	751.4
1.0	639.0	478.0	291.2	311.7	688.0
1.5	588.0				639.0
2.0	541.0		182.8	211.0	
2.5	500.0				
3.0	462.0	268.4	116.6		
3.5	428.0				

Graphic Reproduction of the Molecular Conductivity.

In the curves shown in Fig. 1, the values for $\frac{k}{m}$ are used as ordinates and the values for $m^{\frac{1}{2}}$ as abscissæ. Thus we can represent the course of the curve for all possible degrees of concentrations in a comparatively small space. The curves are constructed directly from the results obtained, excepting in a few cases where they are made a little more regular than the observations give, although the change is very slight.

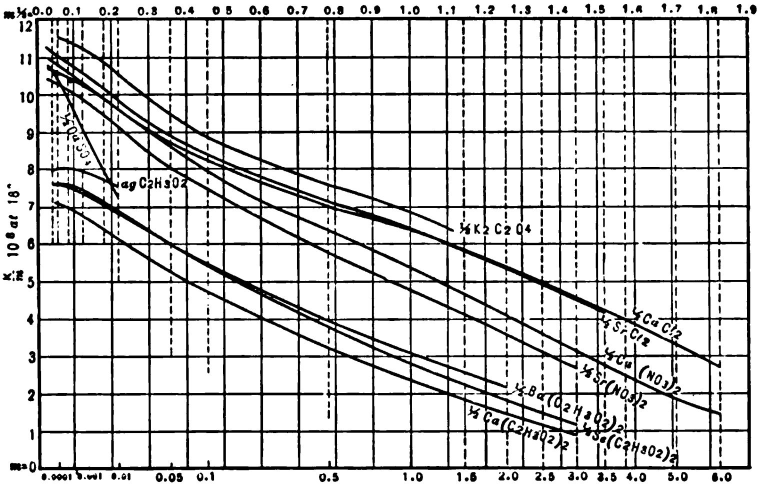

Fig. 1.

Temperature Coefficient.

The following tables contain the temperature coefficient for the normal solutions excepting for $CaSO_4$, where the value is for $m = 0.01$.

The values for $CaCl_2$, $Ca(NO_3)_2$, and $SrCl_2$ are those obtained by Professor Kohlrausch.

TABLE III.

Salt.	$\frac{\Delta k}{k_{18}}$.	Salt.	$\frac{\Delta k}{k_{18}}$.
½ $CaSO_4$	0.023	½ $SrCl_2$	0.0212
½ $CaCl_2$	0.0213	½ $Sr(NO_3)_2$	0.0224
½ $Ca(NO_3)_2$	0.0219	½ $Ba(C_2H_3O_2)_2$	0.022
½ $Ca(C_2H_3O_2)_2$	0.023	½ $K_2C_2O_4$	0.0209

General Remarks.

The curves giving the molecular conductivity are, for the most part, entirely regular, and even in the most dilute solutions there is no tendency in the curve to change from its upward course.

In the case of $AgC_2H_3O_2$ three separate determinations were made, and each time the curve changed from an upward to a downward course. Professor Kohlrausch assisted in one of the series at which the dilutions were carried to $m = 0.00001$, and then we obtained the value $\frac{k}{m} = 782 \cdot 10^{-8}$. This change in direction is quite characteristic for free acids or bases, but why it should occur with $AgC_2H_8O_2$, never having been observed with any other salt, is something I am unable to explain.

The salt had almost no odor of acetic acid. Perhaps a more thorough study of the acetates will explain this peculiarity.

The curve for $CaSO_4$ agrees in form very nearly with the curves for $MgSO_4$, $ZnSO_4$, and $CuSO_4$, as given by Professor Kohlrausch.

COLGATE UNIVERSITY, PHYSICAL DEPARTMENT.

THE APPARENT FORCES BETWEEN FINE SOLID PARTICLES TOTALLY IMMERSED IN LIQUIDS. II.

By W. J. A. Bliss.

Part II. Theoretical.

§ 14. In Part I. the phenomena of flocculation have been shown to be capable of close reproduction between two pieces of glass, at least so far as the effect of KOH is concerned. The evidence in the case of NaCl is indirect, but still strong. As these two substances are typical of the two classes as to effect on flocculation, these experiments furnish a very conclusive argument in favor of a purely physical explanation of the phenomena in question, rather than the one usually opposed to it, that there is a chemical change, at least in the coating, of the flocculated particles. The following discussion of the manner in which such phenomena might result from molecular forces may be divided into five heads: —

(1) Previous state of the theory. Sections 15–17.

(2) Proof that the force which tends to hold two submerged particles apart may be considered to be proportional to the difference between the coefficients of molecular force between a unit volume of solid and a unit volume of liquid, on the one hand, and two unit volumes of liquid on the other. Sections 18 and 19.

(3) Expression of these coefficients in terms of the densities of the solid, liquid, and salt in solution, and the constants of attraction between unit masses of these substances. Sections 20–23.

(4) Rough determination of the relative magnitudes which it is necessary to assign to these last constants in order to explain the phenomena of flocculation. Sections 24–27.

(5) Suggestions as to further investigations along these lines. Section 28.

§ 15. Barus[1] has advanced as a possible factor in a physical explanation, that the bombardment of two particles by the molecules of salt in solution forces them together in a manner similar to Lesage's theory of gravitation. This explanation is supported by the close resemblance between the order of substances in producing flocculation and their order as to electrolytic conductivity,[2] which suggests that both might alike depend on the number of ions. It fails to explain, however, the action of alkalies, since it would be difficult to suppose that in this case the ions were more active between the particles and so produced a repulsion.

§ 16. More probable explanations are those based upon the molecular forces between solid-and-solid, liquid-and-liquid, solid-and-liquid. Barus[3] suggests that the size of a particle of precipitate depends on these forces, the first two aiding, the other opposing, its growth. There is then a critical size, which determines the coarseness of the precipitate. Barus and Schneider[4] suppose each particle surrounded by an adherent envelope of liquid. The thickness of this shell depends upon the intensity of the attraction between solid and liquid. If the shells are large or the number of particles great, the latter cannot gather together or settle in a mass to the bottom without parting from these adherent envelopes. The shells must then shrink before flocculation or settling can take place. Such shrinkage is brought about by a decrease in the attraction between solid and liquid, due to rise in temperature; by the drawing together of the liquid molecules in freezing; or by a portion of the shell leaving the particles to adhere to salt molecules when such are introduced. Any of these causes should aid flocculation, as they actually do. Bodländer, in the paper already mentioned, has a similar but vaguer explanation.

§ 17. The most complete theory the writer has found is that of

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[1] Bull. 36, U. S. Geol. Surv.

[2] Barus, *loc. cit.*, and also Am. Jour. Sci., February, 1889. Also, Bodländer, Gött. Nach., 1893, pp. 267–276.

[3] Barus, *loc. cit.*, p. 127.

[4] Zeitschrift für Phys. Chemie, VIII. 3, 1891, p. 290.

Professor Milton Whitney.[1] As variously stated in treatises on surface tension, if *AB* is the surface of separation of a liquid and some other substance, a molecule at *P*, Fig. 4, has a different potential from a molecule at *Q*, the circles about these points denoting the distance from them to which the molecular forces are operative. If the substance to the left of *AB* is air, the potential of *Q* is less than that of *P*, and surface tension results. If, on the other hand, this second substance is a solid, *Q* may be either at a greater or at a less potential than *P*. Hence, —

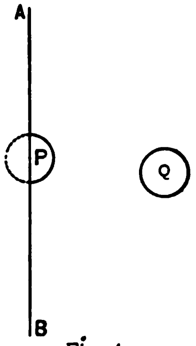

Fig. 4.

"If two small grains of clay, suspended in water, come close together, they may be attracted to each other or not, according to the potential of the water particles on the surface of the clay. If the potential of the surface particle is less than that of the particle in the interior of the mass of liquid, there will be surface tension, and the two grains will come together and be held with some force, as their close contact will diminish the number of surface particles in the liquid. If, on the other hand, the potential of the particle on the surface of the liquid is greater than that of the particle in the interior of the mass, the water surface around the grains will tend to enlarge, as there will be greater attraction for the particles there than in the interior of the mass of liquid, and the grains of clay will not come together, and will even be held apart, as their close contact would diminish the number of surface particles in the liquid around them."

K. Fuchs[2] has stated this last point very clearly and more generally: —

(1) If the molecules of the liquid are attracted more strongly by those of the solid than they are by other molecules of the liquid,

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[1] U. S. Weather Bureau Bull. 4, 1892, pp. 19–20. The author states that the theory was suggested by Lord Rayleigh's article in the Phil. Mag., November and December, 1890, and also acknowledges advice and suggestions from Dr. A. L. Kimball. Notes of these hints, as well as correspondence with other scientific men, have been lent to the present writer, who acknowledges his own indebtedness thereto.

[2] Exner's Rep., XXV., pp. 735–742.

or than the molecules of solid attract one another, the potential energy will be a minimum, when each particle of solid is surrounded by a shell of liquid of the thickness of the radius of molecular force. The two particles will then repel each other if brought closer together than within twice this distance.

(2) If either of the forces, solid-solid or liquid-liquid, is greater than the force solid-liquid, the potential energy will be least when the two particles are approached as closely as possible. They will then attract.

These theories, as well as the treatment about to be given, require that the particles influenced be not very great compared with what is usually called "the sphere of molecular force." The radius of the latter is, according to Quincke, 0.000005 cm. The diameters of the clay particles are, according to Whitney, between 0.0005 and 0.00001 cm., *i.e.* from 100 times to twice the radius of attraction. According to Barus and Brewer they are ultra-microscopic, and the former considers 0.000005 a fair value, *i.e.* the same as the radius in question. Noting the dependence of the theory on a size of particles not much exceeding these values, we will now seek to pass from the general conception of a diminution in Potential Energy by a change in the relative position of the particles to the actual forces by which such change is brought about. For simplicity we limit our attention to two particles.

§ 18. Let the particles be rigid, homogeneous spheres, and let us assume the liquid about them homogeneous, and of the same density within the range of molecular forces. We will consider the effect of the solid on a unit volume of liquid at P, whose sphere of molecular action impinges on both spheres. The unit volume must, of course, be taken extremely minute.

(Fig. 5.) Let ABC and $A'B'C'$ be the sections of the two solid particles by a plane through their centers and through P. Let the circles $EFE'F'$ and $LMNS$ be drawn on P as a center, and cutting ABC and $A'B'C'$. They enclose a section of a spherical shell, of which P is the center. Draw the arcs EF and $E'F'$ symmetrical with BC and $B'C'$ as regards P. Then the resultant of the attraction of the spherical shell upon a small unit volume of liquid at P is made up of the attractions of several pairs of equal volumes symmetrically situated as regards P, as follows: —

(1) $F'S'L'G'$ and $EMND$, whose resultant is zero, for they are both composed of solid, and are directly opposed to one another.

(2) $D'N'LG$ and $E'M'SF$, whose resultant is also zero, since both are liquid.

(3) $GLME$ and $G'L'M'E'$. For every element of one of these there is one in the other precisely similarly situated as regards P, but the one element composed of solid, the other of liquid. Therefore, if B be the attraction of a unit volume of the given solid for one of the given liquid, at unit distance, and C the similar coefficient between unit volumes of liquid, the resultant of the two segments will be $(B-C)$ into an integral taken over either of the two, the force being taken positive when towards the solid.

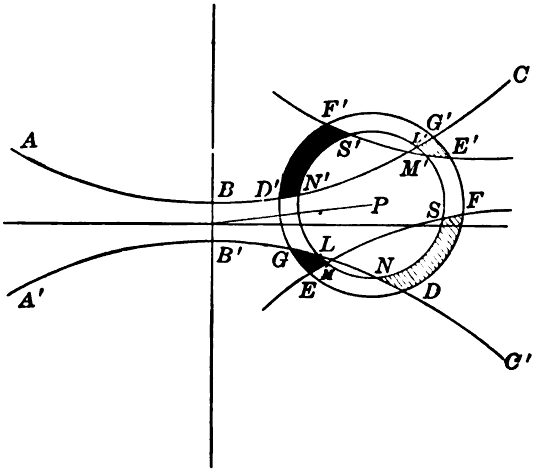

Fig. 5.

(4) Similarly, the resultant of $D'N'S'F'$ and $DNSF$ will be $(B-C)$ into an integral taken over either of these segments.

Hence the resultant of the whole shell will be proportional to $(B-C)$. Taking the sum of all such shells which cut both solids, we get the total force upon the element P due to the presence of the two particles. Hence if a = radius of solid spheres; $e = BB'$, the distance between them; n = the distance of P from O, the middle point of BB'; and ϕ the angle POB, then the force on unit volume at P is

$$p=(B-C)\,F(a,\,e,\,\phi,\text{ and }r). \tag{1}$$

Considering the liquid between the spheres, each shell of liquid of radius r about O produces a pressure on all the liquid within it due to the force p, and hence on the two particles, tending to force them apart. Therefore resolving p along PO and integrating[1] throughout the liquid between them, we get the force Φ tending to separate the particles: —

$$\Phi=(B-C)\,F'(a,\,e). \tag{2}$$

The actual evaluation of F' without a knowledge of the law of variation of molecular attraction with the distance would, of course, be impossible. Even under the assumption that the law is that of the inverse fourth power, the writer has found it far from simple. The variation of the density, and probably the concentration of a salt solution near a solid surface, make it unlikely that B and C are accurately constant throughout the integration; but, while noting this assumption, we may take the above form as an approximation to the force introduced by the pressure of the liquid in trying to form a complete shell about each particle.

§ 19. The distance between the spheres is further affected by the attraction between the spheres themselves. Denoting by (ss) the coefficient of attraction between *unit masses* of the given solid, at unit distance, this force will be

$$\Psi=(ss)n^2F''(a,\,e), \tag{3}$$

where n is the density of the solid. Φ and Ψ, in the absence of electrical forces, may be assumed to be the only ones acting, and therefore the force *holding the particles together* is

$$R=(ss)n^2F''(a,\,e)-(B-C)F'(a,\,e). \tag{4}$$

§ 20. The coefficient $(B-C)$ is the only thing that changes in the value of this force when nothing is changed but the composition of the liquid. For a change in the latter, R increases as $(B-C)$ decreases. Since increased flocculation means an increase of the force holding the particles together, *i.e.* of R, increased flocculation means that $(B-C)$ has decreased. Hence, to explain the facts of flocculation, —

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[1] This part of the process has since been found outlined by Barus and Schneider, Zeit. Phys. Chem. III. 3, 1891.

(1) $B-C$ must decrease when a trace of flocculent substance is added to pure water. Calling σ the mass of salt added to unit volume, —

$$\left[\frac{d(B-C)}{d\sigma}\right]_{\sigma=0}$$

must be negative for such substances.

(2) This quantity must be positive for alkalies such as KOH.

(3) Increase of concentration in the case of flocculent substances produces increased flocculation. In the case of alkalies it reverses the effect of a trace, and also causes strong flocculation. Therefore, $\frac{d^2(B-C)}{d\sigma^2}$ is negative for all substances.

The next step is then to find B and C in terms of σ.

§ 21. C is the measure of the attraction of two unit volumes of liquid. In a salt solution it is probably slightly greater near the solid than in a portion of the same solution further away,[1] and differs still more from the value at an air surface. It is therefore only roughly proportional to the surface tension as usually measured.

On the value of B there is no experimental evidence, so far as I am aware. It is, however, instructive to try the result of a simple hypothesis as to the form of both B and C. Let us start with pure water and denote by (ww) the mutual attraction of two unit *masses* of water. Thus for a density ρ of water we can write $C=\rho^2(ww)$. So, if (sw) denote the attraction between a unit mass of the water and a unit mass of solid, and n is the density of the latter, $B=n\rho(sw)$.

$$\therefore\ B-C=\rho\{n(sw)-\rho(ww)\}, \tag{5}$$

$$\frac{d(B-C)}{d\rho}=n(sw)-2\,\rho(ww). \tag{6}$$

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[1] J. J. Thomson's Applications of Dynamics to Phys. and Chem., p. 191.

Let us see from this equation what the effect of heating on flocculation teaches us as to the relative values of (ww) and (sw). Barus shows[1] that rise of temperature in pure water produces flocculation. Since ρ, the density of the water, decreases with heating, the condition that $B-C$ also decreases is that $n(sw)-2\rho(ww)$ shall be positive, *i.e.* $n(sw)>2\rho(ww)$. Since n for an individual clay particle is not far from 2.6, we have in round numbers, ρ being about 1

$$(sw)>\frac{(ww)}{1.3} \text{ or } \tfrac{3}{4}(ww). \tag{7}$$

Extending the above formulæ for B and C by the simple hypothesis that the effect of salt in very dilute solution is merely additive, we have for B in such a dilute solution,

$$B=\rho' n(sw)+\sigma n(st), \tag{8}$$

where (st) is the attraction of unit masses of salt and solid, and σ the mass of salt in unit volume. Similarly formed constants give for C,

$$C=\rho'^2(ww)+2\sigma\rho'(wt)+\sigma^2(tt), \tag{9}$$

where (tt) is the attraction between two unit masses of the salt, and (wt) between water and salt.

§ 22. A formula analogous to the above has since been found to be used by Volkmann[2] to express the surface tension of a mixture, densities being replaced by the volumes of the solutions mixed, and the constants (ww), etc., by their surface tensions independently. He mentions previous use of it by Poisson, and also quotes from O. E. Meyer[3] a similar expression for the viscosity of a mixture. Sutherland[4] finds a much more complicated expression to agree best with experimental results on surface tension and compressibility, the law of the inverse fourth power being assumed, and also a number of other conditions. As nearly as possible in the present notation his formula is

$$C=\bar{\rho}^2\cdot\frac{(1+n)(ww)(tt)}{(tt)+n(ww)}, \tag{10}$$

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[1] Bull. 36 U. S. Geol. Surv., p. 23.

[2] Wied. Ann., Vol. XVI., p. 321, and Vol. XVII., p. 384.

[3] Pogg. Ann. 113, p. 405, 1861.

[4] Phil. Mag., March, 1893.

where $\bar{\rho}$ is the density of the liquid as a whole, and n the number of salt molecules to one of water. The complexity of this formula may very probably be due, at least in the case of surface tension, to the fact that while the densities and concentration of the solution are those in the interior, the value of C concerned is that for volumes near an air surface, where both the former quantities might be very much modified. The mathematical work, however, of connecting such a constant with experimental results is very complicated, and in the present state of science involves so many assumptions as to be inconclusive, and it seems, therefore, hardly advisable to abandon the simpler expression found in § 21 for the above, at least for the present purpose.

§ 23. The value of ρ', *i.e.* the weight of water in unit volume of the solution after the salt has been put into an amount σ per unit of volume, may be denoted closely enough for present purposes (very dilute solutions) by $\rho'=\rho-\alpha\sigma$, where α is a small constant calculated from densities of salt solutions. A few values[1] are given below: —

Flocculent substances,

NaCl and KCl, $\alpha=0.3$; HCl, $\alpha=0.4$.

Substances which prevent flocculation,

KOH, $\alpha=0.07$; Borax, $\alpha=0.8$; NH_4OH, $\alpha=1.1$.

Introducing this value of ρ' in our values for B and C,

$$B=(\rho-\alpha\sigma)n(sw)+\sigma n(st)=\rho n(sw)+\sigma n\{(st)-\alpha(sw)\}. \tag{11}$$

$$C=\rho^2(ww)+2\rho\sigma\{(wt)-\alpha(ww)\}+\sigma^2\{(tt)-2\alpha(wt)+\alpha^2(ww)\}. \tag{12}$$

$$\begin{aligned}B-C=\rho n(sw)-\rho^2(ww)+\sigma[n\{(st)-\alpha(sw)\}-2\rho\{(wt)-\alpha(ww)\}]\\ -\sigma^2\{(tt)-2\alpha(wt)+\alpha^2(ww)\}.\end{aligned} \tag{13}$$

§ 24. This equation connects B and C with the density of salt in the solution, and in order to find the effect of an addition of salt on the force holding the particles together, we differentiate with regard to σ: —

$$\begin{aligned}\frac{d(B-C)}{d\sigma}=n\{(st)-\alpha(sw)\}-2\rho\{(wt)-\alpha(ww)\}\\ -2\sigma\{(tt)-2\alpha(wt)+\alpha^2(ww)\}.\end{aligned} \tag{14}$$

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[1] Landholt and Börnstein's Tabellen; Röntgen & Schneider, Wied. Ann.

By now putting $\sigma=0$, we get the effect on flocculation of the first trace of salt: —

$$\left[\frac{d(B-C)}{d\sigma}\right]_{\sigma=0}=n(st)+2\,\rho\alpha(ww)-2\,\rho(wt)-n\alpha(sw). \tag{15}$$

But it was shown in § 21, that in order to account for flocculation by rise of temperature,

$$(sw)>\frac{2\,\rho}{n}(ww).$$

$\therefore$ Assuming $\rho=1$, and n, for clay $=2.6$,

$$\left[\frac{d(B-C)}{d\sigma}\right]_{\sigma=0}=2.6(st)-2(wt)-\alpha\times\text{a positive quantity.} \tag{16}$$

If the right hand member is negative, the first trace of salt causes flocculation; *i.e.* salts where $(st)<\frac{3}{4}(wt)$, or which attract the given clay only three-quarters as strongly as they do water, belong to the flocculent class. On the other hand, if (st) is considerably greater than (wt), $B-C$ will increase upon the addition of the first trace of salt, and the latter belongs to the class like KOH, which prevents flocculation when only a trace is present. The value of α here makes a great difference, and KOH and borax, with small values of this constant, need attract the solid very little more than three-quarters as much as they do water; while ammonia, which very much reduces the water in unit volume, must have a very much larger relative attraction for the solid, or it would cause flocculation instead of preventing it.

§ 25. In order to find the conditions under which continued additions of salt always tend to increase the tendency to flocculation, even where the first trace has the opposite effect, we must apply the condition (3) of § 20; *i.e.* $\frac{d^2(B-C)}{d\sigma^2}$ is always negative. Hence, differentiating again in (14),

$$\frac{d^2(B-C)}{d\sigma^2}=-2\{(tt)-2\,\alpha(wt)+\alpha^2(ww)\}, \tag{17}$$

so that the quantity in brackets must be positive for all substances tried, *i.e.* $(tt)+\alpha^2(ww)>2\,\alpha(wt)$.

For KOH, where $\alpha=0.07$, α^2 may be neglected, and in order to account for the production of flocculation by an excess of KOH we need only suppose that $(tt) > 0.14(wt)$; that is, a comparatively weak attraction of KOH for itself as compared with its attraction for water is sufficient.

For NH_4OH, on the other hand, $\alpha = 1.1$, and the condition becomes $(tt) + 1.2(ww) > 2.2(wt)$, which means that the inter-attraction of ammonia and water is weak compared to the attraction of each for itself.

Similarly for flocculating substances, whose effect on flocculation increases *a fortiori* with the quantity present, the same equation holds, and, for example, in the case of NaCl, where $\alpha = 0.3$, becomes

$$(tt) + 0.09(ww) > 0.6\,(wt)\,;$$

i.e. the lowest assignable value for the attraction of unit masses of salt for one another, must be not far from $\frac{1}{2}$ of the attraction between common salt and water.

§ 26. The last paragraph showed how from experiments on flocculation, assuming the present theory, rough relative values may be deduced for the coefficients of attraction between various soluble substances and water. Similarly, from the section before we get relative values of the coefficients between salts and water on the one hand, and salts and solids on the other. The solid specially referred to in the instances given is such clay as gives the results obtained by most experimenters on flocculation. With different solids the effects in flocculation might be very different, since n, the density, as well as the constants (ss), (st), and (sw), in the above equations would vary. For instance, Brewer[1] experimented with Hartford clay, and himself remarks upon the almost exactly opposite results obtained by him from those of most other experimenters. He found that even so small a trace as 0.0008 gr. of KOH per cu. cm. aided precipitation; while, on the other hand, a very dilute mixture of nitric and hydrochloric acid (one part to several thousand of water) held the clay in suspension. In such clay the attraction between the clay and KOH would be less than its attraction for the acids instead of greater. On the other hand, the experiments of the first part of the present paper showed that glass has about the same relative values of the coefficients as the more usual clays.

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[1] Memoir, National Academy of Sciences, February, 1883.

The results of § 25 are entirely independent of the solid, and concern only water and the substance dissolved therein.

§ 38. The most useful outcome of the above discussion seems to be the means afforded of comparing the consequences of the theory given, as stated in the results of the last sections, with our general information concerning the various substances. To this end they are summarized below in round numbers, the forces being measured between unit masses at unit distance. From the clay referred to is to be expressly excluded such clay as the Hartford clay of Brewer, which seems to be exceptional; while, on the other hand, about the same properties may be set down to glass as are here ascribed to clay.

(1) The force between clay and water is greater than three-quarters the attraction of water for itself.

(2) The attraction of ammonia for clay must considerably exceed that of ammonia for water. KOH and the substances of which it is typical must have an attraction for clay *greater* than three-fourths that between the salt and water, while flocculating substances such as NaCl must have an attraction for clay *less* than three-fourths that for water.

(3) The attraction of NaCl for itself must be at least as great as three-quarters its attraction for water. The attraction of KOH for itself need only be from one-tenth to two-tenths its attraction for water. The attraction of ammonia for itself, and of water for itself, must be at least as great as their mutual attraction; or at least if the attraction of ammonia for itself is not as great as its attraction for water, the deficiency must be made up by a very strong attraction of water for itself (relatively to the two other coefficients).

The relative attraction of KOH and salt for water in (3) is interesting in view of the heat evolved when KOH is dissolved, and other parallels between the above and well-known properties of the various substances will readily occur to the reader.

§ 28. The subject of flocculation is one of sufficient practical interest in connection with its effect upon soil texture, as well as its action in sedimentation, to repay a direct study. Data are at present given for the most part only incidentally in describing experiments upon the rate of settling of sediments and precipitates. Its importance in this latter phenomenon has even to a large extent been overlooked, and the effect of traces of salts, ascribed to changes in the viscosity of the liquid and a variety of other causes, many of which are undoubtedly active enough to prevent the rate of settling from being more than a rough guide to the state of flocculation. Direct experiments might be made with a microscope, or perhaps by the colors of light transmitted through the suspended particles, and for merely scientific purposes they should be made on other substances besides soils. The writer proposes at once to try to extend the experiments of Part I. of the present article to a much greater range of salts, and also, if possible, by a modification of the apparatus, to obtain curves connecting the thickness of the liquid film intervening between the glass surfaces with the concentration of the solution.

Quantitative values of the constants in the theory just given could hardly be hoped for from experiments on flocculation, nor could the law of force be determined from them. The writer believes, however, that a process similar to the one here given, applied as rigorously as possible to the calculation of surface tensions of salt solutions and mixtures, would repay the very considerable labor of developing it. He has himself made a beginning, assuming the law of the inverse fourth power, but endeavoring to calculate the density in the surface film instead of assuming a probable value for it.

NOTE.

Since the work given in the above pages was done, there has appeared in the *Philosophical Magazine*, March, 1894, an article by Dr. G. Gore, in which he attacks in a different manner the relative attractions of various solids and salts. His method has the very great advantage of yielding quantitative results, but of a kind not readily interpreted. The experiments measure the rise in temperature of pure water and of various salt solutions when certain solids in the form of powder are poured into them. The heat given to the liquid comes from the combination of several changes of energy.

(1) Heat absorbed by the separation of the solid particles from one another.

(2) Heat liberated by the liquid wetting the particles, *i.e.* replacing air in their surface films. This is the energy Dr. Gore believes he has measured.

(3) Further change of energy as the powder packs together at the bottom of the vessel and the liquid is squeezed out.

The combined effect of (1) and (3) is the change in the potential energy of the solid due to the change in compactness between a dry and a wet powder, which it does not seem proper to neglect. We consequently find a marked difference between his results and the phenomena of flocculation. In the latter pure clay is held in a more separated condition in pure water than in any solutions except extremely dilute ones of certain alkalies. Since Dr. Gore's solutions are 10 per cent, the heat liberated by the wetting of clay by pure water should be greater than when it is wet by any of his solutions. We find, on the contrary, that in the case of both silica and alumina the heat given out in wetting by dropping them into pure water has a medium value exceeded in the case of a number of solutions with each of the two solids. We should not perhaps expect pure silica and alumina to give exactly the same results as clay, but in view of the fact that such strong solutions have been found to flocculate a wide range of materials when in suspension, the discrepancy seems more likely to lie in the cause given above.

MINOR CONTRIBUTIONS.

SURFACE TENSION OF WATER AT TEMPERATURES BELOW ZERO DEGREE CENTIGRADE.

By W. J. Humphreys and J. F. Mohler.

THE ratio of the change of the surface tension of water to the change of its temperature is well known for all values of the latter from 0° C. to 80° C., and the object of the investigation described in this paper was to determine the value of this ratio for temperatures below zero.

The method adopted was that of measuring, at various definite temperatures, the height of a column of water in a vertical capillary tube.

Owing to small unavoidable irregularities in the bore, and the difficulty of determining the exact area and shape of its cross section at the various positions occupied by the free surface of the water in the tube, should its position be allowed to vary, it was found best to keep this surface fixed, and to alter the length of the column by changing the level of the water in which the lower end of the tube was immersed.

The apparatus devised and the method of using it will be readily understood by referring to Fig. 1, in which *T* is a long capillary tube, vertical for some distance, then horizontal, and finally ending in an inverted U, the end of which dips into pure water in the vessel *E*. This vessel rests on a suitable support *G* (a strip of heavy plate glass, a meter in length and five centimeters broad, was used), which in turn rests on the fulcrum *F* and the vernier calipers *V*. The vertical portion *X* of the capillary tube is surrounded by a glass cylinder *K* about three centimeters in diameter and twenty centimeters long. During the investigation the top of the capillary tube was covered by a loosely fitting glass cap *S*, which protected the interior from dust. *R* served to keep *K* filled with cold brine of any desired temperature, the level of which was kept constant by the overflow pipe *O*. The microscope *M* was kept fixed in position, and furnished with a micrometer eyepiece, one of whose cross hairs was made to coincide with the image of the surface of the column of water in the capillary tube. The temperature was taken by two thermometers, the bulb of one being at the bottom of the cylinder *K*, and that of the other on a level with the microscope; but readings were not made when these differed by more than two-tenths of a degree, and in all cases the temperature of the upper thermometer, which read to tenths, was that taken, as it marked the temperature of the surface of the water.

To insure a clean surface, the tube was well washed, before being drawn out into the capillary form, with strong nitric acid, water, alcohol, and ether, in the order named. The water used in the investigation had been redistilled in glass vessels from an alkaline solution of potassium permanganate, corks of all kinds being avoided; and the tube was always refilled just before taking a set of observations by drawing in the water through its top, thus obtaining a surface quite free from dirt of any kind. Previous to each filling the water was freed from absorbed air by being boiled under the receiver of an air pump. Before taking observations the portion *T* of the capillary tube was accurately leveled to avoid errors otherwise arising from the want of uniform density of the water in this part of the tube. The differences between this level and those of the top of the column in *X* and the surface of the water in *E* were measured by a cathetometer. The first of these differences of level gives the length *A* of the tables, and was kept constant during each set of readings; the second was varied by the vernier calipers *V*, so as to keep the top of the column at a definite position in *X*, determined by the cross hairs of the fixed microscope *M*. Before each reading the top of the column in *X* was temporarily raised to

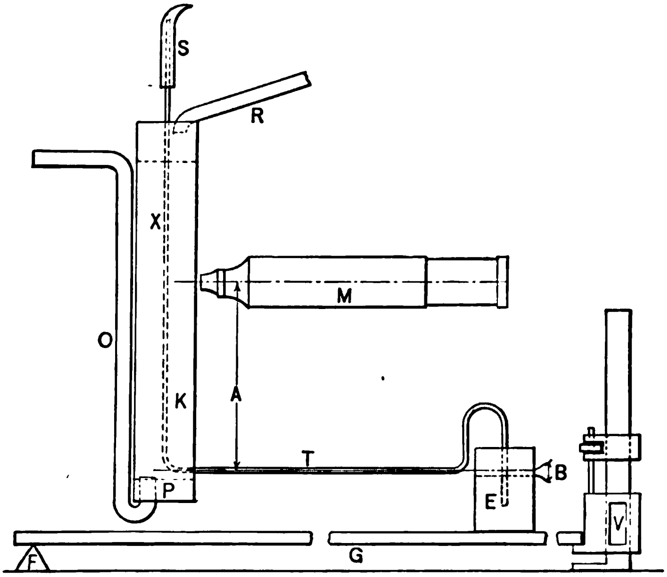

Fig. 1.

insure complete wetting of the tube. It was found that slight tappings of the apparatus often caused the column to more readily take up its final position of rest. This precaution was taken while making the sets of readings marked III. and IV. It was likewise found that small drops of water would occasionally collect in the upper part of the tube X. This led to three sets of observations being discarded on account of complications thus introduced.

Referring now to the tables: T is the temperature of the column of water in X; A is the height of this column; B is the difference in level of the water in E and the horizontal portion T of the capillary tube, and is positive or negative according as the former is below or above the latter; C is A multiplied by the density of water at the corresponding temperatures, as given by Despretz; D is B multiplied by the density of water at the temperature of E — about 16°; h is $C + D$.

I.

T	A	B	C	D	h
+3.0	10.735	0.00	10.735	+0.0	10.735
+1.1	10.735	+0.024	10.734	+0.024	10.758
−0.9	10.735	+0.051	10.733	+0.051	10.786
−2.0	10.735	+0.088	10.732	+0.088	10.820
−3.0	10.735	+0.111	10.731	+0.111	10.842
−5.0	10.735	+0.156	10.728	+0.156	10.884

II.

T	A	B	C	D	h
+6.9	10.92	−0.341	10.920	−0.341	10.579
+4.7	10.92	−0.287	10.920	−0.287	10.643
+1.3	10.92	−0.216	10.920	−0.216	10.704
−0.9	10.92	−0.195	10.918	−0.195	10.723
−2.0	10.92	−0.160	10.917	−0.160	10.757
−4.2	10.92	−0.100	10.914	−0.100	10.814
−6.0	10.92	−0.040	10.910	−0.040	10.870
−7.2	10.92	+0.003	10.908	+0.003	10.911
−8.0	10.92	+0.027	10.905	+0.027	10.934

III.

T	*A*	*B*	*C*	*D*	*h*
+18.5	10.945	−0.508	10.929	−0.508	10.421
+ 5.35	10.945	−0.269	10.945	−0.269	10.676
+ 4.6	10.945	−0.251	10.945	−0.251	10.694
+ 3.5	10.945	−0.235	10.945	−0.235	10.710
+ 3.2	10.945	−0.228	10.945	−0.228	10.717
+ 2.5	10.945	−0.209	10.945	−0.209	10.736
+ 2.0	10.945	−0.201	10.945	−0.201	10.744
+ 1.4	10.945	−0.181	10.944	−0.181	10.763
+ 1.0	10.945	−0.172	10.944	−0.172	10.772
+ 0.1	10.945	−0.148	10.944	−0.148	10.796
− 0.4	10.945	−0.137	10.944	−0.137	10.807
− 2.2	10.945	−0.111	10.942	−0.111	10.831
− 2.8	10.945	−0.095	10.941	−0.095	10.846
− 3.5	10.945	−0.079	10.940	−0.079	10.861
− 4.2	10.945	−0.057	10.939	−0.057	10.882
− 6.0	10.945	−0.009	10.936	−0.009	10.927
− 7.5	10.945	+0.031	10.933	+0.031	10.964
− 8.2	10.945	+0.060	10.931	+0.060	10.991

While taking this set of observations, the upper part of the capillary tube was entirely free from small drops of water, and the column brought to rest by slightly tapping the apparatus before taking each reading.

IV.

T	*A*	*B*	*C*	*D*	*h*
−1.8	10.95	−0.119	10.947	−0.119	10.828
−2.75	10.95	−0.079	10.946	−0.079	10.867
−3.6	10.95	−0.056	10.945	−0.056	10.889
−4.2	10.95	−0.036	10.944	−0.036	10.908
−5.2	10.95	−0.029	10.943	−0.029	10.914
−6.7	10.95	+0.013	10.940	+0.013	10.953
−7.6	10.95	+0.052	10.938	+0.052	10.990
−8.3	10.95	+0.083	10.936	+0.083	11.019

The curves in Fig. 2, plotted from these tables, show the relation of temperature to h, the ordinates being h multiplied by ten, and the abscissæ the corresponding temperatures. These curves very nearly coincide with the straight line $h_t = h_0(1 - .002014t)$, where h_t is h at the temperature t; and, as the surface tension is proportional to h, this gives, for the value of the surface tension of water at temperature $t°$ C., $T_t = T_0(1 - .002014\ t)$, where T_0 is the value of the surface tension at zero degrees Centigrade.

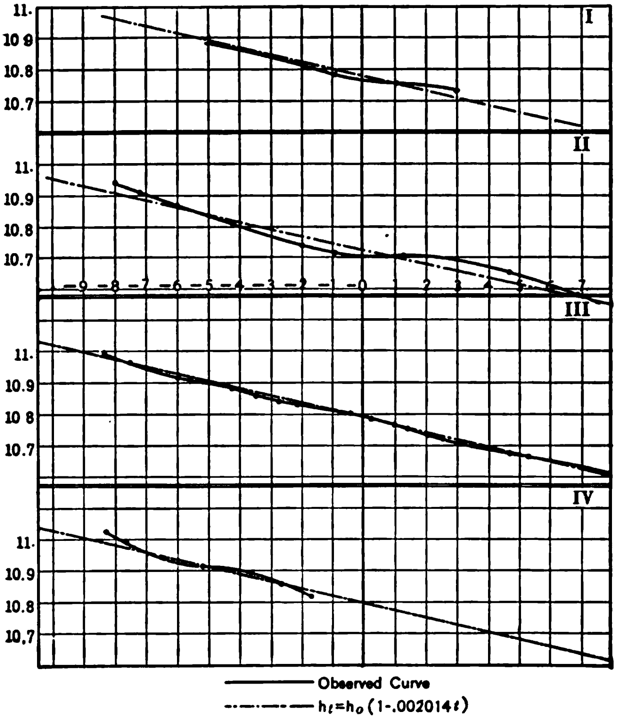

Fig. 2.

Therefore, according to the above observations, the ratio of the change of surface tension of water to the change of temperature is continuous and approximately equal to $.002014T_0$ throughout the range of temperature observed, *i.e.* −8°.3 C. to +18°.5 C.

This result is in close agreement with those obtained by many observers for temperatures above zero. Thus

Volkman gives $T_t = T_0(1 - .001829t)$,[1]
Tinberg gives $T_t = T_0(1 - .002208t)$,[2]
Jäger gives $T_t = T_0(1 - .00232t)$.[3]

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[1] Wied. Ann., 17, p. 353, 1882.

[2] Wied. Ann., 30, p. 545, 1887.

[3] Sitz. Wien. Akad., 100, p. 258, 1891.

JOHNS HOPKINS UNIVERSITY, January, 1895.

VARIATION OF INTERNAL RESISTANCE OF A VOLTAIC CELL WITH CURRENT.

BY HENRY S. CARHART.

IT has long been known that the internal resistance of a voltaic cell, when at a constant temperature, has not a fixed and finite value, but that it is a function of the current flowing through it. The methods commonly employed to measure this resistance are not sufficiently exact to exhibit the relationship in question with satisfactory accuracy. But if the condenser method be employed,[1] together with certain auxiliary appliances to secure very brief and uniform closing of the circuit through different external resistances, while the potential difference between the battery terminals is measured within this short interval, then results can be obtained which plot as a smooth curve.

The resistance measured is a quantity satisfying the equation

$$r = R\frac{E - E'}{E'}, \text{ or } \frac{E}{R + r} = \frac{E'}{R} = I,$$

where r is the internal resistance, E the electromotive force of the cell, and E' the potential difference between the terminals when closed through an external resistance R. To determine the relation between r and I, the potential difference E' must be measured with different external resistances in succession. For this purpose, the operations of closing the circuit, charging the condenser, discharging through a ballistic galvanometer, and finally opening the circuit, must be done by some mechanical device, by which all of them can be repeated in precisely the same interval of time. The disturbing influences due to absorption of the condenser and polarization of the cell are then as nearly eliminated as possible.

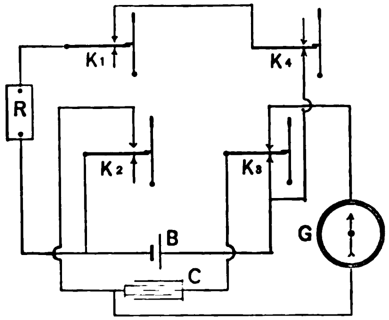

Fig. 1.

For this purpose I have made use of a pendulum consisting of a long narrow frame pivoted at the top, and carrying a heavy bob at the bottom. The bob swings between two curved rails, concentric with the axis of suspension of the pendulum, and each rail supports two keys, which can be clamped on it in any desired position. A vertical lever on each key holds it closed on the lower contact; but when the pendulum sweeps over the rails it throws these levers over, and allows the spring keys to make contact on the upper point. The contacts are all platinum, and the base and pillars of the keys are hard rubber. When care is taken to adjust the contacts properly and to keep them free from dust, their performance is in every way satisfactory.

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[1] Carhart's Primary Batteries, p. 116.

Figure 1 shows the connections for the measurements in hand. Keys K_1 and K_4 are on one rail, while K_2 and K_3 are on the other. When the lever of K_1 is thrown forward while that of K_4 is up, the battery circuit is closed through the resistance R. When the lever of K_2 is thrown over while that of K_3 is up, the battery is charging the condenser C; but as soon as the lever K_3 is thrown over, the cell is disconnected from the condenser, and the latter is instantly discharged through the galvanometer G. The pendulum swings from left to right on removing a detent. If all the levers are up, it closes the battery circuit on reaching K_1, charges the condenser by throwing over lever K_2, discharges it through the galvanometer by means of K_3, and finally opens the battery circuit on passing K_4. The key levers remain forward out of the way of the pendulum bar as it swings back again to the detent. The keys can be set as close together as desired. The length of the pendulum is such that its period is a second. Hence the time during which the battery circuit remains closed can be calculated. To get the deflection proportional to the total electromotive of the cell, the levers of K_1 and K_4 are both left down. The pendulum then charges and discharges the condenser, the circuit through R remaining open.

The change of resistance with the current is exhibited in a marked degree by "dry cells." The table contains the data of a series of measurements

d_1	d_2	R	r	I
271	258	400	21.1	0.0028
	249	200	17.7	0.0056
	238	100	13.9	0.0106
	227	50	9.7	0.0203
271	223	40	8.6	0.0249
	218	30	7.3	0.0324
	212	20	5.56	0.0473
	204	15	4.93	0.0607
	194	10	3.96	0.0868
271	172	5	2.87	0.1538
	164	4	2.59	0.1838
270	153	3	2.29	0.2289

on a Gassner cell, whose electromotive force was 1.213 volts. The first column contains the deflections due to charging the condenser with the whole electromotive force of the cell, and it will be seen that the polarization was inappreciable. The second column contains the deflections due to charging the condenser with the potential difference at the extremities of the cell when closed through the resistances in the third column. The fourth column contains the resulting internal resistances, and the last column the currents.

In Fig. 2 these internal resistances and the corresponding currents are plotted as coördinates. It cannot be objected that the relatively low resistances with the larger currents are due to polarization; for, while it was demonstrated that polarization did occur within the small time interval required for the pendulum to swing from K_1 to K_4, yet the effect of such polarization is to diminish the potential difference between the terminals of the cell when on closed circuit, and this effect increases the computed internal resistance.

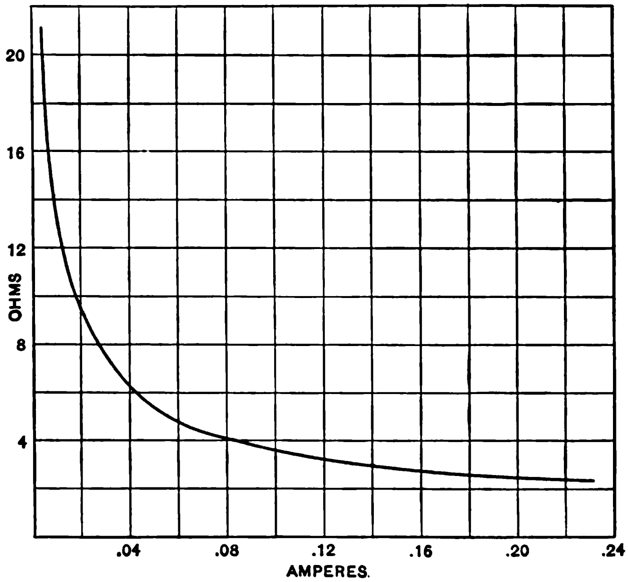

Fig. 2.

The Gessner cell was an old one, and its internal resistance was above the average; but other dry cells show the same characteristic curve, differing only in the relative values of *r*.

A Daniell cell shows a similar decrease in *r* with increase in current, but the decrease is much smaller, and the curve is more nearly a straight line.

UNIVERSITY OF MICHIGAN, January, 1895.

NEW BOOKS.

Manual of Physico-Chemical Measurements. By WILHELM OSTWALD, translated by JAMES WALKER. pp. xii + 255. New York and London, Macmillan & Co., 1894.

The above translation of Professor Ostwald's latest contribution to physical chemistry fills a much felt want in our literature. It is not, as might be inferred from its title, intended as a laboratory guide for the beginner, but rather, as the author states in his preface, for the more advanced student already familiar with physical and chemical manipulation who desires to become acquainted with the various methods now employed in physico-chemical investigations. To the investigator, too, the little work will prove of interest and value, abounding as it does in practical hints and suggestions, the result of the author's own extensive experience. The value of the work is also materially increased by numerous references to original memoirs.

Having in view this more mature class of readers, the author has presented his subject not on the usual plan of a laboratory manual, with descriptions for the performance of set experiments, but rather as a general, and often critical, discussion under separate chapters of the various classes of measurements usually met with in practice. The first chapter on calculation is devoted to an elementary discussion of observations, particular stress being laid on the importance of observing rules for the retention of significant figures. It is only to be regretted that the importance of the graphical method is not brought into more prominence in this connection.

Six chapters follow of an introductory character, four of which are devoted to the usual methods of measuring length, mass, pressure, and temperature. Of the other two, one is devoted to a description and valuable criticism of a number of thermostats, most of which have been tried in the author's laboratory. The other chapter is devoted to glass blowing. Here, as indeed throughout the book, we find emphasized the author's conviction of the importance to the investigator of being able to construct his more simple apparatus. This is of particular importance to the physical chemist, as the necessary equipment for investigations in this branch of science is usually much less elaborate and costly than that required for purely physical researches.

A description of physico-chemical methods proper begins with a discussion of specific gravity determinations in Chapter VIII. Of the methods in general use the author gives preference to the Sprengel-Ostwald form of pyknometer for liquids, and the method of "floating" when applicable for solids. Particularly noteworthy in the following chapter, on methods for determining the physical properties of liquids, are the methods worked out in the author's laboratory for determining the critical constants. The method for critical pressure consists simply in unequally heating the capillary containing the substance under investigation, so that the upper part is above, the lower part below the critical temperature, and observing the pressure at which the meniscus disappears.

Following this are chapters on calorimetric and optical measurements, the latter including calorimetric and spectrophotometric methods, and the essential processes of photography. A special chapter is devoted to solubility determinations, as work along this line has proved, and doubtless will continue to prove, of the greatest value in solving problems connected with the theory of solutions. The well-known boiling and freezing point methods of Beckmann for determining the molecular weight of substances in solution, find, of course, their place in the work.

The chapter on electrical measurements is devoted almost wholly to measurements of the electromotive force of voltaic cells, and of the conductivity of electrolytes. In connection with the former will be found many new and valuable suggestions to persons working in this field. First are described the form and construction of various standards of electromotive force, viz., the Clark, Gouy, and v. Helmholtz cells. Following are several special methods, mostly based on Poggendorff's method, for measuring electromotive force, in all of which a Lippmann's electrometer is strongly recommended as indicating instrument. Particular attention is given to the measurement of very small potential differences, and three forms of Lippmann's electrometer, varying in sensitiveness from 0.001 to 0.0001 volt or even less, are described. My own experience confirms completely all the author claims for these very simple and useful instruments.

Another suggestion which it is to be hoped may meet with general acceptance is the adoption as normal electrode against which to measure the absolute potential difference between metals and liquids, of mercury covered with a layer of calomel in a normal solution of potassium chloride. The potential difference between the mercury and the solution is, according to our most accurate measurements, 0.540 volt. This electrode is easily constructed, its potential is very constant, and it possesses the additional advantage of reducing the contact potential at the liquid junction to a minimum, the velocity of migration of chlorine and potassium ions being very nearly equal.

Conductivity measurements are made by the well-known method of Kohlrausch. Convenient tables for calculating dissociation constants are given, and the electrical method of determining the basicity of acids explained. The last chapter is devoted to chemical dynamics, the rate of inversion of cane sugar, and the catalysis of methyl acetate being taken as illustrative examples.

In conclusion a word should be added in regard to the excellence of the translation and general appearance of the book, which is uniform with Dr. Walker's well-known translation of Ostwald's Grundriss der Allgemeinen Chemie.

H. M. Goodwin.

The Work of Hertz and Some of his Successors. By Oliver Lodge, F.R.S. 8vo, pp. 58. London, The Electrician Printing and Publishing Co., 1894.

This work is a reprint, with revision, from the *London Electrician*, and contains the substance of a lecture delivered at the Royal Institution. It is a semi-popular lecture rather than a critical review of the work which has been done along the line of Hertz' discoveries. Of the fifty-eight pages in the book about twenty, including most of the appendices, are devoted to the discussion of the effect of ultra-violet light on electric discharge. Though this is not Hertz' most important discovery, yet it is one possessing much interest, and the review on the subject, though brief, is very helpful. Fifteen pages are devoted to a description of the construction and use of an appliance which the author calls a "coherer." This is of especial interest because it furnishes a simple and effective method for detecting electric oscillations. In the remaining twenty-three pages is given a review of the general subject of electric waves.

The book does not by any means attempt to do for this subject what the very valuable review published by Lodge on Contact Electricity did for that.[1] In fact, the book is disappointing to one who might have hoped for a similar review on electric waves, and the more so, because with the exception of the part on the effect of ultra-violet light, it is quite deficient in references to original sources. Such an article would certainly be of the highest value; for, on account of the wealth of literature on the subject, it is difficult for a person not thoroughly posted to get a fair idea of the work which has been done along this line, of the relative importance of different methods, and of the results obtained. This criticism, however, is upon the title, which may be somewhat misleading, rather than upon the substance of the book.

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[1] Phil. Mag. (5), Vol. 19.

As has been intimated, a most valuable part of the review is the description of an appliance which the author has made useful, and which he calls the "coherer." The "coherer" consists essentially of a poor contact, whose resistance is much diminished when it is subjected to electric oscillations. The easiest thing to use for this purpose is a tube of iron filings.[1] Lodge found that in the case of a tube which he used, the resistance decreased from 2500 ohms to 2100 ohms when the smallest kind of an electric discharge took place in its neighborhood. He explains this somewhat as follows : when two pieces of metal are in poor contact, only a very few of the molecules of one touch those of the other, and therefore the resistance is high; but when the molecules of the two have been once well shaken together by the electric oscillations, they continue to cohere, and the resistance is very greatly decreased. When there are a number of such contacts, as in the case of a tube of iron filings, the effect is proportional to the intensity of the oscillations. Furthermore it was found that a mechanical jar would restore the tube to its original condition as far as resistance was concerned.

If such a tube be connected with a battery and a galvanometer, a change in the resistance can easily be shown. It was found that short oscillations affected the "coherer" more than longer ones, even though the longer ones were of much greater intensity. The author states that he was easily able to detect an exciting spark of high frequency at a distance of forty or sixty yards. In fact, it proved to be more easy to detect electric oscillations than to screen the "coherer" from their effect. But when this was properly done, so that only those waves coming from a given direction affected the "coherer," the reflection of electric radiation from metallic surfaces could be easily shown, also refraction through a paraffin prism, and even the focussing of electric waves by an ordinary nine-inch glass lens. Polarization effects were very easily secured, and in one case a diffraction effect was detected, although diffraction bands could not be shown.

The author hopes to use the "coherer" for quantitative work, but it seems doubtful whether it can be made as useful for this purpose as some other methods. As a simple means for showing qualitative effects, however, it is unsurpassed.

The book will certainly well repay perusal by any one interested in the Hertz phenomena.

C. D. CHILD.

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[1] E. Brunly, C.R. Vol. 111., p. 785, and 112, p. 90.

An Elementary Treatise on Theoretical Mechanics. By A. ZIWET. Part III, Kinetics. 8vo, pp. viii+236. New York, Macmillan & Co., 1894.

The first two parts of Ziwet's *Mechanics* have already been noticed in the PHYSICAL REVIEW.[1] The third part, completing the work, is now before us.

The book is divided into three chapters, which treat respectively of the kinetics of a particle, of a rigid body, and of a variable system.

The first chapter opens with an elementary account of impulses and impact. Then are discussed the general equations of motion of a particle, the principle of energy, and the principle of areas. Extended applications of these principles to particles free to move and to particles under constraint are next given. The chapter closes with a short account of the Lagrange form of the equations of motion of a free particle.

In the chapter on the motion of a rigid body is given an elaborate account of the theory of moments of inertia—much of which belongs rather to the integral calculus. Next follows a very satisfactory treatment of the motion of a rigid body under continuous forces and of the initial motion due to impulses.

In the concluding chapter is considered the motion of a variable system subject to any conditions whatever. This chapter, though short, is very interesting from the use made of the Lagrange equations of motion.

Throughout the book the language is clear, the explanations concise, but sufficient for the student who has had a first-rate mathematical training. In the earlier portions examples with answers are given; in the later and more difficult parts there are references to books, mostly foreign, where examples may be found. It would seem as if the space taken up in referring to the same books, again and again, might, with more advantage to the student, have been devoted to some well-chosen examples.

In Art. 242 is a novelty which is hardly likely to find much favor. It is the substitution of the term *radius of inertia* for the old *radius of gyration*. The book is happily free from innovations of this kind.

The complete work is as a whole admirable for its systematic arrangement, and for its clear treatment of the more important principles of theoretical mechanics, and will be welcomed by students of the higher mathematics in our universities.

T. W. WRIGHT.

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[1] Vol. I., No. 5.

Photographische Optik ohne mathematische Entwickelungen für Fachleute und Liebhaber. By Dr. Adolf Miethe. Large 8vo, pp. viii + 153. Berlin, R. Mückenberger, 1893.

As indicated by the title, this little book is intended for those professional and amateur photographers who desire to obtain a knowledge of photographic optics without the use of mathematics. The author (well known to the photographic world as one of the originators of telephotographic objectives) "assumes nothing, neither physical nor mathematical attainments or conceptions. The sole aim of the work is universal intelligibility on the basis of common sense, clearness, simplicity."

The topics discussed are, in order, fundamental truths of optics, aberrations, diaphragms, the usual types, and the testing of photographic objectives. Of necessity, the treatment is not at all exhaustive, yet the language is, in general, so lucid as to give as good an idea of the subject as is possible, perhaps, without having recourse to mathematics. This clearness is much increased by the neat typography and illustrations; in particular, the two plates are of material assistance in the study of aberrations, including astigmatism.

The book may be heartily recommended to those who do not feel inclined to plunge into the mathematical intricacies of the subject; indeed, it deserves translation into English.

Henry C. Lomb.

Proceedings of the International Electrical Congress, held in Chicago in 1893. New York, The American Institute of Electrical Engineers, 1894.

In this volume the Proceedings of the Congress are given in full, including the various papers presented, together with the discussion, and the official reports of the Chamber of Delegates. A report of this Congress, together with the list of papers presented and the principal business transacted by the Chamber of Delegates, is given in the Physical Review, Vol. I., No. 3.

The American Annual of Photography and Photographic Almanac for 1895. pp. 438. New York, Scovill & Adams Co. (*Received.*)

Volume II. *May–June, 1895.* *Number 6.*

THE

PHYSICAL REVIEW.

THE CAPACITY OF ELECTROLYTIC CONDENSERS.

BY SAMUEL SHELDON, H. W. LEITCH, AND A. N. SHAW.

Introductory.

THE phenomena of electrolytic polarization have long been known. Varley,[1] however, seems to have been the first to appreciate that an electrolytic cell, when subjected to small voltages, could be employed as a substitute for a condenser of great capacity. He made determinations[2] of the capacity of platinum plates in dilute H_2SO_4. About the same time Kohlrausch[3] determined the capacity of platinum electrodes, in order to find whether the error thus introduced, when these electrodes were employed in the measurements of the resistances of electrolytes according to his method, were negligible or not. (We might well call attention to the fact that Fessenden[4] quite recently says that all measurements made by the Kohlrausch method are incorrect, because the impedance, and not resistance, is really measured. While this may be true in the abstract, only ordinary experimental precaution is necessary to make these two quantities practically the same. Kohlrausch[5] has recently shown the magnitude of these errors.) Blondlot[1] and Helmholtz[2] have worked upon the capacity of electrolytic condensers, but the original publications are rather inaccessible, and the abstracts by other authors are wanting in data expressed in absolute measure. During the progress of this investigation two papers have been published which have a direct bearing on the subject. One by Hopkinson,[3] Wilson, and Lydall considers the capacity of electrolytic condensers when traversed by an alternating current; the other, by Bouty,[4] considers the curves of the polarization of charge and discharge as functions of the time after closing or opening the circuit. Many others have worked upon the subject in its different aspects.

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[1] Patent Specification, C. F. Varley, Electric Telegraphs, etc., Jan., 1860.

[2] Varley, Phil. Trans., 161, p. 129, 1872.

[3] F. Kohlrausch, Nachr. der K., Gött. Gesellsch., Sept., 1872, p. 453.

[4] Fessenden, Phil. Mag., Dec., 1894.

[5] Kohlrausch, Annalen der Phys. u. Chem., Band 49, 1893, p. 235.

Method.

Many workers have assumed that a platinum-H_2SO_4 cell, after being short-circuited for several hours, is in a neutral or unpolarized condition. This is not the case. We have not been able to obtain such a condition even after short-circuiting for days. Even new electrodes, which have never been in acid, exhibit marked polarization. To avoid the delay necessary to bring the electrodes into even an approximately neutral condition, we have had recourse to a method similar to the "method of reversals" employed in testing iron. A difference of potential, measured by a voltmeter, is impressed for a certain time upon a circuit containing the electrolytic cell in series with a non-inductive resistance equal in amount to the resistance of a ballistic galvanometer about to be used. By means of a commutator, the polarity of the difference of potential is reversed and the galvanometer is substituted for the resistance. The throw of the galvanometer gives the quantity of electricity necessary to depolarize the cell from a given voltage to zero, plus the quantity necessary to polarize it to the same voltage in an opposite direction. These two quantities are equal. The capacity at that voltage is obtained by dividing half the quantity by the voltage. The method yields quick results, and any measurement may be repeated at will. The voltmeter employed was one of Weston's. The galvanometer was of the d'Arsonval type, with a complete period of 44 seconds, and a resistance of 1065 ohms. The calibration of the galvanometer was obtained with the help of a Clark element and a standard microfarad condenser. The electrolytic condenser acts like an ordinary condenser of very large resistance as far as its effect upon the damping of the galvanometer is concerned. The E.M.F. set up in the moving coil is opposed by the impressed E.M.F. when the swing is in one direction, and by a counter E.M.F. of the cell when in the other direction. The resulting damping is the same as would occur on open circuit; *e.g.* the ratio of one amplitude to the following with the electrolytic condenser in circuit was 1.08; with open circuit, 1.08; with equal wire resistance substituted for the electrolytic condenser, 1.60.

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[1] Blondlot, Thèse de Docteur, Nr. 460. Paris, 1881.

[2] Wiedemann, Lehre der Elek., Vol. 2, p. 451.

[3] Proc. Royal Society, Vol. LIV., p. 407.

[4] M. E. Bouty, Journal de Physique, Nov., 1894, p. 498.

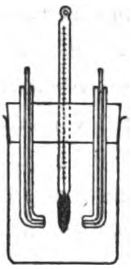

Fig. 1.

We have also employed a method similar to the "step-by-step method" used in obtaining hysteresis curves of magnetic substances. The electrolytic condenser in series with a ballistic galvanometer is subjected to a voltage which is increased by steps from 0 to a maximum; reduced in a similar manner to 0; reversed and carried to a negative maximum and back to 0. The throws of the galvanometer accompanying each change of voltage are observed, and after algebraic summation, as in the magnetic method, are used as ordinates, with voltages as abscissæ.

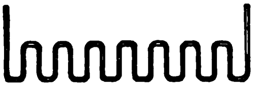

Fig. 2.

The condensers with which we have done the greater part of our work are of two forms, represented respectively in Figs. 1 and 2. The electrodes in the first form were of platinum wire, sealed into capillary glass tubes. The ends of the tubes were bent at right angles, as indicated in the figure, and together with the ends of the contained wires were ground plane and polished. The object in bending the tubes at right angles was to obtain uniform current density — a needless precaution, as explained later. The electrodes, together with a thermometer, were mounted in a cork which fitted in the top of a beaker containing a 1 : 20 solution of H_2SO_4. We have used four such cells with wires of .814 mm. diameter, and four of .41 mm. diameter. The condensers of the second form (Fig. 2) were made from glass tubing 3.2 mm. internal diameter, bent in the form indicated. The electrodes are mercury and the electrolyte dilute H_2SO_4. The condenser consists of several cells placed in series, and the mercury portions which fill the lower bends act as cathodes on one side and anodes on the other. We met with some difficulty in endeavoring to properly dispose of the mercury and acid to the exclusion of air bubbles, but overcame this by employing a force pump. One end of the empty tube was passed through one of two perforations in a rubber stopper which was fitted into a test tube containing equal parts of mercury and dilute acid. Through the other perforation passed a tube connected with the force pump. By slightly tipping the condenser tube its end could be brought at will into either mercury or acid. Then, by manipulating the force pump, any desired volume of acid or mercury could be forced into it.

Effect of E.M.F. on Capacity.

If we consider the capacity, C, of an electrolytic cell to be the ratio of the quantity of electricity, Q, passed through it to the E.M.F. of polarization, p, produced, it is evident that the capacity cannot be constant; for p has a maximum limit, and if an E.M.F. greater than this limit be impressed on the cell a current will flow and C becomes infinite. For very small values of p, C has a definite value, which has been termed by Blondlot the "initial capacity."

The following[1] have been given as the maximum limits of p for platinum electrodes in dilute H_2SO_4 :—

Observer.	Maximum p in Volts.
Wheatstone	2.56
Buff	2.82
Svanberg	2.54
Poggendorff	2.56
	2.62

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[1] Wiedemann, Lehre von der Elek., Vol. 2, p. 695. The assumption is made that E.M.F. of Daniell cell = 1.1 volt.

Fromme[1] states that maximum p varies from 1.45 to 4.31 volts, according to size of electrodes and concentration of electrolyte.

The capacity of a cell varies, then, between its initial value at a low voltage and ∞ at 2.62 volts. The manner in which it varies is represented by the curve in Fig. 3, which was obtained from a cell of the form shown in Fig. 1. The cause of the curvature is undoubtedly what Bouty[2] has termed the "force of depolarization," which he shows increases with p.

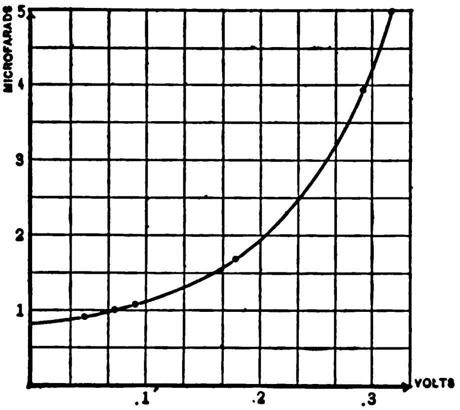

Fig. 3.

Effect of Electrode Surface.

If the two electrodes of a cell expose to the electrolyte areas of S_1 and S_2 cm.^{2} respectively, then the initial capacity C of the cell is obtained from the formula

$$\frac{1}{C}=\frac{1}{K}\left(\frac{1}{S_1}+\frac{1}{S_2}\right),$$

where K is a constant depending upon the material of the electrodes, the character and temperature of the electrolyte. Bouty has shown that the manner of distribution of S_1 and S_2 is of no consequence. If, *e.g.*, the electrodes were two plates placed parallel and opposite to each other, the current density on the opposed surfaces would be much greater than on the reverse sides, and although the polarization might attain for an instant a greater value on these surfaces, local currents would soon equalize it throughout the electrodes.

The values of K vary widely with different specimens of platinum. The following values for platinum in dilute H_2SO_4 are taken from Wiedemann. Blondlot observed that K sunk to about one quarter of the given value after long standing in the electrolyte. Bouty gives $K=20.13$ for platinum in nitrate of soda. We have found for four cells, of the form shown in Fig. 1, with electrodes made from the same piece of wire, fused in the same kind of glass, and used in dilute acid from a common source, the values of K at 20° to be 174, 180, 464, and 656 microfarads respectively. Also for two electrodes of platinum foil 7 × 0.5 cm., fastened by means of heated shellac to two flat pieces of vulcanite, so as to expose one surface only to the action of the current, $K=43.2$ at 20°. Again, for two like electrodes of platinum wire of .0107 cm. diameter and 5.8 cm. length, $K=42.6$ at 21°.7. The very high values for the first four cells we were prone to discredit, but continued redeterminations and recalculations yielded concordant results.

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[1] Annalen d. Physik u. Chemie, XXXIII., p. 80.

[2] Bouty, l.c., p. 505.

Observer.	**K in Microfarads.**
Blondlot	31
Kohlrausch	25.8
Varley	(50)
Helmholtz	39
Colley	314 to 284

In the case of the mercury electrodes (Fig. 2) it is impossible to estimate with any accuracy the area of surface exposed to the electrolyte. The internal diameter of the tubes was 0.32 cm., and the initial capacity of 14 cells in series was 4.14 microfarads at 20°. Assuming all cells to be alike, each would have a capacity of 58 microfarads. Assuming the exposed surface of each electrode to be 0.15 cm.^{2}, gives $K=675$.

Effect of Temperature.

The effect of temperature on the capacity is very marked. If C_{18} represents the initial capacity of a $Pt-H_2SO_4$ cell at 18°, then the capacity C at any temperature t is represented by the formula

$$C=C_{18}[1+(t-18)0.0215].$$

We have obtained this formula from the following observations:—

Temp.	*C.*
17.66	1.60
24.36	1.80
28.16	1.93
33.26	2.14
39.96	2.38

The cell was placed in a water-bath whose temperature could be changed. The temperature of the electrolyte was read from a thermometer, whose bulb was in the cell and near to the electrodes. The electrolyte was not stirred for fear of introducing air.

The $Hg-H_2SO_4$ cells showed a larger temperature coefficient. They were placed in a large water-bath, and the temperature of this was obtained by averaging the readings from several thermometers, placed at various points near the cells. The following are the observations:—

Temperature	13.5	21.4	30.4	35.3	40.0	45.97
Microfarads	3.29	4.29	5.70	6.70	7.55	8.88

These results plotted as a curve show an increase in capacity of about 4 per cent for each degree of rise in the neighborhood of 18°.

Effect of Concentration.

Changes in the concentration of the electrolyte do not have a very great influence on the capacity. This is shown by the following observations, taken at a constant voltage (0.1808 volts) with the same pair of electrodes:—

Volume of Solution. / Volume Acid.	Capacity in Microfarads.	Temp.
3	.376	20 06
6	.387	20.01
12	.400	20.26
24	.404	20.16
48	.450	20.25
96	.465	20.16
192	.491	20.26

Coupling of Cells.

If any number of cells be joined in *multiple arc*, the capacity of the combination is equal to the sum of the capacities of the individual cells. This natural conclusion we have verified by experiment.

If any number of cells, of capacities C_1, $C_2 \cdots$, C_n, be joined in *series*, the capacity of the combination C can be obtained from the formula

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_n}.$$

Inasmuch as C_1, C_2, and C_n vary with the voltage, if cells of different individual capacities be joined in series, the capacity of the combination can be found only by constructing a derived curve from the curves of the separate cells. The manner of doing this may be illustrated by the following example, taken from our observations: —

CELL NO. 1.

Volts	.18	.36	.58	.64
Microcoulombs	.069	.204	.618	.83

CELL NO. 2.

Volts	.090	.18	.29	.319
Microcoulombs	.049	.154	.575	.798

Plotting two curves with microcoulombs for ordinates and volts for abscissæ (Fig. 4), determine from them the voltages of polarization produced in each cell by the passage of different quantities. We have

Microcoulombs	.1	.2	.4	.6	.8
Polarization of No. 1 . . .	.246	.350	.490	.575	.630
Polarization of No. 2 . . .	.146	.210	.266	.299	.319
	.392	.560	.756	.874	.945

From the sums of these voltages construct a new curve, which will represent the relation between volts and quantity which exists for the combination.

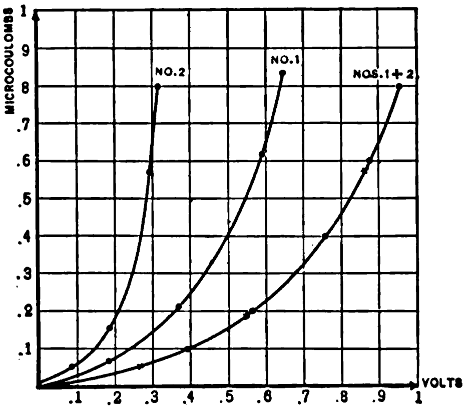

Fig. 4.

The following are the observed values for the cells No. 1 and No. 2 in series: —

Cells in Series.

Volts.	.27	.54	.87	1.01
Microcoulombs . . .	.0595	.187	.573	.917

These values, marked X in the figure, are seen to correspond with the calculated curve.

As the capacities of the cells at any voltage are equal to the slopes of their curves at this voltage, it will be seen that although the capacity of a series combination is smaller than that of the individual elements, it may be considered as constant over a much wider range of voltage.

Dissipation of Charge.

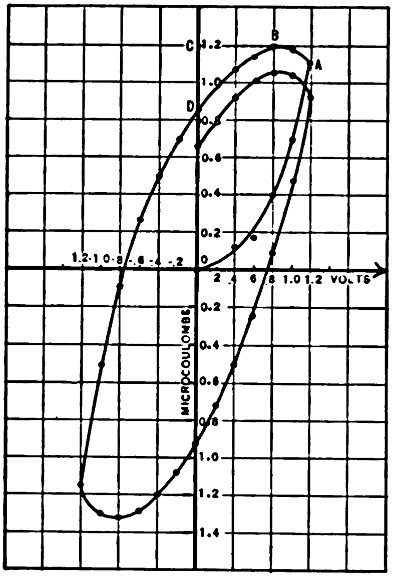

Fig. 5.

The efficiency of an electrolytic condenser cannot be very great, considering the efficiency to be the ratio of the energy of discharge to that of charge. By the step-by-step method we have taken a great number of curves showing the dissipation when the voltage is passed through a cycle. These are very similar to hysteresis curves. We will give but two of these curves. The first (Fig. 5) was taken from two $Pt-H_2SO_4$ cells in series, which were as nearly depolarized at the start as any which we were able to get. The second (Fig. 6) was taken with the $Hg-H_2SO_4$ tubes strongly polarized originally. The method of observing, in each case, was to close the circuit under a given voltage for 11 seconds until the galvanometer throw could be observed, open the circuit for 49 seconds, then close again under the voltage of the next step. The cell was thus on open circuit 49 seconds for each step. (This time was necessary in order to bring the galvanometer needle to rest.) We have increased this time of open circuit in different sets of observations on the same cell, but find little change in size or shape of the curves. Hopkinson has obtained by calculation curves of the same general shape where an alternating voltage was applied and 100 cycles were passed through in a second. The ends of his curves, however, meet. We have been unable to make any of ours do so, and are unable to account for it. We are almost certain that it is not owing to original polarization, to change of temperature during the cycle, to error in the voltage, or to variation of the time of keeping the circuit closed or open at the different steps.

Looking at Fig. 5 it will be observed that the greatest value of q does not correspond to the highest voltage, but to a descending voltage two steps lower. This shows us that in this case the polarization falls on open circuit by an amount greater than the difference between two consecutive steps.

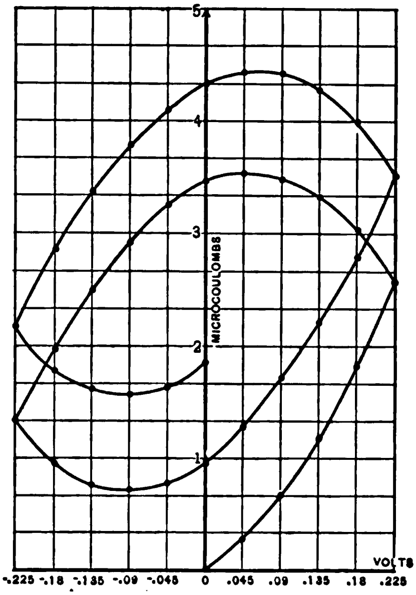

Fig. 6.

The energy expended by the external voltage during the first half-cycle,

$$E = \int v dq,$$

is represented by the area *OABC.* The energy returned by the cell equals the area *BCD.* The ratio of the latter to the former is very small. In the case of the $Hg - H_2SO_4$ cells the efficiency is even lower. The rapid cycles taken by Hopkinson are of the same shape and show equally low efficiency.

In conclusion, we may add that an electrolytic condenser is unsuitable for annulling ferric inductances. While by the adjustment of a proper number of cells in series the capacity might be made to rise with the current in nearly the same manner as the inductance (a thing not true of ordinary condensers), there would be a wide divergence when the current was decreasing. The capacity for decreasing current (slope of curve of quantity and volts) is much less than for increasing. The ferric inductance, owing to hysteresis, is larger on the decrease. Again, the high temperature coefficient and low efficiency are prohibitive to practical usage.

Polytechnic Institute of Brooklyn, December, 1894.

THERMAL CONDUCTIVITY OF COPPER.

By R. W. Quick, C. D. Child, and B. S. Lanphear.

PART I.

CONDUCTIVITY AT HIGH TEMPERATURES.

By C. D. Child and R. W. Quick.

I.

Method Employed.

THE subject of the conduction of heat in metals has received considerable attention during the past three quarters of a century. Many investigations have been conducted on various metals with a view of determining the absolute value of their conductivity, as well as its variation with temperature. Various methods have been employed, both in the manner of determining the value of conductivity and in the measurement of temperature, the latter determination being one of great importance.

In scanning the literature upon the subject one finds that three general methods have been followed in experiments on the determination of conductivity, viz.: —

(1) The metallic wall, whose faces are maintained at different temperatures.

(2) A long bar, one of whose ends is maintained at a constant high temperature.

(3) A long bar, one or both of whose ends may be subjected to periodic changes of temperature.

The first method was employed by Péclét, Christiansen, and Berget; the second by Despretz, Langberg, Forbes, Mitchell, Tait, and others; and the last by Ångström, Neumann, Lorenz, and H. Weber.

Temperatures have been measured by means of the mercury thermometer and the thermo-element. The heating has been accomplished by a flame directly, by hot water or steam, and by plunging the end of the bar into molten metal.

In this investigation, both in dealing with high temperatures and with low temperatures, the second general method of computing the conductivity has been employed, and the results deduced according to the well-known method first outlined and used by Forbes.

Suppose that one end of a metal bar of length a is maintained at a constant temperature either above or below that of the surrounding medium. By calculating the quantity of heat which flows per second through a cross-section of the bar, whose abscissa reckoned from the heated end as origin is x, and equating this value to the quantity of heat which is radiated (or absorbed) by the bar between x and its end a, we get an expression which gives the value of conductivity K. This is easily found to be

$$K = \frac{SD\int_x^a \frac{dv}{dt}dx}{\frac{dv}{dx}}, \tag{1}$$

where v is the temperature excess of the bar at the cross-section considered, t is time, S is specific heat, and D is density. If x is expressed in centimeters, and time in seconds, this gives K in C.G.S. units. If the length of the bar is such that a difference of temperature exists between its end a and the surrounding medium, to the numerator of (1) must be added the quantity of heat which is radiated (or absorbed) per second by the end of the bar. This is seen to be

$$\frac{ASD\frac{dv_a}{dt}}{P}$$

therms per square centimeter, where A is area of cross-section, P is the perimeter, and v_a is the temperature excess at the end.

Now in (1), $\int_x^a \frac{dv}{dt}dx$ represents the area of a curve whose ordinates are the rates of cooling (or absorption) corresponding to different temperature excesses of bar over air, and where abscissæ are distances in centimeters along the bar, reckoned from an arbitrary origin near the heated or cooled end. Thus if the bar be heated or cooled to a uniform temperature and allowed to regain its initial temperature condition while simultaneous observations of time and temperatures are recorded, the curve of cooling can be constructed, and from the tangents to this curve, together with the curve of distribution of temperature along the bar, when one of its ends is maintained at a constant temperature different from that of the surrounding air, the curve whose coördinates are x and $\frac{dv}{dt}$ can be constructed by points, and the area, $\int_x^\infty \frac{dv}{dt}dx$, can be computed.

To obtain $\frac{dv}{dx}$ and $\frac{dv}{dt}$, Forbes assumed v as a function of x in the case of the curve of distribution of temperature along the bar (which for brevity may be called the "statical curve"), and as a certain function of t in the case of the cooling curve. In deducing the results in this investigation, no functions were assumed, but the curve was drawn on a very large scale, utilizing in the best possible manner the points previously located by observed or calculated data; and the tangents at different points were taken by means of a straight edge, which, as determined by trial, could be set with a maximum deviation of 0.2 per cent from the mean setting.

The copper on which the experiments herein described were conducted was electrolytic copper in the form of prismatic bars, having a cross-section of about 5 cm. × 2 cm. One of the bars was a little over a meter in length, and had lines ruled on its surface 5 cm. apart. This will be referred to as the "long bar," and is that which was used for obtaining the statical curve of temperatures, in the experiments both with high and low temperatures. The other bar was about 20 cm. long, and will be designated the "short bar." The original bar from which the two bars were cut was one of a set made for Professor W. A. Rogers, and loaned for the purpose of carrying on these investigations. The short bar had a hole about 1 cm. in diameter bored near the center of its length and perpendicular thereto, parallel to the longest sides of the rectangular cross-section of the bar. This hole, extending not quite through the bar, could be filled with mercury and a thermometer bulb inserted, or, when the bar was wanted as a homogeneous mass, the hole was filled with a tightly fitting copper plug.

II.

Measurement of Temperature.

As has already been stated, the methods that have been employed in measuring temperatures of the bar are the thermometer and thermo-element. The use of the former inserted in holes bored at certain intervals in the bar possesses many disadvantages. The presence of holes filled with mercury and glass destroys the homogeneity of the bar, besides necessitating many uncertain corrections relative to expansion of glass and mercury. When the thermo-junction is inserted in holes in the bar, a similar objection may be offered; and when it is used on the surface, the temperature is obtained at only one point of the perimeter of the cross-section. The method that seemed to be best adapted to the present case is the "resistance" method, and is that which was used throughout this investigation.

In dealing with ordinary and high temperatures, this method involved only a means of detecting very minute changes in the resistance of a very small wire. This wire of insulated electrolytic copper was 0.002 in. in diameter, and wound in a single layer of about twenty turns (close together as possible) upon a copper collar formed of very thin sheet copper $\frac{1}{8}$ in. in width, that fitted the bar just closely enough to admit of movement along it. After the wire was wound and the terminals joined to a coarse wire, which in turn was carefully connected to a flexible cable, the wire was given a coat of shellac to maintain it in place. This collar, bearing the copper coil of fine wire, as shown in Fig. 1, was formed from two sheets of copper. *c* and *c'* are simple extensions of the end faces of the collar turned up at right angles thereto. The cross-pieces *a* and *b* are used to stiffen *c* and *c'*.

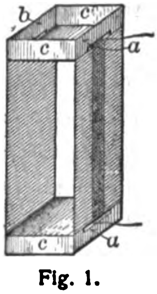

Fig. 1.

The object of these extensions *c* and *c'* was to afford a means of moving the collar on the bar. Their use will be more apparent when we come to the measurement of low temperatures.

The resistances of this copper coil were measured by a Wheatstone slide wire bridge, of which the collar formed one arm, *R* (Fig. 2). The wire *W*, which formed the ratio arms of the bridge, was an alloy of platinum and iridium, and contact was made at any point *P*, by means of a collar *C*, with vernier attachment, sliding on an insulated bar *S*, provided with a millimeter scale. The shaded portions are heavy copper. *G* is a sensitive and almost dead-beat galvanometer, and *R'* is a resistance compensated for temperature, against which the collar *R* is balanced. *r* and *r'* are resistances so chosen that, for the range of temperatures to be measured, *P* moves the whole length of the wire *W*.

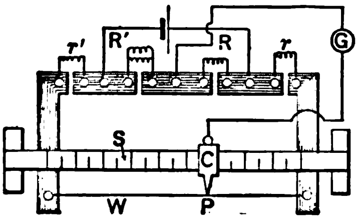

Fig. 2.

In experiments involving ordinary and high temperatures, this bridge was calibrated in the following manner, directly for temperatures of the bar: The collar in electrical connection with the bridge was placed on the short bar beside the hole already referred to, and which contained a thermometer bulb immersed in mercury. On each end of this bar, and insulated from the copper beneath by a layer of asbestos paper, were wound several turns of iron wire, through which was passed an electric current. By suitably adjusting the resistance in series with the solenoid, the bar could thus be heated to any desired temperature.[1] Two sets of simultaneous readings of the thermometer and the bridge were then taken: first, when the bar was slowly rising in temperature; and second, when its temperature was allowed to slowly fall. From these observations two curves were constructed with bridge readings as abscissæ, and temperatures of the bar as ordinates. A curve which was the mean of these two curves was then drawn, and this was used in determining temperatures of the bar from the bridge readings. The calibration was for the range 50° C. to 200° C.

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[1] This method of heating has been employed quite extensively in the Cornell Physical Laboratory. See PHYSICAL REVIEW, Vol. I., p. 144.

III.

Determination of Data for the Statical Curve.

The long bar was placed on non-conducting supports, in a room whose temperature was maintained very constant, and one end was wound with a solenoid of iron wire as above described; in this case three layers were used, insulated from each other by asbestos paper. In series with the solenoid were placed an ammeter and a variable resistance. It was assumed that if the current were constant, the heat supply at the end of the bar would be practically constant; and so, in order to avoid the fluctuations of a dynamo current, the storage battery was used, and the current was maintained at the proper strength by adjusting the resistance. In order to prevent direct radiation from the solenoid and heated asbestos upon the copper coil of the collar, a screen was constructed in the following manner: Two sheets of tin about 50 cm. square were taken, and after the edges of one sheet were turned up at right angles on three sides of the square, the other sheet of tin was soldered to the first, leaving a space of a little over a centimeter between the tin sheets. In the center of this double thickness of tin was cut out a rectangular aperture or tunnel (whose sides were closed with tin) just large enough to admit the insertion of the long bar *without* contact with the tin. This screen was placed in a suitable standard, which supported it in a vertical plane, with one end of the bar protruding through it far enough to admit of the winding of the heating solenoid. The space between the tin sheets was then filled with water of the temperature of the room, and was constantly renewed from a tube leading out of a large jar of water placed on a level above the upper edge of the screen. After circulating through the screen, it emptied into a receptacle placed on the floor of the room. In this way direct radiation was cut off, and the screen never reached a temperature above that of the surrounding air.

After the bar had come to a statical temperature condition, as was shown by the collar in connection with the slide bridge, readings were taken on the latter, together with temperature of the surrounding air (which varied only about 2°), when the collar was placed successively at the different divisions on the bar. Table I. gives the temperature excess of the bar over air at the different divisions on the bar, counted from a point near the water screen. These excesses are obtained by subtracting the air temperature from the temperatures taken from the calibration curve of the bridge.

Table I.

DATA FOR STATICAL CURVE.

Distance from heated end in cm.	Temperature excess of bar over air.	Distance from heated end in cm.	Temperature excess of bar over air.
0	144.2	50	65.0
5	131.2	55	61.1
10	120.4	60	58.0
15	110.4	65	55.8
20	101.6	70	53.5
25	93.6	75	51.4
30	85.8	80	50.3
35	79.6	85	49.4
40	74.0	90	48.7
45	68.9		

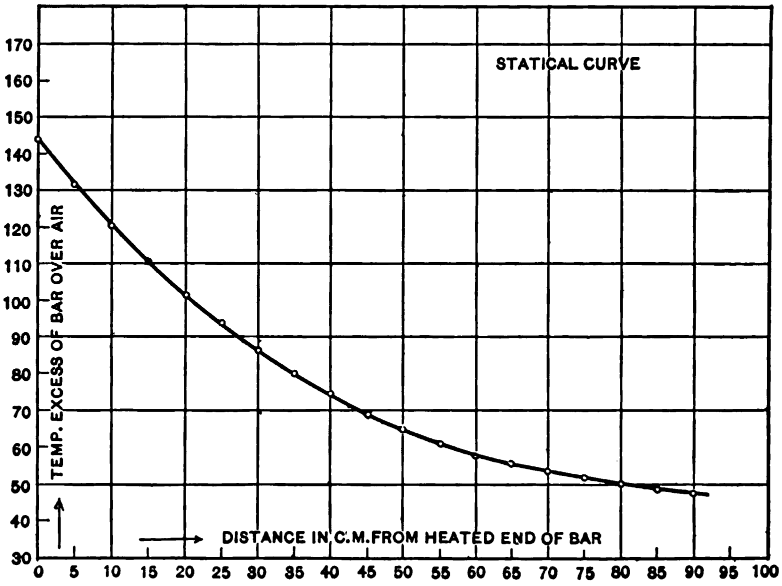

Fig. 3.

From Table I. the curve in Fig. 3 was constructed.

IV.

Determination of Data for Time Curve of Cooling of Long Bar.

One of the great difficulties that have hitherto been encountered in obtaining these observations was that of heating the bar to a uniform temperature. A method employed in this case, and which proved satisfactory, was that of electric heating by means of a solenoid. The collar was first placed on the long bar near its center, and the latter was wound throughout its entire length with iron wire, whose turns were quite close together and insulated from the bar by asbestos paper. After the collar indicated a temperature of about 25° above the highest temperature recorded in the statical experiment, the solenoid was quickly removed from the bar, and simultaneous observations of bridge readings, time, and temperature of the room were taken. Table II. contains the data obtained from these observations, and from which the curve in Fig. 4 was constructed.

Table II.

DATA FOR TIME CURVE OF COOLING OF LONG BAR.

Time.	Temperature excess of bar over air.	Time.	Temperature excess of bar over air.
min. sec.		min. sec.	
0 52	155.6	20 2	92.8
2 10	149.7	22 29	87.3
3 35	143.7	25 3	81.8
5 5	137.9	27 52	76.4
6 35	132.1	30 51	71.1
8 12	126.5	34 5	65.8
9 57	120.9	37 44	60.5
11 43	115.3	41 38	55.2
13 37	109.6	46 7	50.0
15 38	104.0	50 0	45.9
17 46	98.3		

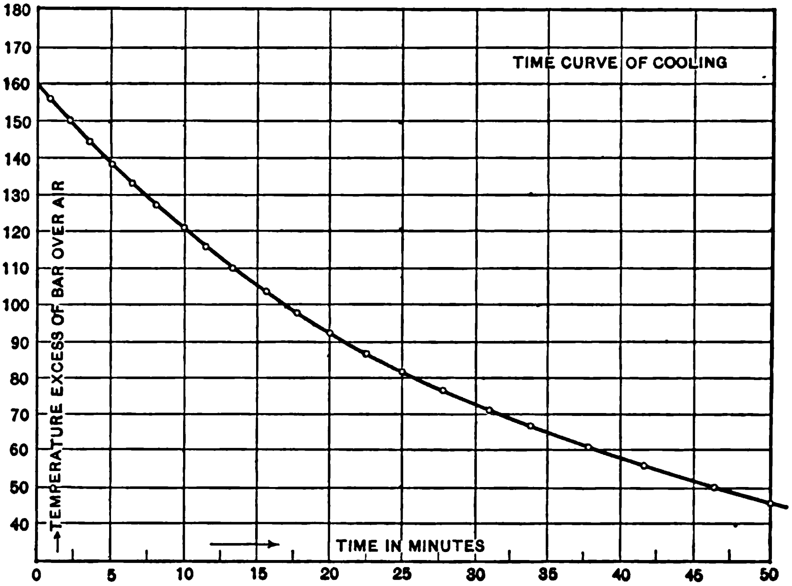

Fig. 4.

V.

Deduction of Results.

It now remains to construct, by aid of the two curves already drawn, the curve whose abscissæ are distances from heated end,

Table III.

DATA FOR INTEGRATION CURVE.

Distance in cm. (x) from heated end of bar.	**Tangents to cooling curve $-\frac{dv}{dt}$**	**Distance in cm. (x) from heated end of bar.**	**Tangents to cooling curve $-\frac{dv}{dt}$**
0	0.0690	50	0.0260
5	0.0598	55	0.0240
10	0.0540	60	0.0220
15	0.0490	65	0.0207
20	0.0450	70	0.0200
25	0.0396	75	0.0190
30	0.0362	80	0.0187
35	0.0325	85	0.0183
40	0.0298	90	0.0178
45	0.0278	92.12	0.0177

and whose ordinates are $-\frac{dv}{dt}$. This curve, for convenience, will be designated the "integration curve"; for the area, $\int_x^a \frac{dv}{dt}dx$, is proportional to the quantity of heat radiated per second by the bar between the neutral end and x. The numbers in the second column of Table III. are obtained by taking tangents to the cooling curve at temperature excesses that are the same as those of the bar at the different distances in column 1 when it was in the statical temperature condition. The statical curve shows this. The curve constructed from Table III., and shown in Fig. 5, was integrated in parts by drawing it on a sufficiently large scale, so that consecutive small lengths of the curve deviated very little from a straight line. The area included between two ordinates, the x-axis, and a small length of the curve, could then be easily obtained, and by adding together such consecutive areas, the area between any ordinate and the end of the bar can be easily obtained. In Table IV. the lower limit of integration x in column 2 is the same x as in column

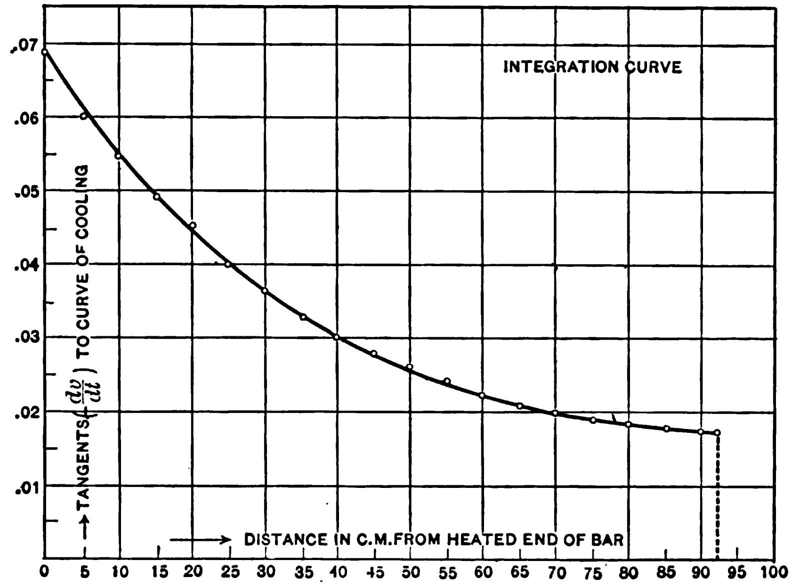

Fig. 5.

1, and a represents the total length of the bar from the assumed origin near the heated end. To each of the areas in column 2 has been added the area that corresponds to the heat radiated per second from the face end of the bar. The numbers of column 3 are the tangents to the statical curve corresponding respectively to distances in column 1. The temperatures in the fourth column are obtained by adding to the temperature excess of the bar the temperature of the room at the time of the statical experiment. The values of K are then computed from equation (1); that is, dividing column 2 by column 3 and multiplying by SD. But since both specific heat and density are functions of temperature, cognizance must be taken of this change. Most writers give the average value of S for copper between 0° and 100° as 0.093, and this coincided with a determination made by Shepard[1] on the same specimen of copper that was used in this investigation. Moreover, as this corresponds to the value given by Bède's formula

TABLE IV.

DATA FOR CURVE OF CONDUCTIVITY (K).

Distance on bar x.	$\int_x^a \frac{dv}{dt} dx$	Tang. to statical curve $-\frac{dv}{dx}$	Temperature.	Conductivity K.
80	0.232	0.211	74.0	0.914
70	0.425	0.385	77.2	0.920
60	0.634	0.550	81.8	0.963
50	0.871	0.768	88.2	0.952
40	1.149	1.06	97.3	0.915
35	1.306	1.21	103.1	0.915
30	1.478	1.35	109.5	0.932
25	1.668	1.51	116.9	0.945
20	1.878	1.71	124.9	0.944
15	2.111	1.88	133.8	0.971
10	2.369	2.05	143.6	1.006
5	2.656	2.32	154.6	1.004
0	2.979	2.57	166.8	1.024

$$S_t = 0.0892(1 + 0.00073\, t),$$

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[1] Thesis, Physical Properties of Copper, Library of Cornell University.

the value of S was obtained from the above in computing absolute values of K. Density also varies with temperature, but as the 5 cm. divisions were marked on the bar itself, a small error in measurement would be introduced, owing to linear expansion of the bar. Therefore in order to correct for this, instead of using D_t, we use $D_t^{\frac{1}{3}} \cdot D_{20}^{\frac{2}{3}}$, where D_{20} is the density at the temperature at which the bar was marked. Now the writers obtained 8.86 for the value of density of the bar at a temperature of 4° C., which, combined with the temperature factor $(1-0.000056\,t)$ as determined by R. W. Stewart,[1] gives

$$D_t = 8.862(1-0.000056\,t).$$

Using the values of specific heat and density as given above, the values of K were computed at the different temperatures, showing an increase in conductivity as temperature increases through the range 70° to 170°. This result will be shown graphically in Part II., which gives the result of the investigation on conductivity at low temperatures.

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[1] Proc. Roy. Soc. London, V. 53, p. 151, 1893.

ON THE ABSORPTION OF CERTAIN CRYSTALS IN THE INFRA-RED AS DEPENDENT ON THE DIRECTION OF THE PLANE OF POLARIZATION.

BY ERNEST MERRITT.

THE fact that many doubly refracting crystals show different absorbing powers for the ordinary and extraordinary rays was doubtless first brought to the attention of physicists by the behavior of turmalin. This substance, in fact, exhibits the phenomena of dichroism in an unusually marked degree, for in specimens of suitable thickness the ordinary ray may often be completely suppressed, while the extraordinary ray is still transmitted in considerable intensity. A turmalin plate may thus be employed as a polarizer, and on account of their simplicity such plates have long been used for this purpose in elementary lectures on polarized light.

Turmalin, however, is by no means the only crystal that exhibits dichroism. Investigations of the absorbing power of doubly-refracting crystals by Brewster,[1] Haidinger[2] and others have shown that a more or less unequal transmission of the ordinary and extraordinary rays may be observed in almost all cases, although the phenomenon is rarely so marked as in the case of turmalin. The absorption of one ray, or in some cases of both rays, is often selective, thus leading to a difference in color between the two transmitted rays. It is to this effect that the phenomenon owes its name. Dichroism in the sense of an unequal transmission of the two polarized rays is often present, however, even when no color difference can be observed. Iceland spar, for example, although colorless and apparently quite transparent to both rays, has been found to transmit the extraordinary ray more readily than the ordinary ray, the difference being about 4 per cent throughout the visible spectrum.

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[1] Edinburgh Philosophical Journal, 4, 348.

[2] Poggendorff's Annalen, 65, 1; also 86, 131, etc.

A theory of the absorption of light in dichroic crystals has been developed by Voigt,[1] by whom a quantitative relation has also been deduced between the coefficient of absorption and the direction of propagation. Qualitative observations by Becquerel[2] are in the main in accord with the predictions of this theory. Chief among the results of these experiments may be mentioned the establishment of the law that absorption is dependent only upon the direction of vibration, and not upon the direction of propagation. Investigations bearing upon the quantitative relation between the coefficient of absorption and the direction of vibration have been undertaken by Becquerel,[3] Schwebel,[4] Carvallo,[5] and others. The results are, however, not numerous, and are in some cases contradictory, so that they cannot be looked upon as affording a satisfactory solution of the problem.

It is because of its bearing upon the general theory of double refraction that the subject of crystalline absorption appears most interesting and important. It seems natural to expect that the intimate relation which has been found to exist between absorption on the one hand, and refraction and dispersion on the other, should be present in the case of doubly refracting crystals, as well as in the case of isotropic substances. If this is true, the fact that the two polarized rays are refracted and dispersed in different degrees would in itself be an indication that they are unequally absorbed. In other words, we should expect *all* doubly refracting crystals to exhibit dichroism. The fact that many crystals are known which appear to be equally transparent to the ordinary and extraordinary rays, cannot be looked upon as proving that this conclusion is false; for the investigations of dichroism which have so far been made have extended only to the region of the visible spectrum. The behavior of crystals in the ultra-violet and infra-red is at present entirely unknown, and it seems not improbable that a selective absorption of the two rays may really occur, even when no dichroism is observable in the visible region. An extension of our knowledge of crystalline absorption throughout a wide range of wave-lengths is therefore desirable. Such data, in connection with a knowledge of the law of dispersion, would afford a means of testing the theories advanced by Kelvin, Helmholtz, Ketteler and others, and might throw light not only upon the subject of double refraction, but also upon the question of the physical structure of crystals.

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[1] Nachrichten der Gesellschaft der Wissenschaft zu Göttingen, 1884, p. 337. Wiedemann's Annalen, 23, 557.

[2] Annales de Chimie et de Physique (6), 14, 170.

[3] l.c.

[4] Zeitschrift für Krystallographie, 7, 153.

[5] Comptes Rendus, 114, 661.

It was with this thought that the following investigation of dichroism in the infra-red was begun. The subject was suggested by Professor Kundt, whose illness and death, almost before the investigation had been begun, deprived the writer of that stimulating interest and timely advice which all who have worked with Professor Kundt have found so valuable. The experimental work was carried on in the Physical Institute of the University of Berlin, where every facility was afforded the writer by those in charge. I wish especially to acknowledge my indebtedness to Dr. Rubens, whose wide experience in bolometric work made his kind advice and assistance of the greatest value.

METHODS AND APPARATUS.

In the following investigation uniaxial crystals only have been studied, and the absorption has been determined for the two principal directions of vibration: viz., parallel and perpendicular, respectively, to the optic axis. For this purpose plates were obtained of the crystals to be studied, ranging in thickness from 1 mm. to 5 mm., the plates being cut in most cases parallel to the optic axis. With the aid of a spectro-bolometer the absorption of these plates was measured in the usual manner. By using polarized light, so as to be able to cut off at pleasure either the ordinary or extraordinary ray, it was possible to determine the absorption for each direction of vibration.

A diagram of the apparatus as first arranged is shown in Fig. 1. Rays from the intense zirconium lamp *Z* fell upon the rock-salt condensing lens *L*, and after reflection at the polarizer *P* were brought to a focus on the slit of the spectrometer. The polarizer consisted of four sheets of glass, the thickness of each sheet being about 0.1 mm. It was set at the proper angle for maximum polarization with the aid of a nicol, and gave almost complete polarization for all visible rays. As will be shown later its action on the infra-red rays was equally satisfactory.

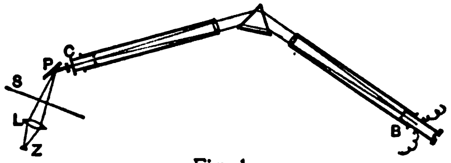

Fig. 1.

The lenses as well as the prism of the spectrometer were of fluorite, the aperture of the former being 2.5 cm. A linear bolometer was mounted at *B* in place of the cross-hair of the observing telescope. The bolometer first used consisted of several thin strips of iron mounted side by side and connected in series, the combined resistance being about 3 ohms. The width of this instrument, expressed in angular measure, was about 10′. The remaining arms of the bolometer bridge were made of German silver, and were protected from sudden temperature changes. Connections in the bridge circuit proper were in most cases made by soldering; where this was impossible the joint was carefully wrapped with several layers of cotton to prevent sudden variations of temperature. It was found that this precaution could in no case be neglected. The writer was fortunate in securing for use with the bolometer the extremely sensitive galvanometer which was constructed by Professor Snow some years since, and which has already been described in these pages.[1]

In making observations the method of procedure was as follows: The plate of crystal to be tested was placed in front of the slit at *C*, the optic axis being vertical, and the throw of the galvanometer was observed on removing the screen *S*. The crystal was now removed so that rays from the lamp would no longer have to pass through it, and the throw of the galvanometer was again noted. The ratio of the first throw to the second was then taken as the percentage transmission for the extraordinary ray. To guard against errors due to unsteadiness in the galvanometer, and other irregularities, each observation was usually repeated from five to twenty times, readings being taken alternately through the crystal and with the latter removed. In all cases the first throw of the galvanometer was used, instead of waiting for the needle to come to rest; in fact, the unsteadiness of the galvanometer made it impossible to observe otherwise. The use of the first throw in such cases is however fully justified, since, as the writer has shown in a previous article,[1] it is strictly proportional to the energy absorbed by the bolometer wire. By placing the crystal with its optic axis horizontal, determinations could be made for the ordinary ray.

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[1] Physical Review, 1, 28.

In computing wave-lengths the values recently found by Rubens[2] for the indices of refraction of fluorite were used. The prism was set each day in the position of minimum deviation for sodium light and was then left unaltered. Although the computations were made slightly more complicated by this arrangement, it was found more convenient, with the apparatus at hand, than to work always at the position of minimum deviation for the particular ray studied.

A highly satisfactory sensitiveness was reached with this apparatus in spite of the very considerable loss of energy at the polarizer. This loss was certainly in excess of 70 per cent. To this must be added a loss of fully 15 per cent due to reflection at the two faces of the prism.[3] In the case of polarized rays, the loss at the prism is twice as great as it would be with ordinary light, for the prism tends to polarize the rays refracted through it in the vertical plane, while the plane of polarization of the rays entering the slit is horizontal. It will readily be seen that when observations are to be made in the extreme infra-red, these losses become the source of serious difficulty.

With the apparatus described observations were made on Iceland spar and quartz, the transmission curves for each being determined out to $\lambda=4\mu$. On plotting these curves and comparing them, indications were however found of some serious error. Each curve showed somewhere between $\lambda=2$ and $\lambda=4$ a rapid increase of absorption, followed in each case by a gradual decrease. The curves were strikingly similar, a fact which seemed to the writer sufficiently remarkable to warrant investigation.

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[1] American Journal of Science, xli., 417.

[2] Wiedemann's Annalen, 51, 381.

[3] du Bois and Rubens, Wiedemann's Annalen, 49, 593.

The first explanation that suggested itself was that reflection might be taking place from the tubes of the collimator and telescope, so that the spectrum at the bolometer was rendered impure. In this case the bolometer would receive not only the rays proper to its position in the spectrum, but also diffuse rays of all wave-lengths, due to irregular reflection from the tubes of the telescope.

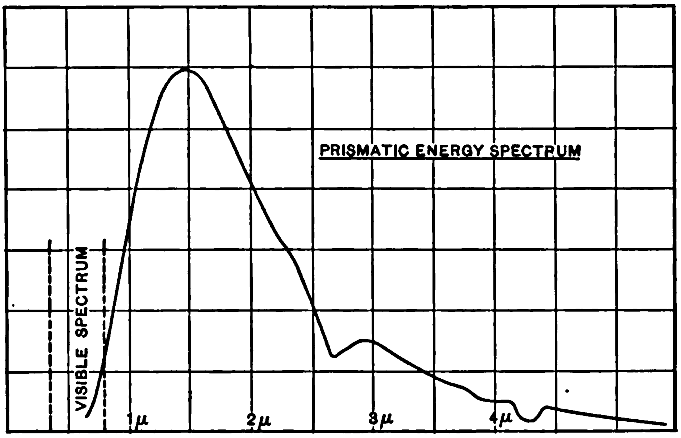

Fig. 2.

The fact that the field of the spectrometer was never entirely dark, even when the telescope was placed far beyond the region of the visible spectrum, seemed to support this explanation of the difficulty, for any cause which would produce an irregular scattering of the visible rays would probably bring about the same effect in the case of those that are invisible. The rays that are most important in causing errors are probably those whose wave-lengths lie between 1.0 μ and 2.0 μ, since it is in this region that the spectrum of the zirconium lamp is most intense. As will be seen by Fig. 2, which represents the distribution of energy throughout the prismatic spectrum, the intensity at $\lambda=1.5\ \mu$ is about seven times as great as at $\lambda = 3.5\ \mu$, and beyond. It is clear that if only a small proportion of these intense rays were for any reason irregularly reflected or scattered, so as to fall upon the bolometer wire when in a position where the energy of the true spectrum is small, serious errors might result. A little consideration shows also that the effect in all such cases would be to make the apparent percentage transmission approach the value which corresponds to the wave-length of the more intense rays: viz., $\lambda = 1.5\ \mu$. In the case of Iceland spar and quartz, whose absorption in this region is only slight, the effect would therefore be to increase the apparent transmission, and this effect would be more marked in the case of the longer wave-lengths. It will thus be seen that the assumption of some cause which will bring about a scattering of the more intense rays, affords a satisfactory explanation of the peculiar form observed in the transmission curves. The importance of this source of error is well brought out in Fig. 3. The upper curve in this figure shows the percentage transmission for Iceland spar (ordinary ray) as first determined, while the lower curve represents the curve obtained after the error had been eliminated.[1]

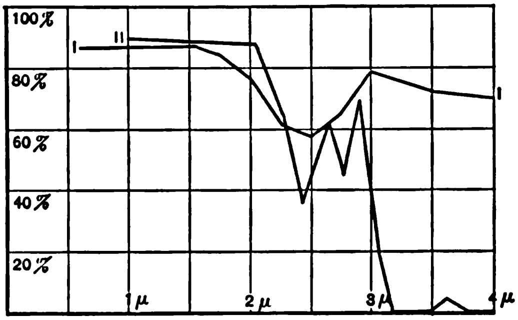

Fig. 3.

Under the impression that the evident impurity of the spectrum was chiefly due to irregular reflection in the telescope and collimator, elaborate precautions were taken to prevent such reflection. With this object the tubes were thoroughly coated with lampblack, while numerous screens were so placed as to make it practically impossible for reflected rays to reach the bolometer. The transmission curve for Iceland spar was now redetermined. In this set of observations especial precautions were taken to adjust the focus of the fluorite lenses for each different wave-length. The lack of achromatism of such lenses, as has been pointed out by Rubens and Paschen, becomes quite marked beyond $\lambda = 1\ \mu$, and is in itself a possible source of serious error. The very property of fluorite which renders it invaluable in the construction of prisms for work in the infra-red, makes it quite unsuited as a material for lenses.

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[1] The difference between the two curves is doubtless due in part also to the fact that a much narrower bolometer was used in making the observations shown in curve II.

The results of this second set of observations showed a partial elimination of diffuse rays, the transmission curve taking a course

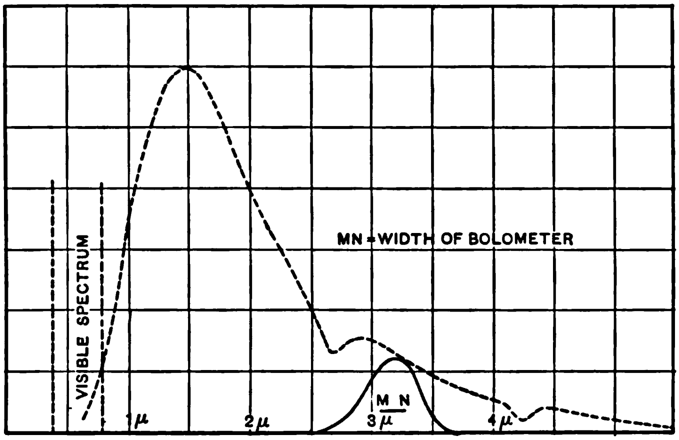

Fig. 4.

intermediate between the two curves of Fig. 3. It seemed however questionable whether the error was entirely removed. The possibility of a slight scattering of the rays at the surface of the lenses and prism still remained, and the fact that a faint illumination of the field of the telescope was still to be observed in the infra-red, was, in fact, conclusive in showing that such diffuse rays were present. After an unsuccessful attempt to find some substance that would absorb the intense rays lying between $\lambda = 1$ and $\lambda = 2$ and so eliminate the error, the following method was devised, by which satisfactory results were finally obtained.

A rather broad slit was mounted immediately in front of the lamp, and by means of a second prism a spectrum was thrown upon the slit of the spectrometer. The length of this spectrum being much greater than the width of the slit, only a small fraction of the rays from the source could enter the spectrometer. By suitable adjustment it was thus possible to illuminate the slit with rays of any wave-length desired, while all other rays were excluded. The energy spectrum under these circumstances assumed a form similar to that shown in Fig. 4. In making observations at $\lambda=2.5$, and beyond, rays from the more intense region of the spectrum did not even enter the spectrometer, so that errors due to diffuse rays were impossible.

Although the arrangement of the apparatus just described gave satisfactory results, it was inconvenient on account of the difficulty of proper adjustment. After sufficient trial to show its efficiency, it was therefore abandoned, and the whole apparatus was reconstructed and arranged as shown in Fig. 5. Instead of fluorite lenses, whose disadvantages have already been referred to, concave mirrors were employed. (M_1, M_2, Fig. 5.) The two lenses were now used in constructing a new spectrometer, which was placed before the collimator of the spectrometer proper. Rays from the zirconium lamp, after passing through a rock-salt lens at *L*, were reflected by the polarizer *P* so as to converge upon the slit of the first spectrometer. The spectrum formed by this instrument fell upon the slit of the second spectrometer at s_2. The crystal to be tested was placed in front of the first slit, while the screen *S* stood immediately back of the condensing lens. In some preliminary observations the crystal was placed at s_2, but it was soon found that this arrangement was likely to introduce errors. If the

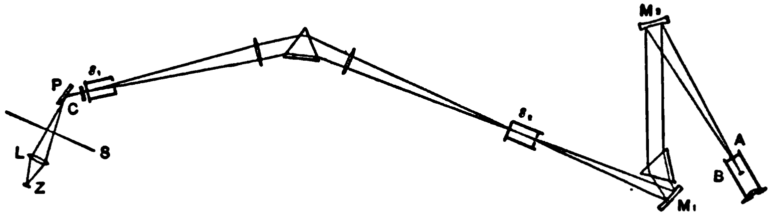

Fig. 5.

crystal was not exactly perpendicular to the axis of the collimator tube, a slight displacement of the spectrum was produced, so that the group of rays entering the slit through the crystal was not the same as that received when the crystal was removed. The effect was to alter the apparent transmission, often in a marked degree. In one case, for example, the result was somewhat startling, the percentage transmission being apparently greater than 100! It should be mentioned that such errors cannot occur when the crystal is placed in front of s_1, provided the slit is completely covered by the image of the source. In this case the effect of setting the crystal in an oblique position would merely be to slightly increase the thickness of the absorbing layer.

The arrangement of apparatus shown in Fig. 5 was found in all respects satisfactory, the adjustments being simple and convenient. It was used throughout the observations which follow.

It may be mentioned here that the cause of the diffuse rays which had made the adoption of this form of apparatus necessary, was finally traced to the prism itself. On using the mirror spectrometer in the ordinary way, the field of the telescope remained as brightly illuminated as before, and the transmission curve for Iceland spar showed the same form as that previously obtained. Yet the fine character of the mirror surface made it highly improbable that any scattering of the reflected rays could occur there, while the tubes of the instrument, which were quite short, were blackened and screened in such a way as to render diffuse reflection from their inner surface impossible. The mirrors were of thick glass silvered by electric deposition *in vacuo*. The surface obtained in this way was excellent, being practically invisible even when illuminated by the brilliant rays of the zirconium lamp. It would thus appear that the diffuse rays, to which reference has so frequently been made, must be ascribed almost wholly to the prism. The prism used, while not perfectly pure, was still a fairly good specimen of fluorite. But even when all visible imperfections were covered, the error still remained. In the opinion of the writer the fault is one which is common to all prisms, even when not so obvious as in the case here cited. Even with the best glass the effect is present to some small extent, for the field beyond the visible spectrum is rarely entirely dark. The effect may be due either to the unavoidable imperfections of the polished surfaces, or to repeated reflection inside the prism. The latter cause would doubtless be especially important when the angle of the prism differs only slightly from 60°. It is because of the serious character of the error introduced, and the difficulty in its detection, that I have described so in detail the preliminary experiments which led to its removal.

OBSERVATIONS AND RESULTS.

In the course of this investigation three crystals were tested, viz., Iceland spar, quartz, and turmalin. The two transmission curves were determined first, in each case, by the use of polarized rays, as described above, the specimens being cut parallel to the optic axis. As a check upon the results, the polarizer was then removed and the transmission curves determined for unpolarized rays. To eliminate the error due to the polarizing action of the two prisms, the specimen was placed during this determination with its axis at an angle of 45° with the vertical. By removing the polarizer and using the direct rays of the lamp, the energy available at the bolometer was greatly increased, so that the curves of average transmission are probably more reliable than the other two. The fact that in all cases they lie almost midway between the curves obtained for the two polarized rays, serves as a very satisfactory check on the accuracy of the original determinations. Still another check on the results was obtained in the case of Iceland spar and quartz by using plates cut perpendicular to the optic axis. In this case also the direct unpolarized rays of the lamp were used.

A more detailed statement of the results is given below.

Iceland Spar.

Plate *A*, parallel to the axis; thickness = 2.99 mm.

Plate *B*, perpendicular to the axis; thickness = 1.98 mm.

Observations were first made with plate *A* as above described, the bolometer used being 11′ wide. In the case of the ordinary ray indications were found of absorption bands in the neighborhood of $\lambda = 2.4\mu$ and $\lambda = 2.7\mu$. The bolometer was, however, too broad to admit of an accurate location of the bands. A narrower bolometer was therefore substituted, the new instrument having a width of about 3′.5. This thoroughly excellent bolometer was kindly loaned to me by Dr. Rubens. It was slightly bent to correspond to the curvature of the spectral lines, and in spite of its small surface showed almost the same sensitiveness as the broad bolometer first used. Its resistance was about 15 ohms. The region lying between $\lambda = 2.0\mu$ and $\lambda = 3.0\mu$ was very thoroughly investigated with this instrument, with the result that two sharp absorption bands were found, as shown in Fig. 1 (Plate I.) curve *O*, at $\lambda = 2.44$ and $\lambda = 2.74$ respectively. Beyond $\lambda = 3.0\mu$ the absorption of the ordinary ray was so great that determinations made with plate *A* were untrustworthy. The portion of curve *O* lying between $\lambda = 3.0$ and $\lambda = 5.5$ was afterwards located by observations on plate *B*.

In determining the transmission curve for the extraordinary ray (curve *E*, Fig. 1), the narrow bolometer was used out to $\lambda = 4.0\mu$. Beyond this point the energy of the spectrum was so small that it was found necessary to substitute a broader wire. In the determination of the transmission curve for unpolarized rays (curve *A*), the narrow bolometer was used throughout.

In Fig. 3 are shown the results obtained with plate *B* (perpendicular to the optic axis). The resemblance between this curve and that previously obtained for the ordinary ray is obvious, although the fact that the thickness was different prevents a perfect agreement. The dotted portion of curve *O*, Fig. 1, was computed from these observations, the difference in thickness being taken into account.

The results plotted in Fig. 1 exhibit in striking form the complete independence of the two transmission curves. No indication can be found that the form of one curve is in the least influenced by that of the other. The absorption bands at $\lambda = 2.44$, $\lambda = 2.74$, afford perhaps the severest test of this conclusion. In the neighborhood of these bands the absorption of the extraordinary ray was determined with the greatest care, but no increase in absorption could be detected. This complete independence of the two curves offers also convincing proof of the practically complete polarization of the rays entering the spectrometer. A deviation of 2 per cent from complete polarization would certainly have been detected by a depression in curve E at $\lambda = 2.5$.

It will be seen that the absorption of the ordinary ray at $\lambda = 3.3$ is almost complete, while for the same wave-length 60 per cent of the extraordinary ray is transmitted. At this particular wave-length, therefore, Iceland spar has the same property as is possessed in the visible spectrum by turmalin. A plate parallel to the axis should act as a polarizer, while two plates placed with their axes crossed should suppress the rays of this wave-length entirely. This conclusion was verified by experiment.

In order to bring the results into a form which should be independent of the thickness of the specimens used, the coefficient of extinction has been computed for each of the two transmission curves. If the intensities of the incident and transmitted rays be represented by I_1 and I_2 respectively, while the intensity of the rays reflected from the first surface is put equal to rI_1, then

$$I_2 = I_1(1 - 2r + \ldots)\, e^{-lp},$$

where l represents the thickness of the absorbing layer. The value of the coefficient of extinction p is therefore

$$p = -\frac{I}{l} \log_e \frac{I_2}{I_1(1 - 2r)}.$$

Terms in higher powers of r may be neglected, since in no case did r exceed 0.06. For the region of the visible spectrum r was computed by the use of Fresnel's formula from the index of refraction, and this value was then used for computations in the infra-red. A small error is here introduced by the assumption that r remains unchanged. Since, however, the correction for reflection was in all cases of only small importance, the influence of this error on the result cannot be great.

The values of p computed in the manner indicated are given in Table I.[1] The centimeter has been used as unit of thickness (l).

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[1] Only a small portion of the observations have been included in this table. But the data given have been selected so as to show the more important features of the curves in question. The same statement applies to Tables II. and III.

Table I.

ICELAND SPAR.

Ordinary Ray.				Extraordinary Ray.			
Wave-length.	**Observed percentage transmission.**	***p***	**Absorption for plate 1 mm. thick.**	**Wave-length.**	**Observed percentage transmission.**	***p***	**Absorption for plate 1 mm. thick.**
1.02 μ	90.5 %	0.0	0.0 %	1.02 μ	94.0 %	0.0	0.0 %
1.25	89.6	0.0	0.0	1.27	91.7	0.0	0.0
1.45	88.9	0.0	0.0	1.80	91.7	0.0	0.0
1.72	88.0	0.033	0.2	2.15	92.0	0 0	0.0
2.07	85.5	0.13	1.0	2.49	89.0	0.14	1.0
2.11	71.0	0.74	7.0	2.87	90.4	0.08	0.5
2.30	50.0	1.92	17.3	3.00	81.5	0.43	4.0
2.44	36.4	3.00	25.7	3.28	62.5	1.32	12.2
2.53	50.0	1.92	17.3	3.38	71.0	0.89	8.2
2.60	62.0	1.21	11.2	3.59	54.5	1.79	16.0
2.65	53.0	1.74	15.7	3.76	50.3	2.04	18.4
2.74	44.2	2.36	21.0	3.90	65.3	1.17	10.8
2.83	60.0	1.32	12.4	4.02	71.0	0.89	8.2
2.90	72.0	0.70	6.6	4.41	67.2	1.07	10.0
2.95	52.0	1.80	16.2	4.67	45.5	2.40	21.7
3.04	21.5	4.71	37.4	4.91	64.0	1.25	11.5
3.30*	0.5	22.7	90.0	5.04	49.0	2.13	19.2
3.47	1.7	19.4	86.0	5.34	24.5	4.41	35.0
3.62	13.6	9.6	61.0	5.50	2.0	12.8	73.0
3.80	2.2	18.6	84.0				
3.98	0.0	∞	100.0				
4.35	24.3	6.6	48.0				
4.52	5.2	14.3	76.0				
4.66	8.7	11.6	68.0				
4.83	27.0	6.1	45.0				
4.96	29.3	5.5	42.0				
5.25	19.0	8.0	55.0				

* For this position and those which follow, plate *B* was used.

In the fourth column of the same table are given values of the percentage transmission in the case of a plate 1 mm. thick.

Quartz.

Plate *A*, parallel to the optic axis; thickness = 3.02 mm.
Plate *B*, perpendicular to the optic axis; thickness = 3.91 mm.

The results obtained in the case of quartz are shown in Fig. 2, Plate I. It will be seen that the curves for the ordinary and extraordinary ray exhibit no striking differences in the region covered by these experiments. Yet the two curves are not identical, and the variations shown in the figure are in most cases greater than could be accounted for by errors of observation.

TABLE II.

QUARTZ.

Ordinary Ray.				Extraordinary Ray.			
Wave-length.	Observed percentage transmission.	*p*	Absorption for plate 1 mm. thick.	Wave-length.	Observed percentage transmission.	*p*	Absorption for plate 1 mm. thick.
0.80 μ	91.2 %	0.0	0.0 %	0.80 μ	94.0 %	0.0	0.0 %
1.26	91.4	0.0	0.0	1.34	92.8	0.0	0.0
1.51	91.8	0.0	0.0	1.55	92.8	0.0	0.0
1.80	91.1	0.0	0.0	1.81	91.5	0.0	0.0
2.05	91.2	0 0	0.0	2.22	91.0	0.0	0.0
2.50	91.7	0.0	0.0	2.50	89.8	0.0	0 0
2.67	91.6	0.0	0.0	2.78	88.5	0.0	0.0
2.72	84.0*	0.198	1.6	2.89	87.8	0.11	1.0
2.83	75.5*	0.474	4.3	3.00	82.3	0.33	3.2
2.95	72.5*	0 573	5.4	3.08	84.0	0.26	2.4
3.07	80.0*	0.315	3.0	3.26	87.9	0.11	1.0
3.17	84.0*	0.198	1.6	3.43	77.8	0.51	4.8
3.38	85.6*	0.153	1.3	3.52	71.9	0.76	7.3
3.67	55.5*	1.26	11.5	3.59	51.3	1.88	16.8
3.82	48.5*	1.61	14.6	3.64	52.2	1.83	16.5
3.96	41.0*	2.04	18.3	3.73	56.0	1.62	14.6
4.12	24.0*	3.41	29.0	3.91	48.6	2.22	10.2
4.50	10.0	7.30	51.5	4.19	33.6	3.35	28.3
				4.36	7.9	8.02	55.0

* Observations marked with a star were made with plate B cut perpendicular to the axis.

At λ = 2.9 the variation is especially noticeable. This region was studied with especial care, as many as twenty observations being frequently made in locating a single point. The absorption band in curve *O*, while not sharp, is still perfectly definite, and does not occur in the curve for the extraordinary ray. In the latter curve some trace of selective absorption is shown, however, at $\lambda = 3.0$. It seems not improbable that experiments with a thicker plate might bring out these bands more prominently.

The results obtained with plate *B* are shown in Fig. 3 and are in good agreement with curve *O*, Fig. 2. These observations give a wave-length of 2.95 μ for the absorption band instead of 2.90 μ. On account of the breadth and indistinctness of the band the data appear insufficient to determine which result is correct.

Values of the coefficient of extinction have been computed as in the case of Iceland spar, and are given in Table II.

Turmalin.

Plate *A*, parallel to the optic axis; thickness = 2.22 mm.

The plate of turmalin used was an unusually fine specimen of the green Brazilian variety, quite transparent and free from flaws. The suppression of the ordinary ray was practically complete

Table III.

TURMALIN.

Ordinary Ray.				Extraordinary Ray.			
Wave-length.	**Observed percentage transmission.**	p	**Absorption in plate 1 mm. thick.**	**Wave-length.**	**Observed percentage transmission.**	p	**Absorption in plate 1 mm. thick.**
1.17 μ	0.0 %	∞	100.0 %	0.759 μ	11.5 %	9.20	60 %
1.34	3.5	14.6	77.0	0.885	23.0	6.17	46
1.54	13.5	8.6	57.0	1.00	14.5	8.30	56
1.73	34.0	4.45	36.0	1.25	6.0	12.1	70
1.94	52.5	2.43	21.0	1.50	23.0	6.17	46
2.09	44.0	3.24	27.5	1.76	59.0	1.91	17
2.30	27.5	5.30	41.0	1.89	66.0	1.40	13
2.40	26.0	5.50	42.0	2.04	62.0	1.68	15
2.50	29.0	5.08	40.0	2.23	39.0	3.78	31
2.76	9.0	10.3	63.0	2.40	12.0	9.1	60
2.83	7.0	11.4	67.0	2.76	2.0	16.6	81
2.88	15.0	8.05	55.0	3.04	6.0	12.1	70
3.20	26.0	5.50	42.0	3.32	18.0	7.40	52
3.53	12.5	8.97	59.0	3.94	15.0	8.05	55
3.93	0.5	20.1	86.0				

throughout the visible spectrum. As will be seen in Fig. 4 (curve *O*) it is not until a wave-length of 1.34μ is reached that this ray begins to be transmitted in measurable amount. One absorption band occurs in the case of the ordinary ray at $\lambda = 2.82\,\mu$, but with this exception both curves are smooth. The band indicated in curve *E* at $\lambda = 1.68$ depends upon a single observation, so that its existence cannot be looked upon as definitely proven. The check afforded by the transmission curve for unpolarized light (curve *A*) is at most points quite satisfactory.

By far the most interesting point in connection with the results obtained for turmalin is the fact that the two transmission curves intersect. There are, in fact, two intersections, one at $\lambda = 2.30$ and the other at $\lambda = 3.48$. In the region lying between these points the dichroism is reversed: *i.e.*, the ordinary ray is transmitted, while the extraordinary ray is nearly suppressed.

Values of the extinction coefficient for turmalin are given in Table III.

SUMMARY OF RESULTS.

The chief results of the foregoing investigation may be summarized as follows: —

1. Observations extending to $\lambda = 4.5\,\mu$ in the case of quartz and turmalin, and to $\lambda = 5.5\,\mu$ in the case of Iceland spar, show that the absorption of polarized rays is greatly influenced by the direction of the plane of polarization in all three crystals. The transmission curves for the ordinary and extraordinary rays appear, in fact, to be entirely independent of one another. The difference between the two curves is most marked in the case of Iceland spar and turmalin.

2. Absorption bands were found in the infra-red transmission spectra as follows: —

Iceland spar. — Ordinary ray: $\lambda = 2.44\mu$ and $\lambda = 2.74\mu$. These bands are quite sharp. Broad bands are shown at $\lambda = 3.3\,\mu$, $4.0\,\mu$, and $4.6\,\mu$.

Extraordinary ray: $\lambda = 3.28\,\mu$, $3.75\,\mu$, and $4.66\,\mu$.

Quartz. — Ordinary ray: $\lambda = 2.90\,\mu$.

Extraordinary ray: $\lambda = 3.00\,\mu$. Neither band is very sharp.

Turmalin. — Ordinary ray: $\lambda = 2.82\,\mu$.

3. In the case of turmalin the transmission curves intersect at two points: viz., at $\lambda = 2.30\,\mu$ and $\lambda = 3.48\,\mu$.

4. Determinations of percentage transmission in the infra-red are shown to be liable to error on account of a slight scattering of the rays on passing through the prism. An impure spectrum results from the presence of these diffuse rays, which may lead to serious error at points beyond $\lambda = 2.5\,\mu$. Such errors may be avoided without great loss of sensitiveness by using two spectrometers, one of which throws its spectrum upon the slit of the other.

RESONANCE IN TRANSFORMER CIRCUITS.

By Frederick Bedell and Albert C. Crehore.

THE reactive action of a condenser in an alternating-current circuit produces effects of theoretical interest and practical importance. The resultant observed phenomena under different conditions of operation are many and various; but under whatever particular sets of conditions the condenser is used, the effects produced are closely correlated, and may be grouped together in one class. Electrical resonance is one of the striking phenomena of this class. In a single circuit, resonance is obtained when, inductance and capacity being present, the resultant reactance is zero; that is, the reactance of the condenser is exactly equal and opposite to the reactance due to self-induction. The natural period of the circuit then equals the period of the impressed electromotive force. Complete resonance is thus obtained only when the impressed electromotive force varies harmonically.

Effects corresponding to resonance in a single circuit may be produced by the reactive influence of one circuit upon another, occasioned by the mutual induction of the two circuits. Such effects have been discussed by Dr. M. I. Pupin,[1] who treats the case of a transformer containing a condenser in the secondary circuit; to this class of phenomena he gives the name "electrical consonance." Additional discussion of the same subject has been given by Mr. C. P. Steinmetz.[2]

Resonance, in this case, occurs when the relation between the primary and secondary is such that the apparent reactance of the primary circuit is zero. The elastic influence of the condenser is transferred from one circuit to the other, on account of their mutual relationship, and the natural period of the primary circuit depends not only upon the value of its own constants, but upon those of the secondary as well. There is a surging of energy back and forth between the primary circuit and the secondary condenser, by intervention of their common magnetic field: the period of these surgings determines the period of the system. A condenser in either primary or secondary circuit may give rise to such oscillations.

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[1] Electrical Consonance: M. I. Pupin, Electrical World, Feb. 9, 1895.

[2] Electrical World, March 2, 1895.

In the general sense of resonance, the case here described would be a particular case, but for distinction we may apply the term "simple resonance" to the phenomena observed in a single circuit, the term "consonance," suggested by Dr. Pupin, being used when the resonant action is transferred from one circuit to another by the mutually inductive influence between them.

The conditions necessary for consonance, although not so simple as those for simple resonance, are definite, and may be readily determined. These are in part discussed in the papers already referred to. We have had occasion to investigate the action of condensers with transformers, and to develop their general theory, interpreting the results both analytically and graphically. The application of this general treatment to the case of resonance or consonance throws light upon the nature of the phenomena, and brings out certain important relations. With these results many of the conclusions of Dr. Pupin agree, but not so some of his deductions, — particularly the statement of the conditions under which consonance occurs; namely, when the apparent impedance of the primary circuit is either a maximum or a minimum. To make our discussion complete, we may be pardoned, therefore, if we touch upon some ground in part covered before.

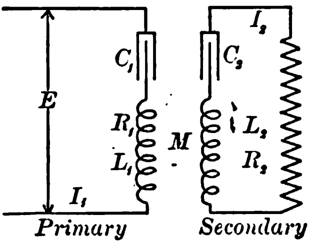

Fig. 1.

Arrangements of Circuits.

Problems of this nature are usually capable of two essentially different modes of treatment, leading to identical results: first, the general analytical treatment; second, the synthetical graphical treatment. Which to use is a matter of preference. It is a choice of which way to go around a block; one way or the other may be shorter, but the same point is reached by either route. These methods are well illustrated in the present instance. In the study of resonance and consonance the graphical method is the more direct. It is replete with obvious conclusions, and at once brings out relations not simply shown by analytics.

After a brief analytical treatment we will pass to the graphical.

Analytical Treatment.

Let us use the following notation: —

I_1 = primary current.
R_1 = resistance of the primary circuit.
L_1 = self-induction of the primary circuit.
C_1 = capacity of the primary circuit.
K_1 = reactance of the primary circuit.
J_1 = impedance of the primary circuit.

Let I_2, R_2, L_2, C_2, K_2, and J_2 represent the corresponding quantities for the secondary circuit, within and without the transformer.

E = harmonic electromotive force supplied to the primary circuit.
ϕ = angle of phase difference between primary electromotive force and current.
M = coefficient of mutual induction between the primary and secondary circuits.
$\omega = 2\pi \times$ frequency.

Let R_1', L_1', K_1', and J_1' represent the apparent values of the quantities indicated.

It is to be noted that

$$\text{reactance} \equiv K = \omega L - \frac{1}{C\omega};$$

$$\text{impedance} \equiv J = \sqrt{R^2 + K^2};$$

$$\text{apparent impedance} \equiv J_1' = \sqrt{R_1'^2 + K_1'^2}.$$

For the case in which the secondary alone contains a condenser,

$$\frac{1}{C_1\omega}=0, \text{ and } K_1=L_1\omega.$$

For the general case,[1] in which we suppose that condensers with capacities C_1 and C_2 are located in the primary and secondary circuit respectively, as shown in Fig. 1, we have the solution for the primary current

$$i_1=I_1 \sin(\omega t+\phi),$$

where

$$I_1=\frac{EJ_2}{\sqrt{a^2+b^2}},$$

and

$$\phi=-\text{arc tan}\,\frac{b}{a}+\text{arc tan}\,\frac{K_2}{R_2},$$

$$\tan\phi=\frac{-\frac{b}{a}+\frac{K_2}{R_2}}{1+\frac{b}{a}\cdot\frac{K_2}{R_2}}=-\frac{bR_2-aK_2}{aR_2+bK_2}.$$

Here a and b are abbreviations which, in this general case, have the values

$$a=-\omega^2(L_1L_2-M^2)+\left(\frac{L_1}{C_2}+\frac{L_2}{C_1}+R_1R_2\right)-\frac{1}{C_1C_2\omega^2},$$

$$b=\omega(R_1L_2+R_2L_1)-\left(\frac{R_1}{C_2\omega}+\frac{R_2}{C_1\omega}\right);$$

or

$$a=R_1R_2-K_1K_2+M^2\omega^2,$$

$$b=R_1K_2+R_2K_1.$$

Substituting these values of a and b, we obtain

$$\tan\phi=-\frac{R_2^2K_1+K_1K_2^2-M^2\omega^2K_2}{R_1R_2^2+R_1K_2^2+M^2\omega^2R_2}.$$

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[1] Discussed at length by the writers in a paper "On Mutual Induction and Capacity," page 100, Proceedings of the International Electrical Congress, Chicago, 1893; also, PHYSICAL REVIEW, Vol. I., page 193. The letters a and b in the present paper correspond to α and β in the paper referred to, with $-\omega^2$ and ω^2 factored out. The conditions are there shown under which oscillations in one circuit may occur, on account of a condenser placed in the other. The period of the oscillations in a primary circuit is given for the case in which the secondary alone contains a condenser.

By rearrangement we obtain our results in the following useful form,

$$\tan\phi = -\frac{K_1-\gamma^2K_2}{R_1+\gamma^2R_2} = -\frac{K_1'}{R_1'},$$

and

$$I_1 = \frac{E}{\sqrt{(R_1+\gamma^2R_2)^2+(K_1-\gamma^2K_2)^2}} = \frac{E}{J_1'},$$

where

$$\gamma = \frac{M\omega}{J_2} = \frac{I_2}{I_1}.$$

We thus have for the primary circuit: —

apparent resistance $= R_1' = R_1+\gamma^2R_2$;
apparent reactance $= K_1' = K_1-\gamma^2K_2$;
apparent impedance $= J_1' = \sqrt{R_1'^2+K_1'^2}$.

The apparent primary reactance equals the real reactance diminished by a multiple of the secondary reactance. The apparent primary resistance equals the real resistance increased by the same multiple of the secondary resistance. This multiple is the square of the ratio of secondary and primary currents.

For completeness we may add

$$i_2 = I_2 \sin(\omega t - \phi - 90^\circ - \theta_2),$$

where

$$I_2 = \frac{M\omega I_1}{J_2},$$

and

$$\tan\theta_2 = \frac{K_2}{R_2}.$$

These results are independent of any assumption concerning magnetic leakage; they are rigorously correct when the capacities and coefficients of induction are constant, and the impressed electromotive force is harmonic. Where the electromotive force is not harmonic, the solution may be applied to each component of the complex wave; or, we may make use of the idea of an equivalent sine wave, which, although useful and in many cases capable of giving approximately correct results, is not rigorous for this case.

The conditions for consonance are readily obtained by noting the conditions under which the apparent reactance of the primary is zero; these will be discussed after the graphical treatment of the problem, from which these results analytically obtained are independently derived from geometrical considerations.

Graphical Treatment.

In the graphical treatment of resonant phenomena, or of any alternating-current problem, there are four correlated methods of representation. We may plot *electromotive forces*, as in Fig. 2; *impedances*, as in Fig. 3, obtained by dividing our electromotive force diagram by current; *admittances*, as in Fig. 4, the reciprocal of our impedance diagram; and *currents*, as in Fig. 5, obtained by multiplying the admittance diagram by an electromotive force. Figures 4 and 5 are reciprocal to Figs. 2 and 3. There is a close relation between all of these diagrams. We will build up Fig. 2, step by step, and discuss it in detail; the full discussion of the other diagrams will scarcely be called for. Our particular study is to determine the action of the condenser in the secondary circuit.

In Fig. 2 let $\overline{\mathrm{OA}} = I_1$ represent the primary current, which for the present is assumed constant; $\overline{\mathrm{OB}} = M\omega I_1$, drawn ninety degrees behind $\overline{\mathrm{OA}}$, represents the electromotive force induced in the secondary by the primary current, the instantaneous value of this electromotive force being $M\frac{di_1}{dt}$. Positive direction of rotation is taken as counter-clockwise.

The secondary current, $\overline{\mathrm{OC}}$, lags behind $\overline{\mathrm{OB}}$ by an angle θ_2, whose tangent is $K_2 \div R_2$. The primary impressed electromotive force, E, is the sum of three components: $\overline{\mathrm{OH}} = R_1 I_1$, to overcome primary resistance; $\overline{\mathrm{HT}} = K_1 I_1$, to overcome the reactance of the primary circuit; and $\overline{\mathrm{TF}} = M\omega I_2$, to overcome the back electromotive force induced in the primary by the secondary current. These components are in the direction of the primary current, and at right angles to the primary and secondary currents respectively. When iron is present, R_1 is strictly a power coefficient, hysteretic and ohmic; $\overline{\mathrm{OH}}$ is the power electromotive force for all expenditure of power excepting in the secondary. Eddy currents and hysteresis are thus taken into consideration in so far as they occasion an expenditure of energy. Hysteresis would likewise cause a lagging of magnetization behind the magnetizing current, on account of which $\overline{OB}$ would be a little more than ninety degrees behind $\overline{OA}$.

The vector sum of the active electromotive force, $\overline{OH}$, and the reactive electromotive force, $\overline{HT}$, determines $\overline{OT}$, the electromotive force to overcome the impedance proper of the primary. The impressed electromotive force, $\overline{OF}$, is the sum of $\overline{OT}$, and the back electromotive force, $\overline{TF}$.

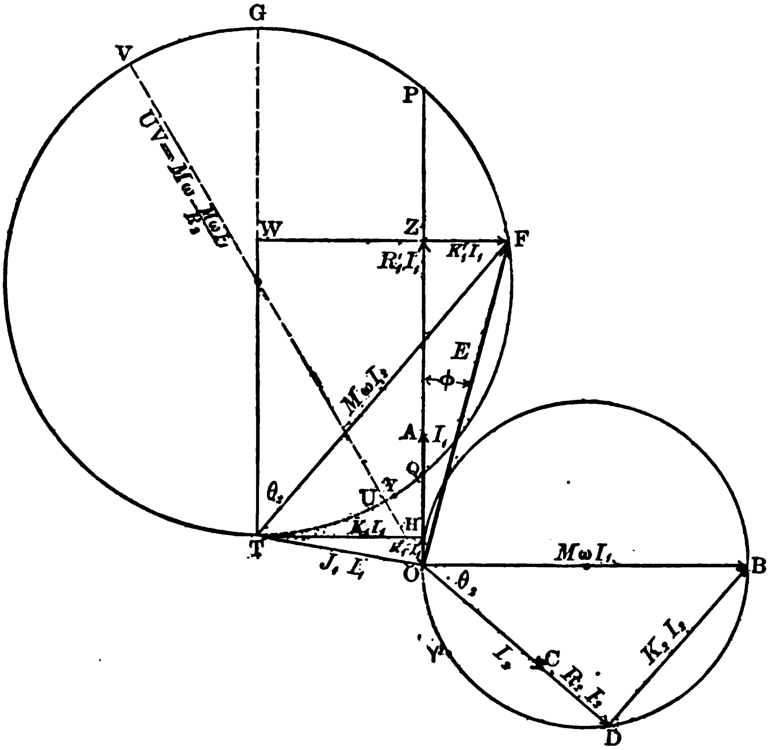

Fig. 2.

Electromotive Force Diagram: Positive Rotation is Counter-clockwise.

Under the assumption that I_1 is constant, evidently $\overline{OB}$ is constant. As the secondary capacity or reactance is varied, the point D moves on the circle ODB, and the point F moves similarly on the circle TQFPG. This determines the phase of the primary electromotive force $\overline{OF}$, which may be either in advance of the current or behind it, being in phase with it at two points P and Q. In the case represented in the figure, the electromotive force is behind the current.

The line $\overline{\mathrm{TF}}$, showing the back electromotive force due to the secondary current, may be resolved into

the active electromotive force $\overline{\mathrm{TW}}=\overline{\mathrm{TF}}\cos\theta_2=\left(\frac{M\omega}{J_2}\right)^2 R_2 I_1$;

the reactive electromotive force $\overline{\mathrm{WF}}=\mathrm{TF}\sin\theta_2=\left(\frac{M\omega}{J_2}\right)^2 K_2 I_1$.

These are the components of the back electromotive force. Adding these to $\overline{\mathrm{OH}}$ and to $\overline{\mathrm{HT}}$ respectively, we obtain the total values of the active and reactive electromotive force in the primary; viz.:

$$\overline{\mathrm{OZ}}=R_1 I_1+\gamma^2 R_2 I_1=R_1' I_1;$$

$$\overline{\mathrm{ZF}}=K_1 I_1-\gamma^2 K_2 I_1=K_1' I_1.$$

The diameter of this primary circle is the value of $\overline{\mathrm{TF}}$, when in the position $\overline{\mathrm{TG}}$, which occurs when $K_2=0$. The secondary current is then in the direction of $\overline{\mathrm{OB}}$, and we have $I_2=M\omega I_1\div R_2$. The diameter of the primary circle is then

$$M\omega I_2=M\omega\frac{M\omega I_1}{R_2}.$$

A primary condenser changes the length of $\overline{\mathrm{HT}}$, the point T moving toward H, as the condenser reactance is increased; in fact, it may be moved to the right of H. For a case in which the primary reactance is due to inductance alone, the diagram as drawn applies.

With the secondary condenser removed, the point F would move to some point as Y, and the point D to a corresponding point Y′, depending upon the resistance in the secondary circuit, and upon the amount of magnetic leakage, a small amount of which is here assumed present. The limiting positions, therefore, of the point F are Y and T. As $\overline{\mathrm{TY}}$ would be the position of $\overline{\mathrm{TF}}$ in the absence of the secondary condenser, it follows that a line drawn from Y to F in any position shows the component of the primary electromotive force due to the secondary condenser. The point F could not lie on the dotted portion of the curve.

If we divide the electromotive force diagram, Fig. 2, by the value of the primary current for which it was constructed, we obtain (Fig. 3) an impedance diagram, independent of current or electromotive force. Here $\overline{\mathrm{OH}}$ represents ohmic resistance [or a power coefficient where other losses are involved]; $\overline{\mathrm{HT}}$ is the primary reactance; $\overline{\mathrm{OT}}$ is the primary impedance. The apparent primary impedance $\overline{\mathrm{OF}}$, is the sum of the impedance $\overline{\mathrm{OT}}$ and of $\overline{\mathrm{TF}}$, an impedance component due to the reaction of the secondary; of this, $\overline{\mathrm{YF}}$ is due to the presence of the secondary condenser.

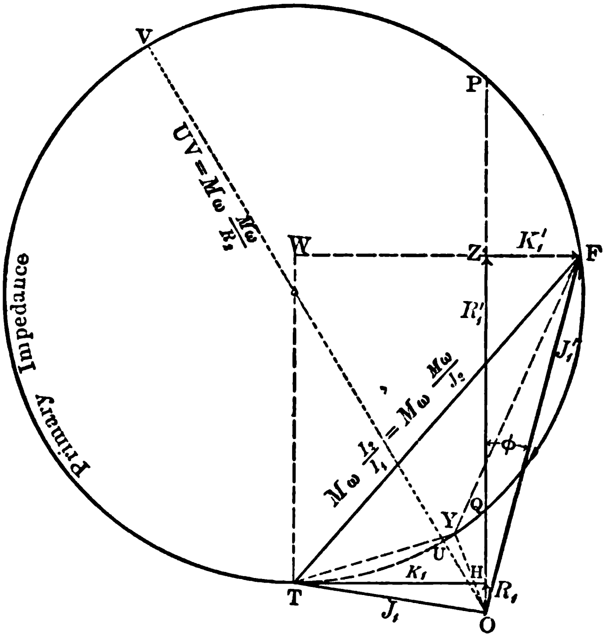
V
P
$UV = M\omega \frac{M\omega}{R_2}$
W
Z
K_1'
F
R_1'
J_1'
$M\omega \frac{I_2}{I_1} = M\omega \frac{M\omega}{J_2}$
ϕ
Primary Impedance
Q
Y
U
K_1
H
R_1
T
J_1
O

Fig. 3.

Impedance Diagram (obtained by dividing Electromotive Force Diagram, Fig. 2, by I_1).

From Fig. 3 we have

$$\overline{\mathrm{OZ}} = R_1' = R_1 + \gamma^2 R_2;$$

$$\overline{\mathrm{ZF}} = K_1' = K_1 - \gamma^2 K_2;$$

$$\overline{\mathrm{OF}} = J_1' = \sqrt{R_1'^2 + K_1'^2}.$$

From Fig. 2 we also have

$$E=I_1\sqrt{R_1'^2+{}_1'K^2}=I_1\sqrt{(R_1+\gamma^2R_2)^2+(K_1-\gamma^2K_2)^2}.$$

We have thus obtained by geometrical methods the results derived above by analytical processes. In Fig. 3, for the maximum and minimum values of the apparent impedance, we have $\overline{\text{OV}}$ and $\overline{\text{OU}}$; for consonance we have the values $\overline{\text{OQ}}$ and $\overline{\text{OP}}$. We may note that, while for consonance the secondary condenser decreases the apparent reactance of the primary to zero, it gives rise to a corresponding increase in the apparent resistance.

If, in Fig. 3, $\overline{\text{OF}}$ is a vector representing the apparent impedance of the primary, we may obtain the value of the admittance by taking the reciprocal of $\overline{\text{OF}}$. Now $\overline{\text{OF}}$ is a vector drawn from the origin O, and lies on the circle TQFPG as the secondary reactance is changed. By the principle of reciprocal vectors it follows that, if the locus of the impedance vector is a circle, the locus of the admittance vector (its reciprocal) will likewise be a circle. In Fig. 4, $\overline{of}$ is a vector representing admittance, being always equal to the reciprocal of $\overline{\text{OF}}$, Fig. 3. The circle *vpfqu*, in Fig. 4, is reciprocal to the circle VPFQU in Fig. 3. We may note the reciprocal points *u*, *v*, *p*, *q*, and U, V, P, Q. The points of minimum and maximum impedance U and V correspond to the points of maximum and minimum admittance *u* and *v*. The admittance diagram is evidently independent of current or electromotive force.

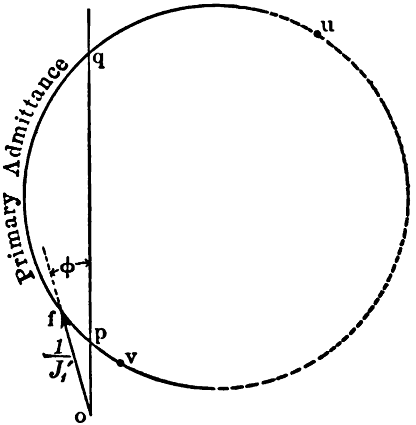

Fig. 4.
Admittance Diagram (the Reciprocal of the Impedance Diagram).

Let us now assume an alternating electromotive force *E*, constant in value, supplied to the primary circuit. Multiplying our admittance diagram [which for simplicity is not complete] by *E*, we obtain the current diagram, Fig. 5. Here $\overline{oh}$ is the power current in the direction of the electromotive force; $\overline{ht}$ is the reactive current, and represents no work. The resultant is $\overline{ot}$, the open-circuit current, which is denoted by I_0. It is to be observed that $\overline{oh}$ is the current representing the power expended in all ways other than in the secondary circuit. The actual current I_1, which flows in the primary when the secondary is closed, is the resultant of the open-circuit current $\overline{ot}$ and the component $\overline{tf}$, due to the reaction of the secondary, which is equal to the secondary current multiplied by the ratio of secondary to primary turns $(S_2 \div S_1)$.

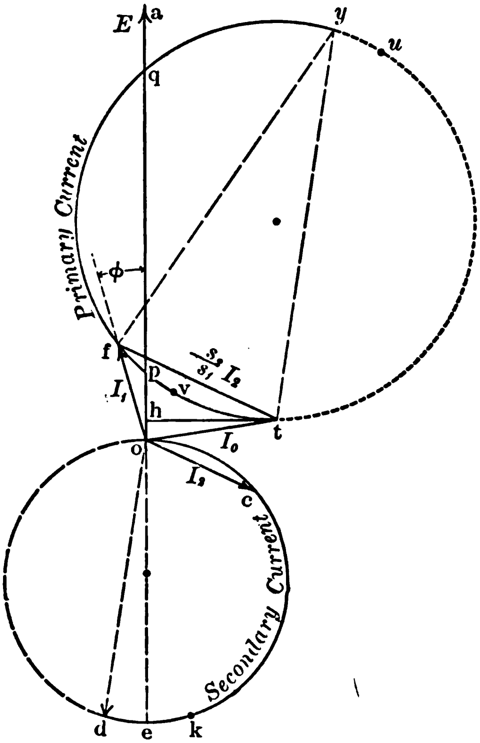

Fig. 5.
Current Diagram (Admittance Diagram multiplied by Primary Electromotive Force, *E*).

As the secondary reactance is varied, the primary current changes in magnitude and direction according to the reaction of the secondary, and lies upon the circle, as shown, as a locus. The dotted portion of the circle corresponds to the dotted portion in the previous figures. This diagram corresponds to that of Mr. Steinmetz.

The points of consonance, p and q, correspond to P and Q above. If consonance can be obtained at all, there are two values of the primary current for which it may be obtained.

The secondary current, I_2, varies proportionally to $\overline{tf}$, and is drawn as $\overline{oc}$, with its locus on the circle c, k, e, d. In the present case (which assumes a certain amount of magnetic leakage), the secondary current would take the position $\overline{od}$, if the secondary condenser were removed from the circuit. It could only move on to the dotted part of the curve in case self-induction were added to the secondary circuit, — or, what is the same thing, the magnetic leak increased. We thus see the action of the condenser and of magnetic leak are opposed, and that magnetic leak may be thus compensated for. In fact, the secondary circuit may be given a capacity which will overcompensate for magnetic leakage; as a constant potential transformer is loaded the secondary electromotive may rise, despite the magnetic-leakage drop and the fall in potential due to ohmic resistance. In the absence of condenser, magnetic leak and open-circuit losses (that is, if $\overline{oh}$ were zero), the secondary current would be in the direction $\overline{oe}$. Losses on open-circuit, eddy currents, hysteresis, etc., give to the open-circuit current the power component $\overline{oh}$, and in the absence of magnetic leak the secondary current would assume the position $\overline{ok}$.

In the present instance the point of minimum impedance U, and maximum current *u*, are on the dotted portion of their respective circles, and so could not be actually obtained. This, however, is not necessarily so. By the principle of reciprocal points, it follows that the product of the maximum and minimum currents is equal to the product of the two consonant values of current; that is,

$$\overline{ou} \times \overline{ov} = \overline{op} \times \overline{oq}.$$

We may best examine the conditions of consonance by returning to Fig. 2. When the primary circle does not intersect $\overline{\mathrm{OP}}$, there can be no consonance; analytically we would obtain imaginary values. If there is one condition for consonance, there must be two, corresponding to the points P and Q, except in the limiting case when the circle is tangent to $\overline{\mathrm{OP}}$. Then $\overline{\mathrm{HT}}$ would equal the radius of the circle, and

$$K_1 I_1 = \frac{M\omega}{2} \; \frac{M\omega I_1}{R_2}$$

For consonance to be possible, we must therefore have the relation

$$M^2\omega^2 > 2 K_1 R_2.$$

The points of consonance are determined by equating the value of primary apparent reactance to zero; thus

$$K_1' = K_1 - \gamma^2 K_2 = 0;$$

or

$$\frac{K_1}{K_2} = \frac{M^2\omega^2}{R_2{}^2 + {}_2K^2}.$$

Two values of K_2 will satisfy this condition. Frequency is an important factor in this criterion. The conditions for consonance are obtained when $\overline{\text{OD}}$ is perpendicular to $\overline{\text{TQ}}$ or to $\overline{\text{TP}}$.

A particular case, which may be termed "pure consonance," is obtained when the natural period of each circuit independently is equal to the period of the impressed electromotive force. The period of the system in this case is evidently the same as the period of either circuit. We have K_1 and K_2 each equal to zero. The point T coincides with H, and $\overline{\text{OD}}$ takes the position $\overline{\text{OB}}$.

We have seen that all our conclusions concerning consonance may be obtained independently, by analytical or by graphical methods; the results by the two methods being easily identified. The graphical method, however, appeals to us as the more evident.

Although it is readily shown that the loci of primary impedance and admittance are circles, as the secondary reactance is changed, the conditions are not easily obtainable for a wide range of variation. The diagrams here drawn are for an assumed case, and the secondary capacity is varied from zero to infinity. Ordinarily, the range of variation would be small, as Dr. Pupin has pointed out, and as has been the result of our own experiments. It is to be hoped that Dr. Pupin, who has experimented so faithfully on simple resonance, will bring out equally valuable results from his experiments in electrical consonance.

In addition to the effect of consonance, the action of a condenser to counterbalance leakage has been pointed out above. Although scarcely germane to the present discussion, we may call attention to another effect which may be produced by means of a properly proportioned condenser in the secondary circuit. If the secondary reactance be made zero (inside and out of the transformer), simple secondary resonance is obtained, and a constant primary current will transform to a constant secondary electromotive force.

ON THE SECULAR MOTION OF A FREE MAGNETIC NEEDLE.[1]

By L. A. Bauer.

A MAGNETIC needle, suspended so as to move freely in all directions, will set itself tangent to the lines of terrestrial magnetic force. At any particular time it will assume a definite direction. It will make a definite angle with the true meridian, which, measured in the horizontal plane, is known as its *declination* (D); also a definite angle with this plane, which, measured in the vertical plane passing through the magnetic meridian, is termed its *inclination* (I) or dip. The *intensity* of terrestrial magnetic force is usually determined from a measurement of the horizontal component (H), so that for any particular time D, I, and H will have definite values. Now, as is well known, the direction and intensity of the geomagnetic lines of force are in a continual state of flux. About any assumed mean position of equilibrium a variety of small periodic variations (daily, seasonal, annual, 11⅛ year, etc.) take place, accompanied at times by fitful or irregular ones, which occasionally become really considerable. These changes are usually small. In consequence of the so-called *secular variation*, however, geomagnetism suffers in the course of time most remarkable changes.

The fact of a secular variation has now been known for two and a half centuries.[2] It has been the cause of no end of fruitless speculations. It has engaged some of the best minds and given rise to most ingenious theories, but the riddle is still unsolved.

One of the greatest of geomagneticians has truly said: "Viewed in itself and its various relations, the magnetism of the earth cannot be counted less than one of the most important branches of the physical history of the planet we inhabit." It is indeed being recognized more and more that gravitation is not the only bond that binds us in friendly union with our sister planets and our parent sun. The astronomer has encountered phenomena partly of electric nature, some of which, making themselves felt in geophysical changes, have defied all his present methods of research. His gravitation hypothesis fails him here. His only hope of explanation appears to lie in geophysical investigations; and among the great geophysical enigmas, whose unraveling promises to unlock many another of nature's secrets, this question of the cause of the secular variation of terrestrial magnetism plays a prominent part. It is for that reason that we find that so many brilliant minds have been engaged in attempting to discover the cause. With but few exceptions, however, it has seemed to escape all theorists that the problem does not consist in determining *what* will cause a secular variation, but rather in determining *which* of all the causes that can be assigned is *the* one. Qualitatively, causes innumerable and plausible can be given; quantitatively — "there lies the rub." It is not then that we are embarrassed by a paucity, but rather by a multiplicity, of causes. The present attitude of an investigator of the secular variation must then be rather that of a judge between the various causes that have already been assigned. With this in view the writer has thus far studiously avoided the formulation of any theory of his own, so that an unbiased and impartial investigation of this perplexing phenomenon could be undertaken, and facts that might serve as criteria between the various causes be critically established.

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[1] A paper read before the National Academy of Science, Washington, April 16, 1895.

[2] Discovered in 1634 by Gellibrand, from the fact that the declination at London had changed from 11¼ E. in 1580 to 4° 4′ E. in 1634.

The general problem of the secular variation of geomagnetism, *i.e.* the determination of the change in direction and in *intensity* of the geomagnetic lines of force, cannot as yet be undertaken with success, since we possess reliable intensity observations for hardly more than half a century. The investigation of the change in *direction*, however, on account of the accumulation of declination and inclination data for over three centuries, can be attempted, since for this purpose a knowledge of only declination and inclination, not of force, is required. We will then, in the present communication, formulate our first problem as follows: *What is the secular motion of the geomagnetic lines of force?* or more concretely, since, as above stated, a free magnetic needle sets itself ever tangent to the lines of force: *What is the secular motion of a free magnetic needle?* or still better: *What curve in space does the north end of a free magnetic needle describe in the course of centuries?*

The usual method of procedure in discussing the secular variation of the direction of geomagnetic force has been to discuss *separately* the secular variation of the different magnetic elements, declination or inclination, as though they were *different* effects of forces acting instead of *component* effects. It is due to this fact chiefly that all attempts at a discovery of a single law of the secular variation applying to the whole earth have failed.

Among those who have been engaged with a determination of the secular variation curve, chief mention must be made of A. T. Kupfer, E. Quetelet, R. Wolf, C. A. Schott, and J. Liznar. Kupfer drew the curve for a few months for St. Petersburg and Peking; Quetelet, the Brussels curve 1828–1878; Wolf, the London curve 1580–1876; and Schott, a mean New England curve 1820–1885. Quetelet and Wolf made the arbitrary assumption that the cone described by the free magnetic needle was a circular one, and upon this hypothesis determined the curves by least square adjustment of the observations. Liznar (1891) appears to have established first the general formulæ, free from all assumptions as to the geometric nature of the curve. He, however, made no use of them, as his purpose was to investigate the *daily* and *annual* curves, for the construction of which only very rough formulæ are needed. Before Liznar's paper had fallen into the writer's hands, other and simpler formulæ had been established by the latter, and the secular variation curves for London and Paris constructed therewith. These curves were exhibited at the meeting of the American Association for the Advancement of Science in August, 1892.[1] The following curves were roughly constructed and shown at the same time, viz.: Rome, Berlin, St. Petersburg, Cape Comorin (India), Tokyo, Pelropawlowsk, San Francisco, Mexico,

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[1] See "Abstract" in Science, Vol. 20., No. 506, p. 218.

Washington, New York, Cambridge (Mass.), Azores Is., Ascension I., St. Helena I. and Cape Town. From an inspection of these curves a surprising and beautiful law revealed itself with regard to the *direction* of the secular motion, viz.: *that at all the stations both in the northern and the southern hemispheres, the north end of a free magnetic needle, as beheld by an observer standing at the center of suspension of the needle, moves in the direction of the hands of a watch.* Up to that time this simple law had not been known. As this fact cannot be established *a priori*, much less the actual direction and curve be given for various parts of the earth by theory, it was deemed highly essential that the investigation of the secular curves be continued and amplified. There was another fact which disclosed itself at that time which made it very desirable that the investigation be continued; namely, that the secular variation curves seemed to develop themselves more and more by going around the earth eastwardly, *i.e.* it appeared as though the secular variation curve might be constructed by getting a composite of all the parts of curves described at stations lying somewhere near the same parallel of latitude. If this were true, it would imply that the curve described by the north end of a free magnetic needle in making an instantaneous circuit of the earth in an easterly direction along a parallel of latitude, should bear a resemblance to the secular variation curve. Other preliminary conclusions with regard to the period could also be announced.

Since the presentation of the above paper, the writer has had opportunity to continue and extend his researches, and carefully test his preliminary conclusions. The present result of his labors has been laid down on the two charts appended,[1] which have been taken from a recent publication.[2] In the preparation of the latter, the writer enjoyed the use of the Washington and Berlin libraries, and the encouragement of Professors Neumayer, von Bezold, and Planck. For the warm interest shown by these gentlemen, the writer tenders his heartfelt thanks.

In the following, the necessary steps for drawing the secular variation curve will be outlined, and the main results be briefly given. For more explicit information reference must be made to the publication cited.

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[1] The charts referred to will accompany Part II. of the present article.

[2] L. A. Bauer: Beitraege zur Kenntniss des Wesens der Saecular Variation des Erdmagnetismus. Berlin, 1895. Mayer und Müller.

First, all observations of declination and inclination made at any one station must be carefully collected and critically discussed. The collection of the material, owing to its being scattered throughout many publications, and owing to the unsatisfactory state of the literature on geomagnetism, is by far the greatest and most unenjoyable task in the construction of the curve. To illustrate: In order to gather the London material, the writer went through all the volumes of the Philosophical Transactions and Proceedings of the Royal Society, and gleaned besides from other publications. This material must then be subjected to careful scrutiny. Sometimes no date is given, or but roughly so; sometimes no detailed description of station is at hand; sometimes the observation is utterly defective, due either to local disturbances or defect in instrument or method, etc., etc., so that to gather, sift, and weigh the material lying outside one's own country requires considerable time and patience. As the declination and inclination observations have not generally been made simultaneously, nor by the same observer, the next step is to construct suitable interpolation formulæ. In the absence of a knowledge of the function expressing the law of the secular variation, resort must be made to empirical formulæ, and, as is customary in such cases, a harmonic series is employed. Thus to express the secular variation of the declination, for instance, we shall employ a series of the form:—

$$D=D_0+r\sin(\beta\tau+C)+r_1\sin(\beta_1\tau+C_1)+\cdots,$$

where D stands for the declination at any time t, D_0 the mean declination about which the periodic fluctuations take place, $r, r_1, \cdots$ the semi-ranges or amplitudes of the fluctuations, $\beta, \beta_1, \cdots$ coefficients corresponding to the oscillation periods, *i.e.* $\beta=360^\circ\div\text{period}$, τ = time interval in years between some adopted epoch, say 1850, and that of observation, *i.e.* $\tau=(t-1850)$ and $C, C_1, \cdots$ the epochal constants of the several periodic terms. A similar formula is employed for the secular variation of the inclination. In general, two periodic terms are amply sufficient, and in most cases one will suffice to give the greater part of the secular variation during the time interval for which we at present possess data. The interpolation formulæ for each element are then established, according to the well-known method for the treatment of such observations.

In the tables below, an abstract of the material for London is given in a concise form, to enable one to see how well the interpolation formulæ employed represent the observations.

Δ_F is here the difference between observation and computation (*O.* – *C.*) when using Dr. Felgentraeger's formula,[1] as transformed by the writer, viz.: —

LONDON: DECLINATION.

Date.	Observer.	*D* observed.	Δ_F	Δ_B
		°	°	
1540.	van Bemmelen Chart . . .	– 8(?)	–0.8(?)	+3.0(?)
1580.79	Boroughs and Norman . .	–11.25	–0.30	–0.28
1622.45	E. Gunter	– 5.92	+0.92	+0.72
1634.45	Gellibrand	– 4.07	+0.39	+0.67
1640.	H. Bond, Sen.	– 3.11	0.00	+0.55
1657.	H. Bond, Sen.	0.00	+0.10	+0.43
1665.94	H. Bond, Sen.	+ 1.48	–0.09	+0.15
1666.44	J. Seller.	+ 1.57	–0.09	+0.15
1670.40	Millet	+ 2.10	–0.24	–0.15
1672.	E. Halley	+ 2.50	–0.19	–0.21
1680.	Hooke	+ 4.50	+0.54	+0.09
1683.	E. Halley	+ 4.50	+0.06	–0.57
1692.	E. Halley	+ 6.00	+0.16	–1.02
1698.	Cassini	+ 7.00	+0.18	–1.32
1700.	——	+ 8.0(?)	—	—
1722.	G. Graham.	+14.2(?)	—	—
1746.50	G. Graham	+17.31	+0.70	–0.42
1759.29	J. Canton	+18.97	–0.65	–0.72
1773.5	Heberden	+21.15	–0.63	–0.33
1779.02	Royal Society.	+22.36	–0.14	+0.30
1788.55	G. Gilpin	+23.48	+0.05	+0.60
1793.62	G. Gilpin	+23.82	+0.06	+0.59
1798.57	G. Gilpin	+24.03	+0.03	+0.52
1803.57	G. Gilpin	+24.13	–0.01	+0.39

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[1] Dr. W. Felgentraeger: Die laengste nachweissbare saeculare Periode der Erdmagnetischen Elemente. Teil I. Declination. Inaug. Diss., Un. Göttingen, 1892.

LONDON: DECLINATION (*continued*).

Date.	Observer.	D observed.	Δ_F	Δ_B
1808.75	G. Gilpin	+24.18	−0.01	+0.29
1814.59	G. Gilpin and Royal Society	+24.29	+0.09	+0.34
1814.66	M. Beaufoy	+24.29	+0.09	+0.34
1820.45	Royal Society.	+24.22	+0.13	+0.24
1837.50	J. C. Ross	+23.60	+0.22	+0.13
1842.50	Kew and Greenwich . . .	+23.23	+0.17	+0.05
1845.50	Greenwich Observatory . .	+23.22	+0.4	+0.36
1850.50	Greenwich Observatory . .	+22.70	+0.2	+0.14
1855.50	Greenwich Observatory . .	+22.06	+0.03	+0.05
1860.50	Kew and Greenwich . . .	+21.44	−0.06	−0.16
1865.50	Kew and Greenwich . . .	+20.76	−0.15	−0.29
1870.50	Kew and Greenwich . . .	+20.10	−0.16	−0.31
1875.50	Kew and Greenwich . . .	+19.49	−0.02	−0.25
1880.50	Kew and Greenwich . . .	+18.79	+0.11	−0.20
1885.50	Kew and Greenwich . . .	+18.22	+0.48	−0.02
1890.50	Kew and Greenwich . . .	+17.63	+0.83	+0.24

$$D = +6^\circ.964 + 16^\circ.975 \sin a - 0^\circ.374 \sin (2a + 45^\circ.11)$$
$$+ 0^\circ.603 \sin (3a + 153^\circ.76) - 0.810 \sin (4a + 43^\circ.78). \qquad (1)$$

Where west declination is +, east declination −, and

$$a = \frac{360^\circ}{477}(t - 1695.95) = 0^\circ.755\,(t - 1850) + 116^\circ.28.$$

$\Delta_B = (O. - C.)$, employing the writer's approximate formula,[1] viz.: —

$$D = +6^\circ.24 + 17^\circ.75 \sin [0^\circ.7\,(t - 1850) + 112^\circ.73]. \qquad (2)$$

A comparison of differences Δ_F and Δ_B is instructive in that an opportunity is afforded thereby to see how closely one periodic term will represent the secular variation. In the establishment of his formula, Dr. Felgentraeger made use of observations 1580–1882, so that the 1885 and 1890 computed values are extrapolations. The author obtained his formula with the aid of observations 1580–1890.

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[1] Science, Vol. 20, No. 506, p. 219.

LONDON: INCLINATION.

Date.	Observer.	I observed.	I computed.	Δ O.-C.	w
		°	°	°	
1576.	Norman	71.83	71.19	+0.64	0.5
1600.	Gilbert	72.00	72.29	−0.29	0.5
1613.	M. Ridley	72.50	72.81	−0.31	0.5
1676.	Bond	73.50	74.34	−0.84	0.5
1720.5	Whiston	74.48	74.08	+0.40	0.5
1723.29	Graham	74.70	74.03	+0.67	0.5
1746.5	Graham	73.5	73.42	+0.08	0.5
1773.0	Nairne and Heberden . .	72.32	72.44	−0.12	0.5
1777.72	Royal Society	72.43	72.25	+0.18	0.5
1795.0	Gilpin	71.30	71.47	−0.17	0.5
1821.10	Kater, Sabine, and Christie	70.32	70.20	+0.12	1.0
1829.82	Sabine, Kater, and Segelcke	69.68	69.79	−0.11	1.0
1837.42	Lloyd, Fox, and others . .	69.35	69.42	−0.07	1.0
1847.50	Greenwich Observatory . .	68.90	68.96	−0.06	1.0
1854.65	Welsh and Sabine . . .	68.51	68.64	−0.13	1.0
1860.12	Greenwich and Kew . . .	68.37	68.40	−0.03	1.0
1866.12	Greenwich and Kew . . .	68.07	68.16	−0.09	1.0
1872.12	Greenwich and Kew . . .	67.87	67.92	−0.05	1.0
1878.00	Greenwich and Kew . . .	67.70	67.71	−0.01	1.0
1884.00	Greenwich and Kew . . .	67.58	67.50	+0.08	1.0
1888.12	Greenwich and Kew . . .	67.52	67.36	+0.16	1.0

Here Δ = observed inclination − computed inclination, employing writer's formula,[1] viz.:—

$$I=70^\circ.40-3^\circ.98 \sin\,[0^\circ.7\,(t-1850)+23^\circ.02)]. \qquad (3)$$

It will be seen by glancing over column Δ that this formula gives a satisfactory representation of the observations. The weight w of each observation need be taken but very roughly. The author in establishing his D and I formulæ found that in both cases the best representation of the observations was obtained by employing the period 514 years, *i.e.* for $\beta=\frac{360^\circ}{514}=0^\circ.70$. This, however, must not be regarded as a proof that such a secular variation period really underlies the observations. The latter extend only a little over one-half this period, and furthermore, the early observations are usually defective to the extent of 1°–2°, so that at present a determination of the period, if there be such, cannot be undertaken. The writer's object was solely to build up *interpolation formulæ*, and the period giving the best agreement was hence adopted.

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[1] Science, Vol. 20, No. 506, p. 219.

With the aid of formulæ (1)[1] and (3), the tabular values given below were then computed. As the values for 1540 and 1560 are extrapolated ones, they have been marked questionable. The value of D for 1890 was obtained from the observations and not from formula (1). We now possess the elements for the construction of the secular variation curve.

As the needle assumes different positions for different epochs, it gradually sweeps out in space a cone, the vertex of which is at the center of suspension of the needle. Or, if a sphere be described about the center of suspension of the needle, having as radius the distance of the north end of needle from the center of suspension, then will the north end in the course of time describe some tortuous curve lying on the sphere. How shall we draw this curve? Undoubtedly the best method will be to project this curve *centrally* upon a plane tangent to the sphere at that point, corresponding to the intersection of the axis of the secular cone with the sphere. We shall thus get a good representation of the curve unfolding itself to an observer standing at the center of the needle, and looking, for instance, toward the *north* end. Or, in other words, the projected curves obtained thus are the curves of intersection of the secular cone with a plane perpendicular to the axis of the cone. Upon this principle the secular variation curves here given have been constructed. The half-length of the needle, or the distance of the perpendicular plane from the center, has been taken throughout as 20 cm. (7.88 inches).

A word must be said with reference to the point of tangency of the plane of projection. As the geometric nature of the secular variation curve is not yet known, the actual center, if there be such, of the curve cannot be ascertained. All that we can do at present is to determine from the material at hand a mean declination (D_0) and a mean inclination (I_0). The point on the sphere corresponding to these mean elements is taken as the point of tangency. These mean values are given by the constants in above periodic formulæ. Thus, for London, D_0 was taken as 7° West, and I_0 as 70°.4. Hence the line defined by these two angles is the direction to which the supposed plane of intersection with the cone is drawn perpendicular. These mean values need be known only approximately, for our sole purpose is to get a tolerably clear representation in a plane of the real curve described in space. It remains only to make brief mention of the system of coördinates employed, and then the steps necessary for the construction of the curves have been sketched. The projection formulæ cannot, of course, be developed here, but must be taken from the publication cited. On the sphere described as above, the declination of the needle as observed is measured along that small circle parallel to the plane of the horizon, which corresponds to the observed inclination of the needle. The observed inclination is measured along a great circle of the sphere perpendicular to the plane of the horizon, or along a meridian. This will be made clearer if we regard declination as corresponding to longitude, and inclination to latitude. The longitude is measured along a parallel of latitude, and the degrees of longitude are different for every latitude, while the latitude is measured along a meridian, but the degrees of latitude are everywhere the same. Just so it is with degrees of declination and those of inclination, — the latter have always the same absolute value, the former, however, are different for every inclination assumed by the needle.

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[1] Adopted in preference to (2) as giving the better agreement with observation.

This latter fact is continually being overlooked, even by geomagneticians. Changes in inclination are directly compared with changes in declination, without any attempt being made to reduce them to a common scale. Just so it is also in the discussion of the probable errors of observation of both elements. Before they can be compared, they must be reduced to a common standard. To avoid confusion we will say in speaking of our supposed sphere, instead of parallel of latitude and meridian, circle of inclination and circle of declination respectively, the declination being measured along the first circle, and the inclination along the latter. The most natural system of coördinates, then, for the projected curves would be the projected circle of inclination and the projected circle of declination. For the secular variation changes that we have encountered thus far, it has been found that for the scale used the projected circles of inclination can be regarded as arcs of circles. The projected declination circles are, of course, straight lines. On the projection, then, the declination changes will be measured along the circular arcs, while the inclination variation will be measured along the radial lines. It has been found that in the northern hemisphere the convex sides of the arcs are turned upwards, while in the southern they are turned downwards. This is due to the following reason. In the northern magnetic hemisphere the north end of the needle points *below* the horizon, or we might say has a negative inclination; in the southern magnetic hemisphere the north end points *above* the horizon. Custom has decreed that inclination below the horizon be designated as positive. If now the inclination decreases in the northern magnetic hemisphere, this means that the north end of the needle is moving toward the plane of the horizon, or is *rising*, and hence the inclination numbers decrease upwards. If, on the contrary, the inclination decreases in the southern hemisphere, the *north* end of the needle is, to be sure, again approaching the plane of the horizon, but is now *falling*, and hence the inclination numbers now decrease downwards.

(*To be concluded.*)

MINOR CONTRIBUTIONS.

A New Method for Testing the Magnetic Properties of Iron.

By W. S. Franklin.

1. *Determination of B and H.*

A ROD, of sectional area q, of the iron to be tested, is bent into the form of a long narrow ∩, and suspended from the (left) pan of an ordinary balance. The legs of the ∩ are surrounded by longer stationary coils, each having n turns of wire per centimeter, these coils being connected as the two coils of a horseshoe magnet. The balance is tared, a weight equal to F dynes placed on the right pan, and the current in the coils is slowly increased (or decreased) until the balance is in equilibrium, when the current, i, is measured; then

$$H = 4\pi ni, \tag{1}$$

and

$$B = \frac{2\pi F}{qH} + H. \tag{2}$$

Proof. — Equation (1) is the well-known expression for the magnetic field in a long coil. Equation (2) may be established as follows. There are two poles, each of strength m, neglecting sign; these poles are pulled, both in the same direction, by the field H, the total force being $F = 2mH$; but $B = \frac{4\pi m}{q} + H$, whence (2) follows at once.

2. *Experimental Results.*

This method for B and H has been applied by Messrs. Lee Campbell and E. C. Dickinson.[1] The coils, $n = 16.3$, were $23\frac{1}{2}''$ long and $\frac{1}{2}''$ in diameter. Table I. gives the results of the observations. F is the observed force in dynes, and i the observed current in C.G.S. units.

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[1] Thesis 1894. MS. in Iowa Ag. College Library.

TABLE I.

First Cycle.				Second Cycle.				Third Cycle.			
F	*i*	*H*	*B*	*F*	*i*	*H*	*B*	*F*	*i*	*H*	*B*
1177	+0.224	45.8	19330	1226	+0.229	46.8	19680	1226	+0.229	47.0	19620
785	0.152	31.2	18930	785	0.150	30.8	19140	783	0.156	32.0	18400
490	0.094	19.3	19141	487	0.097	19.9	18392	490	0.100	20.4	18030
194	0.041	8.4	17270	196	0.042	8.5	17280	189	0.042	8.5	16640
147	−0.047	9.7	11430	167	−0.049	10.2	12330	168	−0.049	10.0	12630
444	0.097	19.9	16760	542	0.113	23.2	17540	510	0.108	22.1	17340
833	0.166	34.0	18460	788	0.158	32.5	18220	784	0.159	32.1	18060
1226	0.232	47.5	19425	1228	0.235	48.2	19170	1253	0.236	48.4	19460
784	0.152	31.1	18990	772	0.150	30.8	18820	785	0.150	30.6	19230
392	0.079	16.2	18180	372	0.097	19.8	14070	490	0.096	19.6	18750
196	0.042	8.6	17160	189	0.042	8.7	16370	196	0.044	8.9	16490
169	+0.049	10.1	12603	154	+0.048	9.8	11790	167	+0.049	9.9	12450
539	0.112	23.0	17630	488	0.102	21.1	17420	540	0.112	23.0	17630
838	0.164	33.5	18800	783	0.158	32.3	18210	787	0.161	33.0	17900
1226	0.231	47.3	19480	1226	0.205	48.2	19150	1246	0.235	48.1	19490

3. *Direct Determination of Hysteresis Loss.*

The iron to be tested is taken in the form of a long straight rod, and suspended vertically from the pan of a balance. A coil of wire of medium length, of n turns per centimeter, surrounds the rod, and is arranged so that it can be moved up and down conveniently. It being required to find the energy loss per cubic centimeter of iron during a cycle in which the magnetizing field changes from $+H$ to $-H$, and back to $+H$ again, a measured current i is passed through the coil. The coil is moved up and down over the whole rod a number of times, the current being reversed each time the coil reaches top or bottom, until the changes in the rod have become cyclic. The rod is then weighed during the upward motion of the coil, and during the downward motion of the coil. Let F be the difference of these two weights in dynes; then

$$W = \frac{F}{q}, \tag{3}$$

in which q is the sectional area of the rod, and W is the energy per cubic centimeter lost in the iron in one cycle. The magnetizing field is calculated from equation (1).

4. *Proof of Equation* (3).

Consider a very long rod, of sectional area q, which has been magnetized by a field $+H$, this field being allowed to drop to zero. Start a magnetizing coil, giving a field $-H$ in its interior, moving slowly along the rod. Those portions of the rod well ahead of the moving coil will be in the condition in which they were left by the field $+H$; as these portions pass into the coil, they will be gradually brought under the influence of the field $-H$, and as they pass out of the coil, this field will gradually drop to zero. The iron has thus been carried over the left-hand portion of Ewing's curve for B and H, and an amount of work $\frac{W}{2}$ has been spent in each cubic centimeter of the iron, W being the energy per cubic centimeter, represented by the entire area of Ewing's curve. It remains to determine the source of the work thus spent. Referred to the moving coil, the magnetic state of the rod is stationary, and the induction through the coil is constant, therefore there can be no counter E.M.F's in the coil, and the only electrical energy spent in the coil is that due to its electrical resistance; this is of course not true when a stationary core is being magnetized by a current in a coil. The work spent in the iron must then be *done* in moving the coil. Let $\frac{F}{2}$ be the force required to drag the coil along the rod, F being the same as in equation (3); then $\frac{F}{2}l$ is the work *done* in dragging the coil through the distance l, during which movement ql cubic centimeters of iron will have been carried through half a cycle; so that $\frac{W}{2}ql$ of work has been *spent*, which is, of course, equal to $\frac{F}{2}l$, whence $W = \frac{F}{q}$. Q.E.D.

5. *Experimental Results.*

This method for determining hysteresis loss has been applied by Messrs. Campbell and Dickinson (*loc. cit.*) to annealed iron wire. The rod consisted of three strands of No. 18 B. & S. iron wire, $q = 0.0247$. The movable coil was 1″ external diameter and 4″ long, having 93 turns of wire per centimeter. The results are given in Table II.

TABLE II.

i	H	F	W
±0.156	±182	757	30600
0.105	123	747	30300
0.049	59	653	26700

The inaccuracy of equation (1) for so short a coil is not considerable. The sensitiveness of the balance would have permitted the accurate measurement of energy loss for very much smaller ranges of H.

AMES, IOWA, January, 1895.

NOTE ON A PHENOMENON IN THE DIFFRACTION OF SOUND.

BY W. S. FRANKLIN.

CONSIDER a train of sound waves, of wave length λ, passing an obstacle of width x. The intensity of the sound at a point p, at a distance P behind the obstacle, is very small when x is large compared with $\sqrt{P\lambda}$.

If a train of sound waves be reflected from a grating formed of parallel bars each of width x, and at distances y in the clear, distinct beats will be heard as the ear moves in a direction perpendicular to the bars and parallel to the grating, at a distance P therefrom, providing neither x nor y is small compared with $\sqrt{P\lambda}$. The same result will be produced if the ear is stationary, and the grating moved.

Let one hold the ear near to one side of a revolving spoked wheel, and produce a sound of high pitch. This sound, after reflection from the spokes of the wheel, will be heard to beat with great distinctness. If the ear is moved farther from the wheel, or if a sound of low pitch be produced, the beats will become scarcely audible.

Ames, Iowa, January, 1895.

NEW BOOKS.

The Theory of Sound. (In two volumes.) Vol. I. By JOHN WILLIAM STRUTT (BARON RAYLEIGH), Sc.D., F.R.S. Second edition, revised and enlarged. 8vo, pp. xiv+480. Macmillan & Co., 1894.

The interest of this classical work, as its readers well know, is not solely due to its exhaustive treatment of the phenomena of Acoustics. The principles which it establishes for the general vibrating system with n degrees of freedom (in which n may be finite or infinite) have applications in every branch of Physics, and furnish illustrations of many beautiful (if sometimes difficult) mathematical and dynamical theories.

A large part of the first edition, published in 1877, consisted of articles that had been previously contributed by Lord Rayleigh to different periodicals. The first volume of the second edition, which is the subject of the present notice, appeared last fall, and contains an account of the more recent contributions of the author, and others, with complete references in all cases to the original sources for fuller information.

The introductory chapter stands as in the first edition. Chapter II., on Harmonic Motions, contains a new article (32*a*) on beats arising from approximate consonance, instead of approximate unison; and another (42*a*) on the resultant of a large number (n) of vibrations of equal periods (τ), and equal amplitudes (a), with phases accidentally distributed over the entire period; giving an expression for the probability of a resultant amplitude between r and $r+dr$, and hence of an amplitude between any assigned values. That the former Senior Wrangler has still a warm side for purely mathematical speculations, is shown, as in many other instances, by his giving the corresponding problem in space of three dimensions, although (as he states) it has no bearing on the theory of vibrations.

In Chapter III., on Vibrations with One Degree of Freedom,[1] there is inserted in Article (59), a statement of the observations of McLeod and Clarke, on the dependence of pitch on temperature, and another on the relative importance of slow and quick beats; while to Article (63) is added a description of the Phonic Wheel, invented independently by M.

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[1] It would seem desirable to have a single adjective to express the number of independent variables involved in a problem. Perhaps "univariant," etc., would serve; *e.g.* a univariant system, a bivariant vibration, a trivariant function.

La Cour and Lord Rayleigh, for regulating the speed of a revolving body by means of an intermittent electric current. A new article (65*a*) shows how the theory of intermittent vibrations may be illustrated by electrically driven forks, and gives the most general expression for a vibration of frequency n, whose amplitude and phase are slowly variable with frequency m. In connection with this the author shows to be inadmissible the assumption sometimes made in optical speculations, that a train of simple waves may begin at a given epoch, continue for a certain time, and ultimately cease. Article (66*a*) gives a more general treatment to the problem of free vibrations, in which the coefficient of damping is not regarded as small, while Articles (68*a*) and (68*b*) treat of maintained vibrations, the latter article giving a brief account of the problem of Maintaining Vibrations with Forces of Double Frequency, more fully treated in a paper by Lord Rayleigh, to which reference is made. This paper is the more interesting, in that it contains an application of the new method of analysis introduced by Dr. G. W. Hill, of Washington, in connection with a problem in the Lunar Theory;[1] and physicists will be gratified to find that Lord Rayleigh has so soon turned the "new instrument" to account in the theory of vibrations. Article (68*c*) treats of the determination of absolute pitch by the methods of Koenig, McLeod and Clarke, Professor Mayer, and of the author himself, while Article (68*d*) gives a method that requires only a common harmonium and a watch.

In Chapters IV., V., on the General Vibrating System, Article (92*a*) examines the effect of introducing constraints, defined by linear equations among the n normal coördinates, and gives a general proof of Routh's "separation theorem," that a system subject to one constraint has its $n-1$ periods *separating* the n periods of the original system; while Articles (103*a*, *b*) give Routh's theorems on the "harmonic determinantal equation" for the natural periods. Article (111*a*) cites references to Helmholtz, Betti, and Lamb, for special applications of the "Principle of Reciprocity," first established in its generality by Lord Rayleigh; and Article (111*b*) investigates an expression for the "reaction at a driving point," giving, as a special case, Maxwell's formula for the reaction upon the primary circuit due to the electric currents generated in a neighboring secondary circuit.

In Chapter VI., on Transverse Vibrations of Strings, an addition is made to Article (140), showing how to determine a variation of density that shall produce assigned displacements in the respective natural periods of the string, while at the end of Article (142), the problem of varying density is worked out in the case of the law of the inverse square. Article (142*a*) shows how to trace the motion of the nodes when a forced period varies continuously. In Article (148*a*) the author investigates the reflection at the junction of two strings, or at a point where the density is discontinuous, and treats in (148*b*) of the reflection due to a *gradual* change of density, again working out the result for the law of the inverse square, while he shows in (148*c*) that considerations of stiffness introduce results analogous to optical dispersion.

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[1] *Acta Mathematica*, 1886. Speaking of this new method at the Mathematical Congress of 1893, Professor Felix Klein remarked that Dr. Hill had made an advance on the theory of the linear differential equation, by means of an instrument new to analysis,—the admissibility of which has been confirmed by subsequent writers,—the infinitely extended, but still convergent determinant.

Chapter VII., on Longitudinal and Torsional Vibrations of Bars, contains a new article (156*a*) on the reflection at a junction, illustrating the difficulty in obtaining transmission of sound from air to metal, or metal to air, in the mechanical telephone.

In Chapter VIII., on the Lateral Vibrations of Bars, Article (171) gives new numerical formulas for the pitch of tuning forks, and (192*a*) treats of the vibrations of a circular ring.

Chapter IX., on the Vibrations of Membranes, contains an addition to Article (204), on the motion of a circular drumhead under a periodic force applied at the center, a problem that involves a Besselian function of the second kind; and a new article (213*a*) on the nodal curves of forced vibrations.

Chapter X., on the Vibrations of Plane Isotropic Plates, has a new article (221*a*) on the vibration of a telephone plate, *i.e.* a thin circular plate clamped at the boundary, a problem that involves the Besselian function with pure imaginary argument. An addition to Article (233) gives Fenkner's observations on the tones of thin metallic cylinders open at one end; while Article (235*a*) gives the results of experiments on bells of glass, and on church bells.

A new chapter (X. A) deals with the vibrations of very thin curved plates or shells. The results are worked out for the general vibrations of an infinitely long cylindrical shell (with a reference to Love's *Elasticity* in the case of finite length); also for the flexural (inextensional) vibrations of a finite cylindrical shell; the radial vibrations of a spherical shell (with a reference to Lamb for the more general extensional vibrations treated by Spherical Harmonics); and finally for the inextensional vibrations of the spherical shell.

The next following chapter, which is also new, contains a discussion of Electrical Vibrations. The introduction of this class of problems in a treatise on Sound is explained by the author as being made necessary by the recent advances in the development of the telephone. Apart from their bearing upon this mode of sound transmission, however, the electrical problems appear equally important as illustrating the generality of the discussion of vibrating systems in Chapter IV. The first case treated is that of the free vibrations of a circuit containing inductance and capacity. Two mutually inductive circuits are next considered, first for the case of the instantaneous establishment of current in one circuit, and afterwards on the assumption of an harmonic impressed electromotive force. In this connection the mutual reactions of the induced currents in a series of independent circuits are considered. As an example of electrical vibrations with two degrees of freedom, the case of two conductors in parallel is treated, and the interesting fact that the (alternating) currents in both branches of a divided circuit may exceed the current in the mains is explained. Article (235*o*) deals with the Hughes Induction Balance, while the next four articles discuss the theory of the Wheatstone Bridge, when used with interrupted or oscillatory currents. Valuable hints will be found here in regard to the action of automatic interrupters. The problem of the induction of currents in conducting cores is briefly discussed in Article (235 *u*). Here, as well as in the case of the following article on the distribution of oscillatory currents over the cross-section of a wire, the reader will regret that the problem has not been treated more in detail. The remaining ten pages are devoted to the problem of current propagation in a cable, when the impressed electromotive forces are harmonic, and to the general theory of the telephone. Among the interesting numerical data, in which these pages abound, may be mentioned the experimentally determined values of the minimum current which will produce an audible sound in a receiver.

Possibly to allay the feeling that may arise in some minds that a sufficiently difficult treatise has become more difficult by the revision, the author has kindly inserted in his preface the statement that "the more difficult parts of the subject will be passed over by the reader on a first perusal." As this sentence seems to be mandatory in its form, Lord Rayleigh would no doubt grant a properly presented request from his readers, to insert in the second volume a preface marking out what seems to him a suitable course for a first reading of the entire work. This would be a special boon to those who have so many other claims on their attention that the first reading must also be their final one.

JAMES MCMAHON.

Les Oscillations Électriques. By H. POINCARÉ, Membre de l'Institut de France. 8vo, pp. 337. Paris, Georges Carré, 1894.

M. Poincaré, the eminent professor of mathematical physics at the Faculty of Science of Paris, has for the past six years been publishing his courses of lectures, and the present volume forms the sixth of the series.

It is a very valuable work, for it discusses the electromagnetic theory in a lucid, original, and penetrating manner.

Although the quantities treated of are almost all vectors, M. Poincaré does not use a vector notation; he mitigates the prolixity and unwieldiness of the coördinate notation by various artifices, such as the use of Σ, writing out only one of a set of three equations, using letters as abbreviations for expressions, and introducing arbitrary symbols such as Δ both for a determinant and for $\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}$. By the coördinate usage, three independent symbols are usually assigned to each vector, as, for instance, X, Y, Z to the electric force, in consequence of which the memory is burdened and the available letters are all soon taken up. In the volume before us, ξ, η, ζ are assigned at different places to two different vectors. Mere economy of notation requires a reform; but more than that, there is need of a logical notation which shall show clearly the analytical relations of the several vectors, in order that the attention may be concentrated upon the physical relations.

In the electromagnetic theory we have four fundamental vectors, —

the electric force,	$Xi+Yj+Zk,$	(1)
the magnetic force,	$Li+Mj+Nk,$	(2)
electric induction,	$\epsilon(Xi+Yj+Zk),$	(3)
magnetic induction,	$\mu(Li+Mj+Nk).$	(4)

From these respectively four other vectors are derived by space differentiation, namely, —

$$\left(\frac{dZ}{dy}-\frac{dY}{dz}\right)i+\left(\frac{dX}{dz}-\frac{dZ}{dx}\right)j+\left(\frac{dY}{dx}-\frac{dX}{dy}\right)k, \tag{5}$$

$$\left(\frac{dN}{dy}-\frac{dM}{dz}\right)i+\left(\frac{Ld}{dz}-\frac{dN}{dx}\right)j+\left(\frac{dM}{dx}-\frac{dL}{dy}\right)k, \tag{6}$$

$$\left(\frac{d\cdot eZ}{dy}-\frac{d\cdot eY}{dz}\right)i+\left(\frac{d\cdot eX}{dz}-\frac{d\cdot eZ}{dx}\right)j+\left(\frac{d\cdot eY}{dx}-\frac{d\cdot eX}{dy}\right)k, \tag{7}$$

$$\left(\frac{d\cdot \mu N}{dy}-\frac{d\cdot \mu M}{dz}\right)i+\left(\frac{d\cdot \mu L}{dz}-\frac{d\cdot \mu N}{dx}\right)j+\left(\frac{d\cdot \mu M}{dx}-\frac{d\cdot \mu L}{dy}\right)k. \tag{8}$$

Another vector, Poynting's radiant vector, is obtained by deriving the vector which is perpendiclar to (1) and (2), namely, —

$$(YN-ZM)i+(ZL-XN)j+(XM-YL)k. \tag{9}$$

There are two other vectors, —

conduction current, $ui + vj + wk,$ (10)
and, potential vector, $\xi i + \eta j + \zeta k.$ (11)

The former is defined by

$$ui + vj + wk = \lambda \{(X - X')i + (Y - Y')j + (Z - Z')k\},$$

where $X'i + Y'j + Z'k$ denotes the electromotive force of contact, and λ the conductivity.

The latter is such that

$$Li + Mj + Nk = \left(\frac{d\zeta}{dy} - \frac{dy}{dz}\right)i + \left(\frac{d\xi}{dz} - \frac{d\zeta}{dx}\right)j + \left(\frac{dy}{dx} - \frac{d\xi}{dy}\right)k.$$

The velocity of a point of the circuit is also denoted by

$$\xi i + \eta j + \zeta k, \tag{12}$$

and from (3) and (12) we obtain the perpendicular vector,

$$e\{(Y\zeta - Z\eta)i + (Z\xi - X\zeta)j + (X\eta - Y\xi)k\}, \tag{13}$$

and, similarly, from (4) and (12),

$$\mu\{(M\zeta - N\eta)i + (N\xi - L\zeta)j + (L\eta - M\xi)k\}. \tag{14}$$

By space differentiation of (13) another vector is obtained, which we shall denote by (15), and by space differentiation of (14) a vector which we shall denote by (16).

In the first chapter M. Poincaré investigates, after the manner of Hertz and Heaviside, the fundamental equations of electrodynamics for bodies which are not in motion. He rests the first equation upon the experimentally established law of electromagnetic induction; namely, that the electromotive force in a closed circuit is equal to the rate of change of the magnetic induction through a surface bounded by the circuit. This is expressed by

$$A\mu \frac{d}{dt}(2) = (5).$$

Here the quantity A is introduced in order to make the two sides have the same dimensions. By means of the above equation and the principle of the conservation of energy, he deduces the other equation; namely, —

$$Ae\frac{d}{dt}(1) = (6) + 4\pi A(10).$$

In the second chapter, M. Poincaré takes up the discussion of the Herzian waves. He starts from Sir William Thomson's investigation of the discharge of a Leyden jar, and describes the experimental methods which were devised to observe the oscillatory discharges. He then considers the apparatus by which Hertz produced much shorter waves, capable of being measured indirectly. He also discusses the apparatus of M. Blondlot; and shows how to calculate the period and damping of the waves.

In the third chapter we have a theoretical study of the Herzian waves, M. Poincaré finds the integral of the differential equations for the case of an exciter placed in a vacuum or air of indefinite extent, and works out fully the special case of a spherical exciter, such as Lodge's, when placed in a uniform electric field, and the field suddenly released. In finding the solution of the differential equations, M. Poincaré employs an artifice, which is said to be frequently useful, namely, "If we find an imaginary solution having no meaning in itself, by retaining the real part, we obtain a solution answering to the physical reality." How can this be possible, unless the artifice has not only a meaning, but a meaning which corresponds to physical reality? An explanation of this paradox will be found in my paper on the *Analytical Treatment of Alternating Currents.*

In the fourth chapter M. Poincaré describes the experiments which have been made on electrical resonance, and on the propagation of waves along a wire, and examines the results in the light of the theory. In the fifth chapter he discusses, in a similar manner, the experiments which have been made on the propagation of electric waves in air. In the sixth chapter he applies the general equations to form a complete theory of the phenomena described in the two preceding chapters. The seventh chapter is devoted to the propagation of waves in dielectrics, other than air; he describes the different methods of measuring the specific inductive capacity, and compares the results with the theoretical relation, $e = n^2$, required by Maxwell's theory.

In the last chapter M. Poincaré establishes, after Hertz, the general electrodynamical equations for the more general case of bodies in motion. The former of the pair of general equations becomes, —

$$A\frac{d}{dt}(4) + A(16) + 4\pi Am(12) = (5),$$

where $4\pi m$ denotes the cosine product, —

$$\frac{d\cdot\mu L}{dx} + \frac{d\cdot\mu M}{dy} + \frac{d\cdot\mu N}{dz}.$$

The latter of the pair of general equations becomes, —

$$A\frac{d}{dt}(3)+A(15)+4\,\pi Ae(12)=(6)-4\,\pi A(10),$$

where

$$4\,\pi e=\frac{d\cdot eX}{dx}+\frac{d\cdot eY}{dy}+\frac{d\cdot eZ}{dz}.$$

The third term of the first equation does not appear in Maxwell's equation, and it is doubted whether it can be different from zero. The existence of the first term of the second equation was established by Hertz's experiments, that of the second by Rowland's experiments, and that of the third by Roentgen's experiments.

It is to be regretted that in a work of so great importance the formulæ are not more free from typographical errors; and it is also true that the diagrams could be improved. Notwithstanding these defects the work is a splendid addition to the literature of mathematical physics, and is one of the best contributions to science of a very prolific author.

Alexander Macfarlane.

University Physics. By H. S. Carhart. Part I., pp. 344. Boston, Allyn & Bacon, 1894.

The volume here reviewed is the first part of a text-book on physics for university students. It treats of Mechanics, Sound, and Light; the second part, not yet published, will be devoted to Electricity and Magnetism.

Part I. contains about 350 pages, and is "an extension of one written several years since for the sole use of the author's classes." Professor Carhart follows the usual custom of combining text-book work with lectures as a first course in physics. The book is apparently an outgrowth of a course of lectures probably greatly expanded on its mathematical side. In the preface the author says: "It leaves room for the personal equation in instruction; but it will relieve the student of a large part of the labor of taking notes, and, it is hoped, will secure for him more accurate statements than he would be likely to obtain from listening to lectures without the aid afforded by a text-book of principles."

The subjects of Sound and Light are treated quite satisfactorily in a manner but little differing from that of most books of this class. One excellent feature not to be found in other such text-books is frequent specific references to a number of standard text-books, such as "Daniell's Physics" and "Preston's Light."

The subject of sound is treated in two chapters on "Nature and Motion of Sound," "Physical Theory of Music." Rather more attention is given to the latter subject than is usual. In the case of sound waves the character of motion of the particles is very early discussed. This is not true in the case of light waves. The direction of vibration of the particles is first mentioned, under the head of polarization, in the last chapter. The first two chapters on light treat the subject largely from the standpoint of geometrical optics. In these chapters nothing is said about different kinds of light; differences in wave length, period, or direction of vibration are not referred to. In the following chapter on dispersion the student learns that "a thin beam of sunlight passed through a prism, . . . is resolved into a number of colors of different refrangibility. . . . The color is the physiological character of light, the refrangibility is its physical character." This is doubtless Newton's way of looking at the matter, but many teachers would prefer explaining differences in wave length, period, and perhaps direction of vibration, when light is first treated as wave motion.

Mechanics is treated in chapters on Kinematics, Kinetics, and Mechanics of Fluids. The discussion of simple harmonic motion is much more complete than is usual in such text-books. On the other hand, several important subjects have been entirely omitted. The author seeks to disarm criticism by saying that "no attempt has been made to secure completeness." Nevertheless, it would seem that this lack of completeness is a serious fault in a text-book termed "University Physics."

The law of gravitation is only barely mentioned under the head of capillarity. The relation between mass and weight, so difficult to beginners, is nowhere discussed. The equation $W = Mg$ may be logically included in the more general equation $F = Ma$, but all beginners need to have this troublesome relation very fully and carefully explained.

Stress and Strain are not treated by themselves. The student is told, under the head of fluids, that "a perfect fluid would offer no resistance to a shearing stress"; but no definition or explanation of a shear or a shearing stress is given. Similarly the terms "rigidity" and "elasticity" are nowhere defined, much less adequately treated, although they are used in discussing the mechanics of fluids, and in the subject of Sound, as though the student had studied them before.

In this part of the book the statement of definitions and principles is not always as concise and accurate as it should be. For example, the student is led to believe that *work* consists in the overcoming of a *resisting force;* later, the student will not be able to see that work is done when a force simply changes the kinetic energy of a body, since it is always difficult for a beginner to obtain a clear appreciation of the existence and magnitude of the resisting force of inertia. Again, after defining the dyne satisfactorily, the statement is made that "it produces unit acceleration of unit mass *in* *unit time.*"[1] In several places, principally in problems, an acceleration is spoken of as so many meters or centimeters per second, ignoring the fact that it depends on the square of a time. This is a mistake that students are very prone to make.

In the first chapter, a distinction is made between a "theoretical" and a "practical" unit; the "theoretical" unit being what the unit was meant to be, *e.g.* the meter being "theoretically" the ten-millionth of the earth's quadrant: a rather unfortunate use of that much-abused term "theoretical." On the same page occurs the following amusing "slip of the pen": "*Prototypes* of the meter and kilogram made by an International Commission are preserved in the Bureau of Weights and Measures at Washington."

The book assumes some knowledge of analytic geometry, but none of calculus, although the methods of the latter are occasionally made use of.

Scattered throughout the book are a number of problems and illustrative examples. The student would find these more useful if a few of them were solved, and a portion, at least, had answers attached.

F. J. ROGERS.

Collected Papers on some Controverted Questions of Geology. By JOSEPH PRESTWICK, F.R.S. 8vo, pp. x + 279. Macmillan & Co., 1895. (*Received.*)

Memoir of Sir Andrew Crombie Ramsay. By Sir ARCHIBALD GEIKIE, F.R.S. 8vo., pp. viii + 397. Macmillan & Co., 1895. (*Received.*)

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[1] The italics are the reviewer's.

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